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Jochen Kühn

Optimal Risk-Return Trade-Offs of Commercial Banks

and the Suitability of Profitability Measures for Loan Portfolios

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With 35 Figures
and 1 Table

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Zurich, June 2006 Jochen Kühn

Contents

1

Introduction

1.1 Problem Statement and Research Question

Active loan portfolio management is becoming more and more important. In the year 2004, European banks sold credits worth EUR 249 billion. Big deals were made by the German banks Hypo Real Estate (EUR 3.6 billion) and Dresdner Bank (EUR 1.2 billion). In addition, credit exchanges were established which made loans more liquid. For example, in October 2004 the German "Deutsche Kredit-Börse" was established, which focuses on trading loans assigned to medium-size businesses.

It is empirically shown that active loan portfolio management can be very profitable.[1] However, a precondition to benefit from active loan portfolio management is having knowledge about valuating loan portfolios. Shareholders can steadily benefit from such transactions only if banks valuate loan portfolios correctly. This is this dissertation's motivation for dealing with profitability measures for loan portfolios.

Nowadays, banks measure the profitability of loan portfolios primarily by calculating the return on risk adjusted capital (RORAC). Here return is the expected profit after refinancing and operational costs. Risk adjusted capital, more frequently called economic capital, is the amount of equity which must be held to guarantee a certain given solvency level of the bank.

However, calculating this ratio is not sufficient when valuating loan portfolios. The calculation of economic capital implies that the bank

[1] See Cebenoyan and Strahan (2004).

already knows which solvency level is optimal. It also presumes that the optimal solvency level is independent of the risk-return profile of the loan portfolio. But this need not be true.

Think about a bank without operational costs that can decide between two risk-return profiles of its loan portfolio (see Table 1.1). The first profile A is characterized by an expected return of 6% and low risk, the second profile B has an expected return of 6.2% and a comparatively higher risk. The bank has equity of 100 and needs to fulfill the condition of requiring less economic capital than the equity it has. The calculation of economic capital is based on a certain given solvency level. Given the risk-return profile of its loan portfolio, the bank can adjust the required economic capital by changing the loan portfolio volume.

First, assume that the bank has a high given solvency level leading to an interest rate on debt of 5%. With the less risky profile A, the bank can hold a portfolio with a volume of 1500. Having the riskier profile B, the bank can only hold a portfolio with a volume of 1200 due to the economic capital restriction. Choosing profile A, this leads to an expected profit after refinancing costs of $6.00\% \cdot 1500 - 5.00\% \cdot (1500 - 100) = 20$ and to a RORAC of 20%. Choosing profile B, an expected profit after refinancing costs of $6.20\% \cdot 1200 - 5.00\% \cdot (1200 - 100) = 19.4$ and a RORAC of 19.4% result. It could be concluded that profile A is preferable.

But now assume that it is actually optimal for the bank to have a lower solvency level with profile B, while at the same time the given solvency level is optimal for profile A. Furthermore, assume that the bank with the lower solvency level has to pay a higher interest rate on debt of 5.1%, but it can also have a higher portfolio volume of 1450, choosing profile B. The expected profit after refinancing costs thus amounts $6.20\% \cdot 1450 - 5.10\% \cdot (1450 - 100) = 21.05$ and the RORAC is 21.05% when profile B and the lower solvency level are chosen. So it is optimal for the bank to decide in favor of profile B.

This example shows that valuating loan portfolios based on RORAC with a given solvency level can be misleading for the optimal solvency level depending on the risk-return profile of the loan portfolio. So it would be a significant improvement to have a sound profitability measure that valuates loan portfolios directly on the basis of its risk-return profile.

Table 1.1. RORAC with a given solvency level

profile	expected return	risk	solvency	refinancing	volume	RORAC
A	6.00%	low	high	5.00%	1500	20.00%
B	6.20%	high	high	5.00%	1200	19.40%
B	6.20%	high	low	5.10%	1450	21.05%

In the literature, two profitability measures are explicitly proposed for optimizing loan portfolios based on its risk-return profile: the Sharpe ratio, which relates the expected excess return over the risk-free rate to the standard deviation of the portfolio return, and the reward-to-VaR ratio, which relates the expected excess return over the risk-free rate to a certain quantile of the excess return.[2] Furthermore, in the context of asymmetric returns, which are typical for banks, reward-to-shortfall ratios are popular. Reward-to-shortfall ratios relate the expected excess return over the risk-free rate to lower partial moments or root lower partial moments of the return.

However, the above reward-to-risk ratios are founded on capital market models, assuming that banks should optimize their loan portfolios the same way as individual capital market investors do. But this need not hold true. Rather, banks should optimize their loan portfolio, targeting at the maximization of the shareholder value. Here banks do not only need to consider the market risk premium, but also additional costs that risk-taking provoke. Thus, it is questionable whether profitability measures, derived from capital market models, reflect optimal risk-return trade-offs of banks.

The dissertation addresses this problem. Its research question is whether reward-to-risk ratios derived from capital market models are suitable for loan portfolios. The approach of the dissertation is to endogenously derive optimal risk-return trade-offs of commercial banks and to compare them to the risk-return trade-offs of the reward-to-risk ratios derived from capital market models. This gives measures such as the Sharpe ratio and the reward-to-VaR ratio a more adequate foundation for valuating loan portfolios.

[2] See Altman and Saunders (1998), pp. 1728-1740, Campbell, Huisman, and Koedijk (2001), and Alexander and Baptista (2003).

1.2 Outline of the Dissertation

The dissertation is divided into nine chapters (see Figure 1.1):

After the introduction, the following two chapters present fundamentals for the derivation of reward-to-risk ratios in the fourth chapter. The second chapter, *Risk Measures*, gives a survey of risk measures quantifying risk in the reward-to-risk ratios. The third chapter, *Asset Pricing*, derives the stochastic discount factor model and provides fundamentals for understanding capital market models, upon which the reward-to-risk ratios are based. Furthermore, the presumptions are pointed out as being necessary for a firm to maximize the market price of equity as a substitute for the shareholders' individual valuations of equity. This justifies the use of the market price of equity as the target figure for a bank.

In the fourth chapter, *Reward-to-Risk Ratios*, several reward-to-risk ratios measuring the profitability of portfolios are derived from capital market models. The starting point is the derivation of the Sharpe ratio from the stochastic discount factor model discussed in the previous chapter.

The next three chapters focus on the derivation of optimal risk-return trade-offs of commercial banks. The derived optimal risk-return trade-offs are based on the effects of risk-taking on shareholder value. They are discussed in the fifth chapter, *Effects of Risk-Taking in Commercial Banks*. The sixth chapter, *Risk-Return Trade-Offs for Commercial Banks*, is the central part of the dissertation. It develops models for endogenously deriving optimal risk-return trade-offs of commercial banks. The models assume that banks only have uninsured debtholders. The seventh chapter, *Deposits and the Risk-Return Trade-Off*, extends the models by taking into account the fact that banks are usually also financed through deposits, which are insured and have a senior credit ranking in most countries.

In the eighth chapter, *Profitability Measures for Loan Portfolios*, the endogenously derived optimal risk-return trade-offs of commercial banks are compared to the risk-return trade-offs of profitability measures derived from capital market models in order to assess their suitability for loan portfolios.

Finally, the ninth chapter, *Conclusion*, provides a summary and points out the implications and limitations of the dissertation.

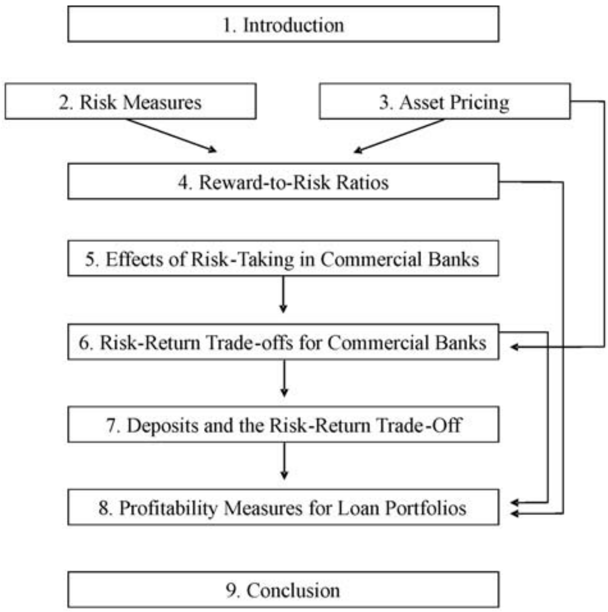

Figure 1.1. Outline of the dissertation.

2

Risk Measures

This chapter discusses risk measures upon which the assessed reward-to-risk ratios are based. It starts with a short presentation of the two most common definitions of risk.

2.1 Defining Risk

Risk is defined in at least two ways, each having a different focus.

The best known definition of risk stems from Frank Knight distinguishing between measurable and unmeasurable uncertainty and defining risk as the former one.[1] This definition emphasizes that risk is closely related to uncertainty and tasks dealing with risk usually require uncertainty to be quantified. Using this definition, uncertainty can be risk, even if it cannot cause harm.

A more intuitive understanding of risk leads to defining risk as a hazard that emanates from uncertainty and is caused by harmful deviations from expectations.[2] According to this definition, risk incorporates two basic elements: uncertainty and harm that can arise, although it is not expected.[3] Using this definition, something might be risky, although the uncertainty is not measurable.

Both definitions have advantages. The first definition is useful, e.g. for describing model risk since it helps to distinguish between uncertainty that is captured by the model and residual uncertainty. The

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[1] See Knight (1921), p. 233.

[2] See Crowe and Horn (1967) and Athearn (1971).

[3] See Holton (2004).

second definition is particularly useful when it is important to distinguish between deviations from expectations leading to harm and those leading to benefits. Since this is important when dealing with risk-return trade-offs, the following discussion of risk measures is based on the second definition.

2.2 Variance and Standard Deviation

The classical risk measure is the variance Var and the square root of it called standard deviation σ. Given a density function f of a continuous square-integrable random variable r, they are defined as

$$Var(r) = \sigma^2(r) = \int_{-\infty}^{\infty} (r - \mathbb{E}(r))^2 f(r) dr \tag{2.1}$$

and

$$\sigma(r) = (\int_{-\infty}^{\infty} (r - \mathbb{E}(r))^2 f(r) dr)^{\frac{1}{2}}. \tag{2.2}$$

Here $\mathbb{E}$ denotes the expectation operator.

When r is the return of a portfolio, the variance of the return is the expected value of the squared deviation of the return from the expected return. It is well known for being used in the seminal work of Markowitz (1952). This work provides a quantitative framework for measuring portfolio risk and for deriving efficient frontiers, which characterize portfolios that maximize the expected return for a given variance of the return or minimize the variance of the return for a given expected return. The advantage of using the variance as a risk measure is that aggregating risk is quite simple using a covariance matrix. The main disadvantage of this mean-variance approach is that it presumes either normally distributed returns or a quadratic utility function of investors. This is discussed in the following.

The mean-variance approach assumes investors who invest an amount I in a portfolio with a payoff X based on the first two statistic moments of the portfolio return r, the expected return $\mathbb{E}(r)$ and the variance of the return $Var(r)$, with

$$r = \frac{X}{I} - 1. \tag{2.3}$$

This is plausible for any risk averse investor when there is no other income if the return distribution is completely specified by the mean and variance, since the variance is a sufficient risk measure in this case. A distribution fulfilling this property is the normal distribution.

Without this distribution assumption, the mean-variance approach leads to correct results if it is assured that investors themselves only care about the first two statistic moments. According to the axioms of von Neumann and Morgenstern (1944), this holds for investors with quadratic utility functions U of the form

$$U(X) = \alpha X^2 + X \text{ with } \alpha \in (-1, 0). \tag{2.4}$$

In this case, the expected utility is

$$\mathbb{E}(U(X)) = \int_{-\infty}^{\infty} U(X) f(X) dX = \alpha \int_{-\infty}^{\infty} X^2 f(X) dX + \int_{-\infty}^{\infty} X f(X) dX, \tag{2.5}$$

and with

$$Var(X) = \int_{-\infty}^{\infty} (X - \mathbb{E}(X))^2 f(X) dX = \int_{-\infty}^{\infty} X^2 f(X) dX - E^2(X), \tag{2.6}$$

the expected utility can be expressed as

$$\mathbb{E}(U(X)) = \alpha(Var(X) + \mathbb{E}(X)) + \mathbb{E}(X). \tag{2.7}$$

Since α is negative, the expected utility of investors increases in the expected payoff and decreases in the variance of the payoff. Thus, the expected utility increases in the expected return and decreases in the variance of the return, too.

However, the assumed utility function has the undesirable property that it implies an absolute risk aversion

$$ARA(X) = -\frac{U''(X)}{U'(X)} = -\frac{2\alpha}{2\alpha X + 1} \tag{2.8}$$

and a relative risk aversion of investors

$$RRA(X) = -X\frac{U''(X)}{U'(X)} = -\frac{2\alpha X}{2\alpha X + 1} \tag{2.9}$$

both increasing in income X.[4] This can be interpreted such that investors who can decide on the partitioning of their initial capital into a risk-free and a risky asset decrease both the absolute and the relative portion of the risky investment in wealth. However, experiments reject the hypothesis of an increasing relative risk aversion and support the hypothesis of a decreasing absolute risk aversion of investors.[5] So this utility function is considered to be implausible.

2.3 Early Downside Risk Measures

Markowitz (1959) had already recognized the limitations of the mean-variance approach. He suggested using downside risk measures rather than total volatility measures, since they focus exclusively on downside losses instead of considering both downside losses and upside gains, making them more intuitive. Measures based on the downside risk concept might be useful, in particular in cases of highly skewed distributions. This is because, in general, skewed distributions need more than the first two statistic moments to be completely specified.

His suggestions for measuring downside risk are the below-mean semivariance SV_m and the below-target semivariance SV_T, which are defined in the continuous case for the random variable r as

$$SV_m(r) = \theta^2_{\mathbb{E}(r)}(r) = \int_{-\infty}^{\mathbb{E}(r)} (r - \mathbb{E}(r))^2 f(r)dr \qquad (2.10)$$

and

$$SV_T(r) = \theta^2_T(r) = \int_{-\infty}^{T} (r - T)^2 f(r)dr \qquad (2.11)$$

with target T.

Analogously, the below-mean semi-standard deviation $SSTD_m$ and the below-target semi-standard deviation $SSTD_T$ are defined as

$$SSTD_m(r) = \theta_{\mathbb{E}(r)}(r) = (\int_{-\infty}^{\mathbb{E}(r)} (r - \mathbb{E}(r))^2 f(r)dr)^{\frac{1}{2}} \qquad (2.12)$$

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[4] See Pratt (1964) and Arrow (1970).

[5] See Levy (1994).

and

$$SSTD_T(r) = \theta_T(r) = (\int_{-\infty}^{T} (r-T)^2 f(r)dr)^{\frac{1}{2}}. \qquad (2.13)$$

Even earlier than Markowitz, Roy (1952) proposed a portfolio theory based on downside risk, namely the probability of falling below a target return T called shortfall probability SP. In the continuous case, the shortfall probability is

$$SP(r) = \int_{-\infty}^{T} f(r)dr. \qquad (2.14)$$

Roy's approach is that investors prefer the investment with the smallest shortfall probability. So given an expected return $\mathbb{E}(r)$, it is likely that it is optimal for investors to choose a portfolio with a low volatility. Roy derives that investors should choose the portfolio with the highest ratio of the expected excess return over the target level to the standard deviation of the return, which is $(\mathbb{E}(r)-T)/\sigma(r)$. However, in order for the maximization of this ratio to result in the exact solution to this problem, returns have to be normally distributed.[6] Roy's portfolio selection rule is also modified by first defining the shortfall probability and then choosing the portfolio with the highest target return.[7] A further portfolio optimization approach based on Roy's framework defines both the shortfall probability and the target return at first and chooses the portfolio with the highest expected return afterwards.[8]

2.4 Lower Partial Moment

In the 1970s, the two-parametric risk class lower partial moment, which generalized the downside risk measures, was defined.[9] The lower partial moment of order k with target T is defined for a random variable r as

[6] See Kaduff (1996), pp. 87-93.
[7] See Kataoka (1963).
[8] See Telser (1955).
[9] See Bawa (1975) and Fishburn (1977). For an even more general class of three-parametric risk measures containing the lower partial moment see Stone (1973).

$$LPM_{T,k}(r) = \int_{-\infty}^{T} (T - r)^k f(r) dr. \tag{2.15}$$

Besides the lower partial moment, the root lower partial moment is defined as

$$RLPM_{T,k}(r) = (\int_{-\infty}^{T} (T - r)^k f(r) dr)^{\frac{1}{k}}. \tag{2.16}$$

The class of lower partial moments contains the below-target semi-variance ($k = 2$) and the shortfall risk ($k = 0$). It can also express the expected return given that the return is smaller than the target T ($k = 1$).

The invention of the lower partial moment class contributed to the discussion about risk measures, since lower partial moments can be linked to utility functions that appear to be more plausible than the quadratic utility function implied by the mean-variance approach.

Fishburn (1977) showed that a decision rule based on expected returns and lower partial moments is congruent with the expected utility theory for any distribution if, and only if, the utility function has the form

$$U(r) = \begin{cases} r & \text{for } r \geq T \\ r - c(T - r)^k & \text{for } r < T \end{cases} \quad \text{with } c \in \mathbb{R}. \tag{2.17}$$

In this case, the expected utility is

$$\begin{aligned} \mathbb{E}(U(r)) &= \int_{-\infty}^{\infty} U(r) f(r) dr \\ &= \int_{-\infty}^{T} (r - c(T - r)^k) f(r) dr + \int_{T}^{\infty} r f(r) dr \\ &= \mathbb{E}(r) - c \int_{-\infty}^{T} (T - r)^k f(r) dr. \end{aligned} \tag{2.18}$$

For $c \in \mathbb{R}^+$ and $k > 1$, the utility function implies investors who are risk neutral above target T and risk averse below target T (see Figure 2.1). Here investors are increasingly more risk averse with lower k. It

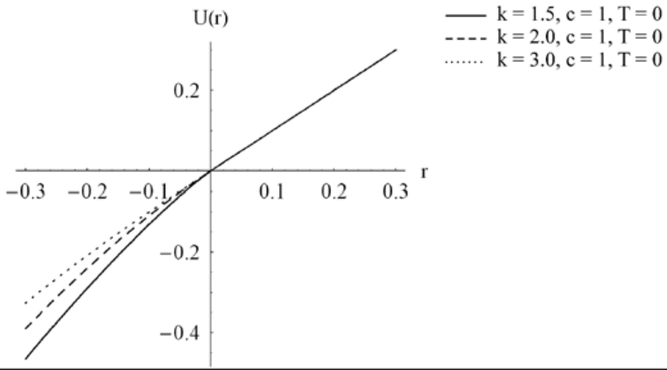

Figure 2.1. Utility functions consistent with the mean-LPM framework.

is thus possible to capture a wide range of relevant risk attitudes using a mean-lower partial moment framework.

Another interesting property of the lower partial moment class is that it can be linked to the very general concept of stochastic dominance.[10] The concept of stochastic dominance enables payoffs or returns to be put into an order based on their distributions, in such a way that the dominant payoffs or returns lead to a higher expected utility of investors. This can be done without concrete knowledge about the utility function of the investors.

Particularly relevant are the first order stochastic dominance, which presumes only an increasing utility function ($U' > 0$), the second order stochastic dominance, which also presumes a decreasing marginal utility ($U'' < 0$), and the third order stochastic dominance, which additionally presumes a positive third derivation of the utility function of investors ($U''' > 0$).[11]

In the following, the criteria for stochastic dominance of the first, second, and third order and their consequences for the expected utility of investors are presented for the return distributions F and G with domain $D \in [a, b)$. Here F FSD G denotes that F dominates G stochastically by the first order, F SSD G denotes that F dominates

[10] For a detailed overview about the development of the concept of stochastic dominance see Kaduff (1996), pp. 19-21.

[11] A positive third derivation of the utility function is a necessary condition for a decreasing absolute risk aversion of investors, since $ARA'(X) < 0 \Leftrightarrow U'''(X) > \frac{(U''(X))^2}{U'(X)} > 0$.

G stochastically by the second order, and F TSD G denotes that F dominates G stochastically by the third order:

- F FSD G if

$$F(r) \leq G(r) \; \forall \; r \in D \text{ and } \exists \; r^* \text{ with } F(r^*) < G(r^*). \tag{2.19}$$

  Under the assumption that $U' > 0$, it follows from F FSD G that $\mathbb{E}(U(r|F)) > \mathbb{E}(U(r|G))$.[12]

- F SSD G if

$$\int_a^r F(p)dp \leq \int_a^r G(p)dp \; \forall \; r \in D$$
$$\text{and } \exists \; r^* \text{ with } \int_a^{r^*} F(p)dp < \int_a^{r^*} G(p)dp, \tag{2.20}$$

  i.e., the area below F up to r must be smaller than or equal to the area below G up to r for all possible r, and there must be at least one r^* for which the described area below F is smaller than the one below G.
  Under the assumptions that $U' > 0$ and $U'' < 0$, it follows from F SSD G that $\mathbb{E}(U(r|F)) > \mathbb{E}(U(r|G))$.[13]

- F TSD G if

$$\int_a^r F_I(q)dq \leq \int_a^r G_I(q)dq \; \forall \; r \in D$$
$$\text{and } \exists \; r^* \text{ with } \int_a^{r^*} F_I(q)dq \leq \int_a^{r^*} G_I(q)dq \tag{2.21}$$

$$\text{with } F_I(r) = \int_a^r F(p)dp \text{ and } G_I(r) = \int_a^r G(p)dp,$$

[12] See Quirk and Saposnik (1962), Fishburn (1964).
[13] See Hadar and Russel (1969), Hanoch and Levy (1969), Hadar and Russel (1971).

i.e., the area below F_I up to r must be smaller than or equal to the area below G_I up to r for all possible r, and there must be at least one r^* for which the described area below F_I is smaller than the one below G_I.
Under the assumptions that $U' > 0$, $U'' < 0$, $U''' > 0$, and $\mathbb{E}(r|F) \geq \mathbb{E}(r|G)$, it follows from F TSD G that $\mathbb{E}(U(r|F)) > \mathbb{E}(U(r|G))$.[14]

The concept of stochastic dominance is important, since it enables the portfolio choice to be restricted to portfolios that are not dominated. The higher the order of stochastic dominance, the higher the number of portfolios that are dominated. This is because the portfolios that are efficient by the third order criterion are a subset of the portfolios that are efficient by the second order criterion, also being a subset of the portfolios that are efficient by the first order criterion.

Because of the generality of the concept of stochastic dominance, a risk measure is preferable if it is possible to do the same ordering with the risk measure as with the criteria of stochastic dominance. This is true for the lower partial moment class. To present the relationship between the lower partial moment class and the stochastic dominance concept, it is convenient to rewrite the notation of the lower partial moment as

$$LPM_{T,k}(r) = LPM_{\Omega,k}(T) \text{ for return distribution } \Omega(r). \qquad (2.22)$$

Using this notation, it holds for any return distributions F and G with domain $D \in [a, b)$ that[15]

- F FSD G if, and only if,

$$LPM_{F,0}(T) \leq LPM_{G,0}(T) \ \forall \ T \in D$$
$$\text{and } \exists \ T^* \text{ with } LPM_{F,0}(T^*) < LPM_{G,0}(T^*), \qquad (2.23)$$

- F SSD G if, and only if,

$$LPM_{F,1}(T) \leq LPM_{G,1}(T) \ \forall \ T \in D$$
$$\text{and } \exists \ T^* \text{ with } LPM_{F,1}(T^*) < LPM_{G,1}(T^*), \qquad (2.24)$$

[14] See Whitmore (1970).
[15] See Porter (1974), Bawa (1975), Fishburn (1977), Kaduff (1996), pp. 27-31.

- F TSD G if, and only if,

$$LPM_{F,2}(T) \leq LPM_{G,2}(T) \ \forall \ T \in D \quad \text{and } \exists \ T^* \text{ with } LPM_{F,2}(T^*) < LPM_{G,2}(T^*). \quad (2.25)$$

Thus, a stochastic dominance preselection of portfolios can also be done on the basis of lower partial moments. Nevertheless, it has to be annotated that the described consistency of the lower partial moment class with the stochastic dominance concept is not a suitable foundation for lower partial moments with a fixed target as used in the capital market models of Hogan and Warren (1974) and Bawa and Lindenberg (1977). Indeed, it is a necessary condition for the stochastic dominance of a return that its lower partial moment with a certain target is higher than the corresponding lower partial moments of all other returns. However, this is not a sufficient condition for stochastic dominance, since lower partial moments with all possible targets have to be compared to determine stochastic dominance. Furthermore, the consistency of the lower partial moment class with the stochastic dominance concept also does not provide any reason for arguing that the variance is less founded than a certain lower partial moment. This is because the variance itself is a lower partial moment with the maximum target. It can only be argued that the lower partial moment class is more general than the variance.

2.5 Value at Risk

Currently, however, the most prominent downside risk measure is not the lower partial moment, but, due to its comprehensibility, the Value at Risk VaR. While the risk measures discussed previously are closely related to the portfolio theory, the Value at Risk was developed by major American financial firms in the late 1980s to be a measure of the exposure to market risk in order to allocate risk limits and to facilitate risk adjusted performance measurement.[16]

For a given time horizon and confidence level α, the Value at Risk is defined as the loss of a security or portfolio that is exceeded with probability $1-\alpha$ under normal market conditions.[17] The Value at Risk can

[16] See Pfaff and Kühn (2005).

[17] See Parsley (1995), p. 42, Duffie and Pan (1997), p. 3, Jorion (2000).

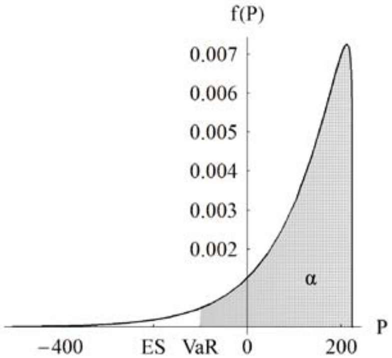

Figure 2.2. Value at Risk VaR and Expected Shortfall ES based on profit P.

equivalently be defined as the maximum loss of a security or portfolio that is not exceeded with probability α under normal market conditions. So the Value at Risk corresponds to the negative α-quantile of the profit distribution (see Figure 2.2).

With f being the density function and F being the distribution function of the profit P, the VaR can formally be defined as

$$VaR = -F^{-1}(1-\alpha) \tag{2.26}$$

$$\Leftrightarrow \int_{-\infty}^{-VaR} f(P)dP = 1-\alpha. \tag{2.27}$$

The Value at Risk is based on the concept of economic capital. Its idea is that financial firms should maintain a high solvency level in order to prohibit costs related to financial distress.[18] There are two variables that influence a firm's solvency level: The firm's exposure to risk and its equity, which serves as a risk buffer. So to maintain a certain solvency level, risk and equity are substitutes. Thus, it makes sense to relate risk directly to equity by quantifying risk in terms of how much equity is necessary to guarantee the desired solvency level. The equity necessary to buffer the risk exposures to maintain a certain solvency level is called economic capital or risk capital. With given equity, the

[18] See Merton and Perold (1993), p. 16, Lehar, Welt, Wiesmayr, and Zechner (1998a), p. 858, Pfaff and Kühn (2005), pp. 185-191.

overall required economic capital should not exceed the equity to assure the desired solvability.

However, this general concept of economic capital does not specify how to quantify economic capital. The quantification of economic capital strongly depends on how the solvency level is defined. One intuitive way is to define the solvency level as the solvency probability of the firm and to assume that the firm is solvent as long as its loss is smaller than its equity. This definition of the solvency level might be suitable if costs, which are tried to be limited, are primarily connected with the jeopardy of insolvency. Using this solvency definition, the economic capital on the firm level corresponds to the Value at Risk with a confidence level equal to the desired solvency probability, since the Value at Risk is the maximum loss which is not exceeded with this probability.

Economic capital is not only used as a risk limit at the corporate level. Economic capital is also allocated to divisions and transactions to quantify to what extent risky transactions require the scarce resource equity. This enables the risk adjusted profitability of divisions and transactions to be measured by calculating the expected return on economic capital, which is known as RORAC (return on risk adjusted capital). Furthermore, the allocation of economic capital is needed to define risk limits at the divisional level and to charge costs of equity.[19]

The allocation of economic capital to single transactions or business lines, however, is only easily solvable if all transactions are completely positively correlated. If this does not hold, a diversification effect occurs, making the required economic capital of a transaction not only dependent on its own risk exposure, but also on the correlations between all other transactions of the firm.

At this point, the Value at Risk is strongly criticized for being used as measure for economic capital, since it has weaknesses mapping diversification effects. To map diversification effects properly, a risk measure needs to have the property of subadditivity.[20] This property would assure that the Value at Risk of a portfolio is never higher than the sum of the Value at Risk of the single portfolio positions. However, the Value at Risk is only subadditive for normally distributed returns. Thus, it

[19] See Pfaff and Kühn (2005), pp. 200-204.

[20] See Artzner, Delbaen, Eber, and Heath (1999).

can happen that a portfolio has a higher Value at Risk, although it is more diversified. This is shown in the following example.[21]

Assume that a bank only has a single loan of 1 million. The interest rate on the loan is 6% and the bank can refinance the loan with 5%. The debtor fully repays the loan with the probability of 99.5%. With the probability of 0.5%, the debtor defaults leading to a complete loss for the bank. So the Value at Risk with a confidence level of 99.5% is negative and amounts -10'000, meaning that with a probability of 99.5%, the profit of the bank is 10'000.

Now assume that the bank decides to split up the investment amount of 1 million equally into several loans that have the same repayment structure as described above. This leads to a diversification effect as long as the insolvency events are not completely positively correlated. Furthermore, as a special case, assume that the insolvency events are statistically independent of each other. In this case, the number of failed loans is binomially distributed and depends on the number of loans granted.

Figure 2.3 shows the relation between the Value at Risk of the loan portfolio and the number of the loans granted. Three jumps, which show that the Value at Risk is not subadditive, attract attention. The first jump of the Value at Risk at three loans occurs because the probability that at least one of the credits fails is above 1% and the associated loss thus becomes relevant for the calculation of the Value at Risk. The jump at 31 loans results from the probability of above 1% that at least two loans fail. Analogously, the third jump is located at 89 loans, since the probability that at least three loans fail exceeds 1% at this number of loans.

Another objection against the Value at Risk is that it insufficiently characterizes the loss distribution. In particular, it does not consider the amount of possible losses given that the loss exceeds the quantile considered. This is primarily relevant for measuring the risk of single departments or transactions because, even if the Value at Risk of a transaction is not high, the transaction can have significant losses in case of a failure and thus compromise the solvability of the entire firm.

[21] A similar example is given by Artzner, Delbaen, Eber, and Heath (1999), pp. 217-218.

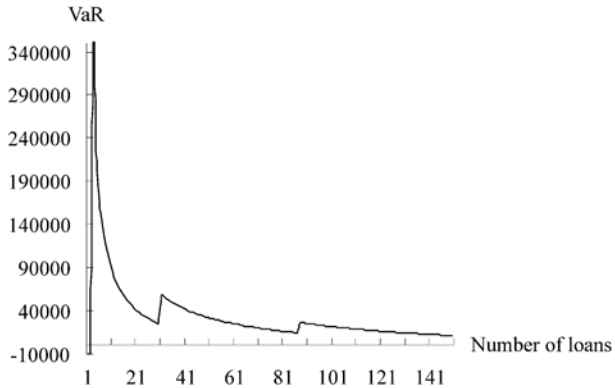

Figure 2.3. Value at Risk based on the profit of a loan portfolio as a function of the number of loans granted.

However, both objections to the Value at Risk resulting from its lack of subadditivity and the fact that the Value at Risk insufficiently characterizes the loss distribution are not significant at the corporate level as long as it is assumed, as done so far, that it is appropriate for the firm to maintain a certain solvability and this solvability can be quantified by the insolvency probability. In this case, the Value at Risk is a suitable measure for economic capital, as described above. The criticism is not even relevant for allocating economic capital to departments or transactions. This is because most methods for allocating economic capital are based on corporate economic capital, which is usually allocated using keys such as the marginal contribution to corporate economic capital.[22] For such keys, the risk measure only needs to be positively homogeneous of the first degree. This property is fulfilled by the Value at Risk. So the Euler relation guarantees the full allocation of economic capital based on the marginal contribution to the corporate economic capital.[23]

[22] See Tasche (2000), Theiler (2002), Pfaff and Kühn (2005), pp. 204-208.
[23] See Tasche (2000), pp. 6-7.

2.6 Expected Shortfall

In any event, the criticism on the Value at Risk led to a rethinking of how to measure economic capital. Today the majority of the literature recommends the Expected Shortfall, also called Conditional Value at Risk, as a measure for economic capital.[24] The Expected Shortfall ES is the loss over a certain investment horizon that can be expected under normal market conditions given that the loss is greater than or equal to the Value at Risk VaR (see Figure 2.2). Formally, it can be defined as

$$ES = -\mathbb{E}(P|P \leq -VaR) \tag{2.28}$$

$$= -\frac{\int_{-\infty}^{-VaR} Pf(P)dP}{\int_{-\infty}^{-VaR} f(P)dP}. \tag{2.29}$$

It can be converted to (see appendix A.1)

$$ES = VaR + \frac{LPM_{-VaR,1}(P)}{LPM_{-VaR,0}(P)}. \tag{2.30}$$

Since the lower partial moments of order zero and one are non-negative, the Value at Risk is the lower bound of the Expected Shortfall for a given confidence level. However, the Expected Shortfall is only equal to the Value at Risk if the Value at Risk corresponds to the highest possible loss.

Quantifying economic capital at the corporate level based on the Expected Shortfall stems from a different understanding of solvability than the one underlying the Value at Risk. It does not focus on the insolvency probability of the firm, but on a certain catastrophe scenario, the lower bound of which is quantified by the Value at Risk. Using Expected Shortfall as measure for economic capital implies the belief that a firm's equity should be able to buffer the expected loss in case of this catastrophe scenario.

An advantage of the Expected Shortfall is that it fulfills the property of subadditivity.[25] So in contrast to the Value at Risk, it always

[24] See Artzner, Delbaen, Eber, and Heath (1999), Denault (2001), and Theiler (2002).

[25] See Artzner, Delbaen, Eber, and Heath (1999).

shows the diversification effect. Like the Value at Risk, it is also positively homogeneous of the first degree and thus suitable for allocating economic capital based on the marginal contribution to the corporate economic capital.

However, the interpretation of the lower bound of the catastrophe scenario quantified by the Value at Risk is difficult, since the firm is not insolvent at this loss level. It stays solvent up to the expected loss of the catastrophe scenario. Therefore, in contrast to the Value at Risk, the clear link between the confidence level and the insolvency probability is missing. Furthermore, it is quite intuitive that the negative effects of risk-taking, which are tried to be controlled, are more closely associated with the jeopardy of insolvency rather than with the amount of loss that can be expected if a certain catastrophe scenario occurs. This could particularly hold if insolvency causes costs that significantly reduce the remaining value of the firm.

However, when discussing measures for economic capital, it has to be pointed out that even the underlying concept of fixing a certain solvency level is problematic, since the solvency level itself should be part of a firm's optimization problem and not an exogenous restriction. The solvency level should reflect the optimal trade-off between risk and expected return and depend thus on the firm's profit-earning possibilities, as illustrated in the introduction.

3

Asset Pricing

The objective of this chapter is to analyze the asset pricing of capital market investors. This provides the fundamentals for understanding capital market models upon which the reward-to-risk ratios presented in the next chapter are based.

First of all, the market price of assets is derived using the stochastic discount factor model. Subsequently, it is analyzed for what risk premiums are paid, which is the systematic part of the risk. In the following section, it is discussed under which conditions pricing models using beta representations such as the Capital Asset Pricing Model and the Arbitrage Pricing Theory are consistent with the general stochastic discount factor model. In the last section, it is demonstrated that under certain assumptions a firm can maximize the market price of equity as a substitute for the shareholders' individual valuations of equity. This justifies the use of the market price of equity as the target figure of a bank, when determining optimal risk-return trade-offs in Chapter 6 and 7.

3.1 Market Price of Assets

To analyze the market price of assets, it is convenient to assume that one of S_{t+1} possible states of nature occurs in the uncertain future $t+1$.[1] For the moment, also assume that for each state s_{t+1}, a security called contingent claim exists with a market prize $P_{c,t}(s_{t+1})$ that pays out one currency unit if state s_{t+1} realizes and otherwise, it pays out

[1] See Cochrane (2001), pp. 51-52.

nothing. Contingent claims need not be explicitly traded. It is sufficient if there are enough securities to synthesize all contingent claims. If this holds, the capital market is called complete.

Also assume that an asset j has a stochastic payoff $X_{j,t+1}(s_{t+1})$ in time $t+1$ depending on state s_{t+1}. So in equilibrium, the market price of the asset is equal to the value of the contingent claims, of which it is a bundle:

$$P_{j,t} = \sum_{s_{t+1}} P_{c,t}(s_{t+1}) X_{j,t+1}(s_{t+1}). \tag{3.1}$$

With the probability $\pi_t(s_{t+1})$ that state s_{t+1} occurs conditioning on information at time t, the market price of asset j can be represented as

$$P_{j,t} = \sum_{s_{t+1}} \pi_t(s_{t+1}) \frac{P_{c,t}(s_{t+1})}{\pi_t(s_{t+1})} X_{j,t+1}(s_{t+1}). \tag{3.2}$$

Defining the market discount factor of state s_{t+1} as

$$m_{t+1}(s_{t+1}) = \frac{P_{c,t}(s_{t+1})}{\pi_t(s_{t+1})}, \tag{3.3}$$

the market price of asset j turns out to be

$$P_{j,t} = \sum_{s_{t+1}} \pi_t(s_{t+1}) m_{t+1}(s_{t+1}) X_{j,t+1}(s_{t+1}). \tag{3.4}$$

So using the stochastic market discount factor m_{t+1} and the stochastic payoff $X_{j,t+1}$, the asset price can be written as

$$P_{j,t} = \mathbb{E}_t(m_{t+1} X_{j,t+1}). \tag{3.5}$$

Here $\mathbb{E}_t$ denotes the expectation operator conditioning on information at time t.

The above market pricing formula states that the market price of any asset can easily be determined just by using the expectation operator and the market discount factor, which is the same for all assets. But what are the conditions for a market to allow such a discount factor to take shape? Three theorems are developed to answer this question.[2]

The first theorem states that a stochastic discount factor exists that relates payoffs to market prices for all assets in the economy if, and only

[2] See Ross (1978), Rubinstein (1976), Harrison and Kreps (1979), for proofs see Cochrane (2001), pp. 63-74.

if, the law of one price holds. The law of one price requires that if two portfolios have the same payoff in every state, they must have the same price. The law of one price implies that the pricing function is linear in the payoffs, since investors could otherwise make instantaneous profits by repackaging portfolios.

The second theorem states that without arbitrage opportunities, the above discount factor is positive. The absence of arbitrage opportunities means that if payoff A is always at least as high as payoff B for all states and sometimes even higher, the asset with payoff A must have a higher market price than the asset with payoff B.

The third theorem states that in the absence of arbitrage opportunities, the stochastic discount factor is unique if the capital market is complete. The condition of the capital market being complete is necessary for the stochastic discount factor being unique because otherwise prices cannot be derived from the absence of arbitrage opportunities, since it is not possible to replicate the payoffs of any asset using contingent claims.

However, a complete capital market is not necessary for the existence of a positive discount factor. As the first two theorems state, the only structure needed for a positive discount factor to exist is the law of one price and the absence of arbitrage opportunities. Thus, very few presumptions need to be fulfilled for a market discount factor to exist.

3.2 Systematic Risk

Now the premium is analyzed that is required by capital market investors for investing in risky assets. The risk premium is the expected excess return over the risk-free rate. It can easily be derived using the pricing formula (3.5)

$$P_{j,t} = \mathbb{E}_t(m_{t+1} X_{j,t+1}).$$

With $P_{j,t}$ being positive and with the return $r_{j,t+1}$ of asset j

$$r_{j,t+1} = \frac{X_{j,t+1}}{P_{j,t}} - 1, \tag{3.6}$$

the above equation is equivalent to

$$1 = \mathbb{E}_t(m_{t+1}(1 + r_{j,t+1})). \tag{3.7}$$

Using this equation, it follows for a risky asset j and a risk-free asset f that

$$\begin{aligned} 0 &= \mathbb{E}_t(m_{t+1}(1+r_{j,t+1})) - \mathbb{E}_t(m_{t+1}(1+r_{f,t+1})) && (3.8)\\ &= \mathbb{E}_t(m_{t+1}(r_{j,t+1}-r_{f,t+1})) && (3.9)\\ &= \mathbb{E}_t(m_{t+1})\mathbb{E}_t(r_{j,t+1}-r_{f,t+1}) + Cov_t(m_{t+1}, r_{j,t+1}-r_{f,t+1}) && (3.10)\\ &= \mathbb{E}_t(m_{t+1})\mathbb{E}_t(r_{j,t+1}-r_{f,t+1}) + Cov_t(m_{t+1}, r_{j,t+1}) && (3.11) \end{aligned}$$

with the covariance operator Cov_t conditioning on information at time t.
This leads to

$$\mathbb{E}_t(r_{j,t+1}) - r_{f,t+1} = \frac{-Cov_t(m_{t+1}, r_{j,t+1})}{\mathbb{E}_t(m_{t+1})}. \tag{3.12}$$

Furthermore, since

$$\mathbb{E}_t(m_{t+1}(1+r_{f,t+1})) = \mathbb{E}_t(m_{t+1})(1+r_{f,t+1}) = 1 \tag{3.13}$$

$$\Leftrightarrow \mathbb{E}_t(m_{t+1}) = \frac{1}{1+r_{f,t+1}}, \tag{3.14}$$

the risk premium of asset j can be expressed as

$$\mathbb{E}_t(r_{j,t+1}) - r_{f,t+1} = -Cov_t(m_{t+1}, r_{j,t+1})(1+r_{f,t+1}). \tag{3.15}$$

Here $r_{f,t+1}$ denotes the risk-free return of the capital market.

The risk premium of an asset thus only depends on the risk-free rate and the covariance between the excess return and the stochastic discount factor.[3] The formula can be interpreted as follows:

Discount factors of certain states being high means that the payoffs in these states are very valuable, for example, think about the states of a recession in which it is difficult to generate income. If an asset has high payoffs particularly in these valuable states, implying a high covariance between the excess return and the stochastic discount factor, investors charge a small or even negative risk premium to buy the asset. This is because the asset can be used to smooth the income of investors.

In contrast, if the payoff of an asset is high particularly in states that are less valuable, for example, think about the states of a boom in which it is easy to generate income, the covariance between the excess

[3] See Cochrane (2001), p. 17.

return and the stochastic discount factor is low. This makes the risk premium of the asset high, since buying the asset leads to the investors having even more volatile income.

Therefore, the risk premium of an asset is determined by the covariance between the asset's payoff and the stochastic discount factor and not primarily by its volatility. Only the component of a payoff that is perfectly correlated with the market discount factor generates a risk premium. The priced volatility component, which is market inherent, is called systematic risk. The residual component of the volatility called idiosyncratic risk is not priced, since it can be diversified for free.

3.3 Models Using Beta Representations

From a theoretical point of view, using a stochastic discount factor to calculate asset prices is superior to using the present value formula since the asset pricing does not require an asset-specific risk adjustment. However, the theoretical concept of mapping discount factors to different states comes up against limits for empirical and practical work. Therefore factor pricing models such as the Capital Asset Pricing Model[4] (CAPM) or the Arbitrage Pricing Theory[5] (APT), which specify asset-specific discount factors by determining expected returns of risk-equivalent market alternatives using beta representations, are still prevalent.

The CAPM states that the expected return of an asset j is

$$\mathbb{E}_t(r_{j,t+1}) = r_{f,t+1} + \lambda_{M,t}\beta_{jM,t} \tag{3.16}$$

with the risk premium of the market $\lambda_{M,t}$ and

$$\beta_{jM,t} = \frac{Cov_t(r_{M,t+1}, r_{j,t+1})}{Var_t(r_{M,t})}, \tag{3.17}$$

which can be interpreted as the regression coefficient of the asset return $r_{j,t+1}$ on the market return $r_{M,t+1}$. Here Var_t denotes the variance operator conditioning on information at time t.

The APT states that the expected return of an asset j is

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[4] See Sharpe (1964), Lintner (1965), and Mossin (1966).

[5] See Ross (1976).

$$\mathbb{E}_t(r_{j,t+1}) = r_{f,t+1} + \sum_{k=1}^{K} \lambda_{k,t}\beta_{jk,t} \tag{3.18}$$

with K common risk factors having the risk premium $\lambda_{k,t}$ and with

$$\beta_{jk,t} = \frac{Cov_t(S_{k,t+1}, r_{j,t+1})}{Var_t(S_{k,t})}, \tag{3.19}$$

which corresponds to the regression coefficient of the asset return $r_{j,t+1}$ on the risk factor $S_{k,t+1}$.

However, under which conditions are such factor pricing models consistent with the stochastic discount factor model? This is analyzed in the following.

As will be shown, asset returns can be described by a linear factor model such as the CAPM or APT if the stochastic discount factor is a linear combination of common risk factors.[6] This is consistent with requiring only one period, no other income of the investors, and either a quadratic utility function or normally distributed returns. The consumption in the last period is then equal to the portfolio return and marginal utility is linear in consumption.[7]

Assuming that the stochastic discount factor is a linear combination of K orthogonal[8] common risk factors $S_{k,t+1}$ generating undiversifiable risk, m_{t+1} has the form

$$m_{t+1} = a_t - \sum_{k=1}^{K} b_{k,t} S_{k,t+1} \tag{3.20}$$

with the parameters a_t and $b_{k,t}$ conditioning on information at time t. So the negative covariance between the portfolio return and the stochastic discount factor can be expressed as (see appendix A.2)

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[6] See Campbell (2000), pp. 1525-1526.

[7] For alternative assumptions leading to the CAPM, see Cochrane (2001), pp. 152-161.

[8] Risk factors being orthogonal only assures that the risk premium also depends on K factors and not on less.

$$
\begin{aligned}
& - Cov_t(m_{t+1}, r_{j,t+1}) \\
&= \sum_{k=1}^{K} b_{k,t} Cov_t(S_{k,t+1}, r_{j,t+1}) && (3.21) \\
&= \sum_{k=1}^{K} (b_{k,t} Var_t(S_{k,t})) \frac{Cov_t(S_{k,t+1}, r_{j,t+1})}{Var_t(S_{k,t})} && (3.22) \\
&= \sum_{k=1}^{K} (b_{k,t} Var_t(S_{k,t})) \beta_{jk,t}. && (3.23)
\end{aligned}
$$

Using equation (3.15), the risk premium of asset j turns out to be

$$\mathbb{E}_t(r_{j,t+1}) - r_{f,t+1} = \sum_{k=1}^{K} (b_{k,t} Var_t(S_{k,t}))(1 + r_{f,t+1}) \beta_{jk,t}. \tag{3.24}$$

Calling $\lambda_{k,t} = (b_{k,t} Var_t(S_{k,t}))(1 + r_{f,t+1})$ the risk premium of risk factor $S_{k,t+1}$, equation (3.24) is equivalent to equation (3.18) representing the APT, which collapses to the CAPM in case that only the risk factor market return $r_{M,t+1}$ is considered. Therefore, the APT and the CAPM are special cases of the stochastic discount factor model for the stochastic discount factor being linear in the risk factors.

However, the beta representation is not specific to the CAPM and the APT. As shown in the following, the expected return of an asset j can also be expressed based on the stochastic discount factor model using a beta representation.
Starting from equation (3.12)

$$\mathbb{E}_t(r_{j,t+1}) - r_{f,t+1} = \frac{-Cov_t(m_{t+1}, r_{j,t+1})}{\mathbb{E}_t(m_{t+1})},$$

the expected return can be represented as

$$\mathbb{E}_t(r_{j,t+1}) = r_{f,t+1} + \lambda_{m,t} \beta_{jm,t} \tag{3.25}$$

with the risk premium $\lambda_{m,t}$ of the stochastic discount factor m

$$\lambda_{m,t} = -\frac{Var_t(m_{t+1})}{\mathbb{E}_t(m_{t+1})} \tag{3.26}$$

and

$$\beta_{jm,t} = \frac{Cov_t(m_{t+1}, r_{j,t+1})}{Var_t(m_{t+1})}. \tag{3.27}$$

3.4 Market Price of Equity as the Target Figure of Firms

The sparse structure that is needed for positive market discount factors to exist explains why it is a temptation to use the market price of equity as a substitute for the shareholders' individual valuations of equity and to define the shareholder value of a firm as the market price of equity. Referring to the Fisher-separation theorem, it might be reasoned that in a perfect capital market investors should choose the portfolio with the maximum net present value, which is equivalent to its market price less the invested capital.[9] A firm would thus maximize the market price of equity. However, the Fisher-separation theorem is developed for a world without uncertainty. In a world with uncertainty, in addition to the perfect capital market assumption, further premises need to be fulfilled in order for shareholders to accept the market price of equity as a substitute for individual valuation.[10]

A first condition is that the realization of investments does not influence the pricing of the capital market. This assumption is necessary because otherwise investors could not determine the market price of assets since the market discount factor would not be given. It is approximately fulfilled if there are many investments in an economy such that the volumes of single investments only represent marginal parts of the economy's investment volume.

A further necessary condition for the use of market discount factors for asset valuation is that all stochastic components of the investor's income - including earned income and income of debtholder positions - can be replicated using securities of the capital market. Without this presumption the individual pricing of assets also depends on the payout structure of the claims.

Third, the decision-making of firms and its consequences have to be common knowledge. This premise assures that investors can construct hedge portfolios that reverse unfavorable payout structures. This presumption is not necessary if firms align their strategy directly with the pricing of individual investors.

While the presumption that the realization of investments does not influence the pricing of the capital market is not problematic, the other

[9] See Fisher (1930).

[10] See Kürsten (2000).

two presumptions are usually at best only partially fulfilled. Nevertheless, in spite of this lack of fulfilment, the market price of equity is broadly accepted as the target figure of firms, since it is nearly impossible to align a firm's strategy with the individual preferences of different shareholders.

4

Reward-to-Risk Ratios

Reward-to-risk ratios are measures for portfolio selection and represent optimal risk-return trade-offs of capital market investors. While the capital market only prices systematic risk, measures for portfolio selection consider both systematic and unsystematic risk, even though they are derived from capital market models. This is because unsystematic risk also decreases an investor's utility, although it is not priced.

4.1 Sharpe Ratio

The Sharpe ratio is the most widely-used measure for portfolio selection.[1] The ex ante-form of the Sharpe ratio is defined as the ratio of the expected excess return over the risk-free rate to the standard deviation of the return:

$$SR = \frac{\mathbb{E}(r) - r_f}{\sigma(r)}. \tag{4.1}$$

It results from the portfolio optimization within the mean-variance framework used in the works of Markowitz (1952), Sharpe (1964), Lintner (1965), and Mossin (1966). The portfolio optimization is based on the Tobin separation theorem, which states that finding an optimal portfolio for a given level of risk can be separated into two problems: first, finding an optimal portfolio of market securities, which does not vary with the risk tolerance of investors, and second, combining it with an appropriate amount of cash.[2] Within this framework, the efficient

[1] See Sharpe (1966).
[2] See Tobin (1958).

frontier is linear in leverage, measuring risk as the standard deviation of the portfolio return. So it is optimal to choose the portfolio with the highest expected excess return-to-risk ratio. This is because every portfolio maximizing this ratio can be leveraged to any given level of risk, and by doing this, it has the highest possible expected return. The portfolio that maximizes this ratio is called tangency portfolio.

However, the Sharpe ratio is not only founded in a mean-variance framework. It can also be derived from the stochastic discount factor model.[3] In the following, this is shown for an asset with return r and a stochastic market discount factor m.[4]

Using equation (3.12), the excess return of the asset can be expressed as

$$\mathbb{E}(r) - r_f = \frac{-Cov(m,r)}{\mathbb{E}(m)}.$$

With the coefficient of correlation

$$\rho_{m,r} = \frac{Cov(m,r)}{\sigma(m)\sigma(r)}, \tag{4.2}$$

the expected excess return turns out to be

$$\mathbb{E}(r) - r_f = -\rho_{m,r}\frac{\sigma(m)}{\mathbb{E}(m)}\sigma(r). \tag{4.3}$$

Since $-1 \leq \rho_{m,r} \leq 1$, the following inequality results:

$$|\frac{\mathbb{E}(r) - r_f}{\sigma(r)}| \leq \frac{\sigma(m)}{\mathbb{E}(m)}. \tag{4.4}$$

This relation can best be interpreted if it is visualized (see Figure 4.1, following Cochrane (2001), p. 21).

The means and standard deviations of asset returns lie in a wedge-shaped region. Its boundary is called the mean-variance frontier. The upper boundary describes the possible amount of expected return in the capital market for a given level of standard deviation of the return. It can be realized by combining different assets in a well-diversified portfolio. Since this diversification is free, there is a component of risk that is priced, called systematic risk, and a residual component of risk, called idiosyncratic risk.

[3] See Cochrane (2001), pp. 20-22.

[4] For a consistent representation, time-indices are left a way.

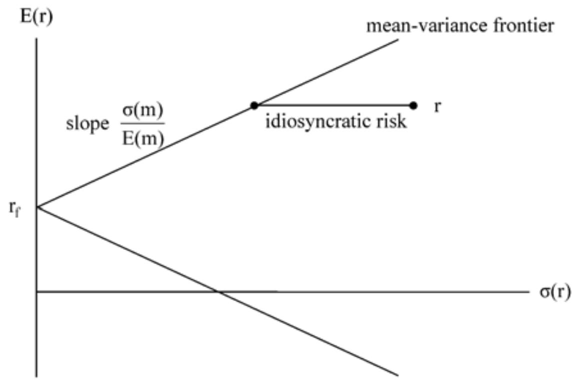

Figure 4.1. Mean-variance frontier.

Since all returns on the frontier are perfectly correlated with the discount factor, all frontier returns are also perfectly correlated with each other. Therefore, it is possible to span or synthesize any frontier return from any two different returns lying on the frontier. Thus, all pricing information is represented by a straight line between the risk-free return and a risky return on the frontier.

What does this mean for portfolio selection by investors? Investors should select their portfolios so that they lie on the upper mean-variance frontier. In other words, they should maximize the ratio of the expected excess return over the risk-free rate to the standard deviation of the portfolio return, which is the Sharpe ratio in its ex ante-form. Thus, within this asset pricing framework, maximizing the Sharpe ratio, is equivalent to coming close to the mean-variance frontier and investing efficiently in the sense of diversifying optimally. Maximizing anything else could lead to unnecessarily bearing a risk.

However, as widely discussed in the literature, the Sharpe ratio appears to be highly misleading, especially if the return distribution is asymmetric.[5] Particularly problematic is that the Sharpe ratio can be improved by truncating the upper end of the return distribution. This is because the Sharpe ratio valuates positive and negative deviations from the mean in the same way, but higher risk premiums are paid for possible negative deviations. In fact, using the above stochastic dis-

[5] See Bookstaber and Clarke (1984), Zimmermann (1994), Bernardo and Ledoit (2000), and Spurgin (2001).

count factor framework, it is even shown that the Sharpe ratio can be maximized by implementing a strategy of selling out-of-the-money calls and selling out-of-the-money puts in a ratio, leading to a truncated right tail and a fat left tail of the return distribution.[6] However, this portfolio maximizing the Sharpe ratio is unlikely to represent the market portfolio. So the stochastic discount factor model is misleading at this point.

4.2 Reward-to-Shortfall Ratios

As an alternative to the Sharpe ratio, reward-to-shortfall ratios are developed, which relate excess return to downside risk, thus valuating upside potential differently from downside risk. Here downside risk is quantified using lower partial moments and root lower partial moments.

The advantages are that they employ risk measures having a higher consistency with the way individuals perceive risk and have increased immunity to manipulation.[7] Furthermore, empirical tests with mutual fund data show that reward-to-shortfall ratios are less statistically biased than the Sharpe ratio.[8] Additionally, capital market models based on the mean-lower partial moment framework such as the ones by Hogan and Warren (1974), Bawa and Lindenberg (1977), and Harlow and Rao (1989) explain empirical data better than the CAPM, which is based on the mean-variance framework.[9]

The best known shortfall ratio is the Sortino ratio.[10] In ex ante-form, it is defined as the ratio of the expected excess return over the target T to the below-target semi-standard deviation of the return:

$$\textit{Sortino ratio} = \frac{\mathbb{E}(r) - T}{\theta_T(r)}. \tag{4.5}$$

However, the problem with this ratio is that it is not consistent with capital market equilibrium theories, since they are based on the excess return over the risk-free rate. In addition, the Sortino ratio is

[6] See Goetzmann, Ingersoll, Jr., Spiegel, and Welch (2002).
[7] See Zimmermann (1994).
[8] See Klemkosky (1973), Ang and Chua (1979), and Nawrocki (1999).
[9] See Harlow and Rao (1989).
[10] See Sortino and Price (1994).

not defined as ex-ante profitability measure using the below-mean semi-standard deviation, as it implies the target $T = \mathbb{E}(r)$ leading to a numerator of zero.

Thus, reward-to-shortfall ratios relating the excess return over the risk-free rate to lower partial moments and root lower partial moments are more plausible. However, using the excess return over the risk-free rate is not sufficient for being consistent with capital market models. For such ratios to be consistent with capital market models, the efficient frontier needs to be linear.[11] This holds, although the linearity of the efficient frontier is not a necessary condition for the portfolio separation and thus for the existence of capital market equilibriums.[12] However, if the efficient frontier is not linear, portfolios with higher reward-to-shortfall ratios will not necessarily be closer to the efficient frontier.

Since efficient frontiers based on lower partial moments are always convex,[13] it makes sense to further restrict the reward-to-shortfall ratios to ratios relating the expected excess return over the risk-free rate to the root lower partial moments of the return. However, the linearity of the efficient frontier based on the root lower partial moment also depends on the target used for calculating the root lower partial moment. This is shown in the following for the case of root lower partial moments of second order, respectively, for below-target semi-standard deviations.[14]

Assume that all investors choose a portfolio P with return r_P that is a mixture of the risk-free asset with return r_f and the same risky portfolio M with return r_M. μ_P and μ_M denote the expected return of the portfolio P and M, respectively. With w being the positive weight of portfolio M, the portfolio return can be expressed as

$$r_P = w r_M + (1 - w) r_f. \tag{4.6}$$

$\theta_T(r_P)$ denotes the below-target semi-standard deviation of the return of portfolio P with target T and is defined as

$$\theta_T(r_P) = \Big(\int_{-\infty}^{T} (T - r_P)^2 f(r_P) dr_P\Big)^{\frac{1}{2}} \tag{4.7}$$

[11] See Pedersen and Satchell (2002), p. 218, proposition I.
[12] See Harlow and Rao (1989).
[13] See Harlow and Rao (1989), p. 289, proposition II.
[14] Pedersen and Satchell (2002) show that the efficient frontier is linear for the target $T = R_f$ and is non-linear for targets $T < R_f$.

with f being the density function of r_P.
Substituting r_P with the right-hand side of equation (4.6) and using

$$w = \frac{\mu_P - r_f}{\mu_M - r_f}, \tag{4.8}$$

$\theta_T(r_P)$ can be expressed as

$$\theta_T(r_P) = \Big(\int_{-\infty}^{B(\mu_P,\mu_M,r_f,T)} (r_f - T + \frac{(r_M - r_f)(\mu_P - r_f)}{\mu_M - r_f})^2 g(r_M) dr_M \Big)^{\frac{1}{2}} \tag{4.9}$$

with boundary

$$B(\mu_P, \mu_M, r_f, T) = \frac{-T\mu_M + r_f(T + \mu_M - \mu_P)}{r_f - \mu_P}$$

and g being the density function of r_M (see appendix A.3).
Differentiating with respect to μ_P leads to

$$\frac{\partial \theta_T(r_P)}{\partial \mu_P} = \frac{1}{\mu_M - r_f} \frac{\int_{-\infty}^{B(\mu_P,\mu_M,r_f,T)} (r_M - r_f) A(\mu_P, \mu_M, r_M, r_f, T) g(r_M) dr_M}{\sqrt{\int_{-\infty}^{B(\mu_P,\mu_M,r_f,T)} (A(\mu_P, \mu_M, r_M, r_f, T))^2 g(r_M) dr_M}} \tag{4.10}$$

with

$$A(\mu_P, \mu_M, r_M, r_f, T) = -r_M(\mu_P - r_f) + r_f(\mu_P - T) + \mu_M(T - r_f).$$

For a linear efficient frontier, $\frac{\partial \theta_T(r_P)}{\partial \mu_P}$ must be independent of μ_P. As shown by equation (4.10), this does not hold in general. Targets T making the efficient frontier linear are, e.g., $T = r_f$, leading to

$$\begin{aligned} \frac{\partial \theta_T(r_P)}{\partial \mu_P} &= \frac{1}{\mu_M - r_f} \sqrt{\int_{-\infty}^{r_f} (r_M - r_f)^2 g(r_M) dr_M} \\ &= \frac{\theta_{r_f}(r_M)}{\mu_M - r_f}, \end{aligned} \tag{4.11}$$

and $T = \mu_P$, leading to

$$\frac{\partial \theta_T(r_P)}{\partial \mu_P} = \frac{1}{r_f - \mu_M} \frac{\int\limits_{-\infty}^{\mu_M} (r_M - r_f)(\mu_M - r_M) g(r_M) dr_M}{\sqrt{\int\limits_{-\infty}^{\mu_M} (\mu_M - r_M)^2 g(r_M) dr_M}}. \tag{4.12}$$

In contrast, the target $T = \mu_M$, which makes the risk measure to the below-mean semi-standard deviation, and the targets $T = c$ for $c \in \mathbb{R}$ and $c \neq r_f$ do not lead to linear efficient frontiers. However, the target $T = \mu_P$ is problematic, since the target would be an endogenous variable of the portfolio selection problem; therefore, the axioms of expected utility theory may not hold.[15]

Therefore, $T = r_f$ is the most appealing target since it fulfills the precondition of leading to a linear efficient frontier and does not conflict with the expected utility theory because it is constant. From equation (4.11), it follows that for portfolio optimization, it is optimal in this framework to minimize the ratio $\frac{\theta_{r_f}(r)}{\mathbb{E}(r) - r_f}$, respectively, to maximize the following reward-to-shortfall ratio:

$$RSR = \frac{\mathbb{E}(r) - r_f}{\theta_{r_f}(r)}. \tag{4.13}$$

The reward-to-shortfall ratio RSR is thus a suitable measure for portfolio selection and consistent with the mean-lower partial moment framework, which does not hold for most other reward-to-shortfall ratios.

4.3 Measures Based on Economic Capital

To develop a bank-specific measures for portfolio selection, Campbell, Huisman, and Koedijk (2001) as well as Alexander and Baptista (2003) constructed a capital market model based on a Value at Risk constraint. This proceeding implies that the perceived risk of all capital market investors is sufficiently represented by the Value at Risk. Furthermore, it needs to be optimal for the bank to price risk the same way the capital market does.

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[15] See Pedersen and Satchell (2002), p. 219.

In this portfolio context, the Value at Risk VaR with confidence level α is not defined as a function of the profit, but as a function of the portfolio return r:

$$VaR(r) = -F^{-1}(1-\alpha) \tag{4.14}$$

with F being the distribution function of the portfolio return r.

However, using this definition, the Value at Risk has a different meaning compared to using the profit-based definition. Although the portfolio return is closely related to the profit, portfolio return and profit cannot be considered to be substitutes. This is because the profit is also determined by costs that are not directly related to the portfolio return and thus not taken into account when only considering the portfolio return. In particular, focusing on the portfolio return neglects that the refinancing costs usually increase in the portfolio risk.

The authors derive the following profitability measure from this capital market model, called the reward-to-VaR ratio $RVaR$:

$$RVaR = \frac{\mathbb{E}(r) - r_f}{VaR(r) + r_f}. \tag{4.15}$$

In the following, this profitability measure is generalized for a risk measure called economic capital EC fulfilling the property of translation invariance and of positive homogeneity of first degree.

The property of translation invariance of a risk measure EC states that, adding a risk-free asset α to a portfolio with payoff X, the risk of the portfolio measured by EC decreases by the amount α:

$$EC(X + \alpha) = EC(X) - \alpha. \tag{4.16}$$

The property of positive homogeneity of first degree states that for all $\lambda \geq 0$

$$EC(\lambda X) = \lambda EC(X). \tag{4.17}$$

Both properties are fulfilled by the Value at Risk and the Expected Shortfall.[16] Here the Expected Shortfall is defined as

$$ES(r) = -\mathbb{E}(r | r \leq F^{-1}(1-\alpha)) \tag{4.18}$$

with F being the distribution function of the return r.

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[16] See Artzner, Delbaen, Eber, and Heath (1999).

Analogously to deriving efficient frontiers in a mean-lower partial moment framework, assume that all investors choose a portfolio P with return r_P that is a mixture of the risk-free asset with return r_f and the same risky portfolio M with return r_M. μ_P and μ_M again denote the expected return of the portfolio P and M, respectively. With w being the positive weight of portfolio M, the portfolio return can be expressed as

$$\begin{aligned} r_P &= w r_M + (1-w) r_f \\ &= r_f + w(r_M - r_f). \end{aligned} \tag{4.19}$$

Due to its translation invariance and its positive homogeneity of first degree, the economic capital $EC(r_P)$ can be represented as

$$\begin{aligned} EC(r_P) &= EC(w(r_M - r_f) + r_f) \\ &= w(EC(r_M) + r_f) - r_f. \end{aligned} \tag{4.20}$$

The weight w can thus be expressed as

$$w = \frac{EC(r_P) + r_f}{EC(r_M) + r_f}. \tag{4.21}$$

Using equation (4.21) and equation (4.19), it follows for the expected portfolio return:

$$\begin{aligned} \mathbb{E}(r_P) &= r_f + \frac{EC(r_P) + r_f}{EC(r_M) + r_f}(\mathbb{E}(r_M) - r_f) \\ &= r_f \frac{EC(r_M) + r_M}{EC(r_M) + r_f} + \frac{\mathbb{E}(r_M) - r_f}{EC(r_M) + r_f} EC(r_P). \end{aligned} \tag{4.22}$$

Thus, in the mean-economic capital framework, the efficient frontier is linear and has the slope $\frac{\mathbb{E}(r_M)-r_f}{EC(r_M)+r_f}$. Therefore, it is optimal to maximize the following ratio

$$REC = \frac{\mathbb{E}(r) - r_f}{EC(r) + r_f}, \tag{4.23}$$

which is called the reward-to-EC ratio. So the reward-to-EC ratio REC is a suitable measure for portfolio selection in the framework considered analogous to the Sharpe ratio in the mean-variance framework and the reward-to-shortfall ratio RSR in the mean-lower partial moment framework.

The reward-to-VaR ratio $RVaR$ derived from the capital market model with the Value at Risk-constraint results as a special case of the reward-to-EC ratio. Besides the reward-to-VaR ratio, a further special case is the ratio quantifying economic capital based on the Expected Shortfall. This ratio is called the reward-to-ES ratio. Both the reward-to-VaR ratio $RVaR$ and the reward-to-ES ratio RES can be defined based on the excess return:

$$RVaR = \frac{\mathbb{E}(r) - r_f}{VaR_{ex}} \tag{4.24}$$

and

$$RES = \frac{\mathbb{E}(r) - r_f}{ES_{ex}} \tag{4.25}$$

with

$$VaR_{ex} = VaR(r - r_f) = VaR(r) + r_f \tag{4.26}$$

and

$$ES_{ex} = ES(r - r_f) = ES(r) + r_f. \tag{4.27}$$

4.4 Measures Based on Portfolio Models of Banks

While the discussed profitability measures reward-to-VaR ratio and reward-to-ES ratio are based on the idea that banks trade assets on the capital market and optimize their asset portfolio like individual capital market investors, Hart and Jaffee (1972) constructed a portfolio model of a bank based on the mean-variance framework that does not only incorporates the assets, but also the liabilities of the bank. It assumes that banks trade assets and liabilities on the capital market and are price-takers. The bank optimizes its portfolio consisting of both assets and liabilities to maximize a given utility function defined over the mean and standard deviation of the portfolio payoff. Bank-specific elements are added by imposing a capital restriction in the form of a certain percentage rate that needs to be invested in cash as well as by imposing a short selling restriction. The equity of the bank is treated like any other liability, implying that the net present value of equity is zero or at least independent of the bank's decision-making. Furthermore, the model assumes that a risk-free security does not exist.

The main results of this model are that the efficient frontier is linear in the mean-standard deviation space and that the efficient portfolios are identical up to a non-negative scalar multiplication. This is because a proportional increase in all assets and liabilities of the portfolio results in a proportional increase in both the expected portfolio payoff and the standard deviation of the portfolio payoff. So the portfolio to be leveraged is the one that maximizes the ratio of the expected portfolio return to the standard deviation of the portfolio return subject to the constraints. This ratio is called the expected return-to-risk ratio

$$ERR = \frac{\mathbb{E}(r)}{\sigma(r)} \tag{4.28}$$

The above setting can also be used to found further expected return-to-risk ratios as long as the risk measures are positively homogeneous of the first degree.

However, this approach is quite problematic. First of all, banks are not price-takers and projects normally have a positive net present value. Thus, equity cannot be treated like other liabilities since its value is sensitive to the portfolio decision of the bank. Moreover, instead of assuming an utility function over the mean and variance of the portfolio payoff, the maximization of the market value of equity would be a more appropriate assumption. Furthermore, the approach fails to map costs that are related to the choice of the asset portfolio and leverage. In particular, these are financing costs, which usually increase in the risk of the asset portfolio and leverage. Finally, the approach neglects the specific payout structure of shareholders, which is usually characterized by their limited liability. In summary, it can be stated that expected return-to-risk ratios are not well founded.

5

Effects of Risk-Taking in Commercial Banks

In the previous chapter, reward-to-risk ratios are derived from capital market models, presuming that the used risk measure sufficiently represents the perceived risk of investors. The derived reward-to-risk ratios represent optimal risk-return trade-offs of individual investors optimizing their portfolios by trading on a perfect capital market.

However, it is questionable whether the derived reward-to-risk ratios are also suited for valuating a bank's loan portfolio. First of all, this is because risk measures that are relevant to capital market investors may not sufficiently represent the risks of banks, e.g., the insolvency risk of banks. Furthermore, banks do not maximize a certain utility function defined over the mean and a risk measure. It is rather plausible to assume that they optimize shareholder value defined as market value of equity. So a bank's optimal risk-return trade-off depends on the market price of risk as well as on further effects of risk-taking on shareholder value. Therefore, profitability measures suitable for loan portfolios may differ from the ones derived from capital market models, even if the capital market models are based on bank-specific risk measures.

There is thus a need to derive optimal risk-return trade-offs of commercial banks, which is done in Chapter 6 and 7. As the basis for this derivation, the effects of risk-taking on shareholder value in commercial banks are analyzed in this chapter.

5.1 The Neoclassical Finance Theory

There is a widespread belief among bankers that risk management increases shareholder value. Although this belief is quite appealing, it contradicts at the very least the results of the neoclassical finance theory. This theory's world is characterized by no bankruptcy costs, transaction costs, information asymmetries, and taxes, and an arbitrage-free and complete capital market in which both equity and debt of a firm are traded. Furthermore, it is assumed that there are no transactions leading to a transfer of value from debtholders to shareholders or from new to old shareholders. In such a world, the transactions of a firm that change its risk structure yet have a net present value of zero do not increase shareholder value, as investors have access to the capital market in which they can replicate or reverse all of the risk management transactions of the firm for free. Thus, although risk averse individuals should manage risk, risk management at the corporate level is irrelevant as long as risk management transactions have a net present value of zero.[1]

Nevertheless, even in the neoclassical world, this does not mean that firms should not care about risk: They still have to consider risk when making investment decisions because shareholders demand a risk premium for systematic risk, as discussed in Chapter 3. What firms have to do is to decide whether a project increases shareholder value by calculating the expected contribution to the payoff of shareholders discounted with the market discount factor and comparing it with the invested capital.

Even today, negative effects of risk-taking on shareholder value are nearly exclusively associated with the systematic part of the risk. However, the fact that the setting of the neoclassical finance theory is not realistic and sufficient to reason about risk management reveals the existence of banks: Under the neoclassical assumptions, there would not be any need for banks, since money could be lent or invested on the perfect capital market itself.[2] However, the proper meaning of the Modigliani and Miller irrelevance proposition is not the description of optimal firm behavior, but rather the detection of assumptions leading

[1] See Modigliani and Miller (1958) and Stiglitz (1969). For a detailed description of assumptions leading to the irrelavance proposition, see Rubinstein (2003).

[2] See Fama (1980).

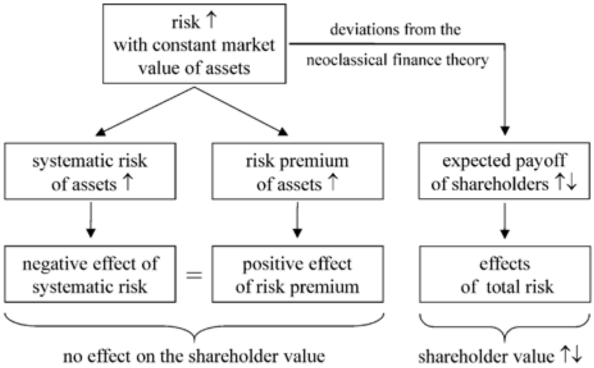

Figure 5.1. Influence of risk-taking on shareholder value.

to the irrelevance of the capital structure and the risk management. Deviations from these assumptions were thus the starting point of analyzing risk management motives.[3]

The focus of the following sections is to identify the effects of risk-taking on shareholder value arising additionally to the described negative effect of taking systematic risk. Positive effects of risk-taking on shareholder value are called benefits of risk-taking. Negative effects of risk-taking on shareholder value are called costs of risk-taking. Benefits and costs of risk-taking, which are related to the date of the decision-making, are distinguished here from gains and harms in the future, which risk-taking can provoke.

As Figure 5.1 illustrates, risk-taking can affect shareholder value not only due to systematic risk, but also by influencing the expected payoff of shareholders. Such effects are searched for in the following starting from the most important risk management motives for commercial banks.

5.2 Benefits of Risk-Taking

Benefits of risk-taking arise if the volatility of internal wealth increases the expected payoff of shareholders. When the payoff of shareholders is a continuous function of internal wealth, the condition for benefits of

[3] For an overview of motives for risk management see Allen and Santomero (1998), pp. 1474-1478, Bartram (2000), and Schroek (2002), pp. 72-128.

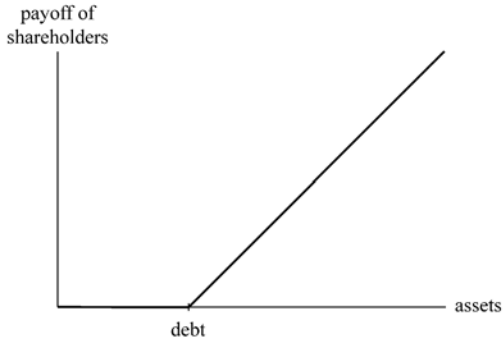

Figure 5.2. Payoff of shareholders with limited liability.

risk-taking is equivalent to the condition that the payoff of shareholders rises convexly in internal wealth, at least in some intervals. This is due to Jensen's inequality, which states that if f is a convex function of the random variable x, the following inequality holds:

$$\mathbb{E}[f(\mathbb{E}(x))] = f(\mathbb{E}(x)) \leq \mathbb{E}[f(x)]. \tag{5.1}$$

Thus, for f being the payoff function of shareholders and x being the internal wealth, it holds that an expected payoff of internal wealth with low volatility - to the extreme that the volatility is zero leading to an expected payoff of $f(\mathbb{E}(x))$ - is smaller than an expected payoff of a more volatile wealth $\mathbb{E}[f(x)]$.

Such a convex relationship emerges from the limited liability of shareholders. In this case, the payoff of shareholders is a linear function of the asset value when asset values are higher than the debt value. However, if the debt value is higher than the asset value, the limited liability will become effective and the payoff of shareholders is zero. This makes the payoff of shareholders a convex function of the firm's asset value (see Figure 5.2).

The benefit of risk-taking due to the limited liability of shareholders can easily be shown, assuming that a firm can decide between two portfolios and its shareholders have limited liability and are risk neutral. The payoffs of the portfolios are the firm's final assets, and both portfolios have two possible equiprobable payoffs. The first one pays out either -50 or 50, the second one with the higher volatility pays out either -100 or 100. So the expected payoff of shareholders when investing in the first portfolio is 25 and the expected payoff of shareholders when investing in the second portfolio is 50, although the expected payoff of both portfolios is zero. Therefore, ceteris paribus, shareholders benefit from higher asset risk.

Shareholders with limited liability especially benefit from higher asset risk if firms are highly debt-financed and have fixed financing conditions. This is because the residual claims of shareholders have the payoff structure of a call option on the assets of the firm with the debt value being the exercise price, and the value of this call option increases in volatility, in particular, if the exercise price is high.[4]

For banks, extending risk is profitable if they are completely financed through insured deposits, and the insurance premium only depends on the volume of deposits. In this case, depositors do not care about the bank's risk-taking, and a risk increase leads to a value transfer from the deposit insurance to shareholders. Again, option price theory can be used as the explanatory method. The payoff structure of the deposit insurance is identical to that of a put option on the bank's assets with the promised payment to depositors being the exercise price. Thus, increasing the volatility of the bank's assets raises the value of the put option, from which shareholders benefit without paying for it, since the rate of the insurance premium is flat.[5]

5.3 Costs of Risk-Taking

Costs of risk-taking arise if the volatility of internal wealth decreases the expected payoff of shareholders. Again assuming that the payoff of shareholders is a continuous function of internal wealth, this is equivalent to the condition that in some intervals the payoff of shareholders is a concave function of internal wealth. This goes along with the Jensen's inequality of f being a positive concave function of the random variable x:

$$\mathbb{E}[f(\mathbb{E}(x))] = f(\mathbb{E}(x)) \geq \mathbb{E}[f(x)]. \tag{5.2}$$

It states that the expected payoff is negatively influenced by the volatility of internal wealth if the payoff of shareholders is a concave function of internal wealth.

[4] See Merton (1974) and Jensen and Meckling (1976).

[5] See Merton (1977).

In the following, different risk management motives are presented, and it is analyzed which of them are based on a concave relation between internal wealth and the payoff of shareholders, thus leading to costs of risk-taking.

5.3.1 Agency Costs

Agency costs have their origin in different interests of the firm's stakeholders. They arise in contractual relationships if one party can act against the interests of its counterparty. The most important agency costs relevant to risk management stem from the relationship between shareholders and debtholders as well as between shareholders and managers. They are analyzed in the following.

Shareholders versus Debtholders

Debtholders face two major problems that both arise from fixed financing conditions and the fact that shareholders are residual claimants: underinvestment and asset substitution.

The underinvestment problem describes the fact that it can be optimal for shareholders to reject risk-reducing actions that are in favor of debtholders even if they have a positive net present value.[6] This is shown in the following example:

Assume that a firm with risk neutral investors having limited liability has debt of 100 and that at the end of the period its asset value is either 50 or 150 with the same probability. The firm has the possibility of investing in a project with positive net present value, which hedges a part of the asset risk such that the asset value at the end of the period is either 100 or 130 with the same probability. From the perspective of the entire firm and of debtholders, the firm should realize this new project. In this case, the value of the firm would increase from 100 to 115 and the expected payoff of debtholders would increase from 75 to 100. However, a realization of this project would lead to a decrease in the expected payoff of shareholders from 25 to 15. So shareholders will reject this project.

[6] See Myers (1977).

But even more problematic for debtholders than this underinvestment is the asset substitution problem that can arise because, as described above, shareholders benefit from higher asset risk after financing conditions are fixed and are thus likely to increase risk by investing in risky projects.[7] However, an increase in asset risk is done at the expense of debtholders since it leads to a transfer of wealth from debtholders to shareholders, as could be shown in an example similar to the one above.

If shareholders do not commit to forbear from activities done at the expense of debtholders, debtholders will anticipate the underinvestment and asset substitution problem. As a first reaction, this leads to higher financing costs of the bank. However, it is problematic that the higher financing costs intensify the underinvestment and asset substitution problem because the higher interest payments make the option of going bankrupt more valuable for shareholders with limited liability. The anticipation of the increased option value by debtholders additionally increases the bank's interest rate on debt. Thus, should a bank not be able to commit to decision-making that is not at the expense of debtholders, the bank has both higher financing costs and higher risk-taking than it would be pareto-optimal.[8]

Therefore, it is in the interest of shareholders to avoid these agency costs resulting from underinvestment and asset substitution by credibly committing not to increase risk, for example, by specifying debt covenants. The fact that banks usually refinance themselves more than once a month and thus expose themselves to market discipline, as described in the next subsection, can be interpreted as another way of using self-commitment to not increase risk and thus as a possibility of avoiding these agency costs.

However, although it is shown that such agency costs lead to a strong risk management motive, they do not lead to costs of risk-taking in the way they are defined above. This is because these agency costs can only be reduced by credibly committing to a certain risk level and not by taking less risk.

[7] See Jensen and Meckling (1976).

[8] See Blum (2002).

Shareholders versus Managers

It is further shown that managerial motives can influence corporate risk management. With a significant cost of hedging on their own account, a full hedge is optimal for risk averse managers who participate linearly in the value of the firm.[9] However, if managers have convex incentive contracts, this hedging decision may be altered according to an argumentation analogous to explaining the condition for the benefits of risk-taking for shareholders.[10] Furthermore, it is shown that where there is asymmetric information, managers may also have an incentive to fully hedge if they intend to signal the quality of the management.[11]

Whether or not managerial motives reduce shareholder value depends on whether the managerial interests diverge from the interests of shareholders. But the shareholder value loss caused by such conflicts is not a cost of risk-taking since it does not directly arise from the firm's risk-taking. However, the contracting cost that is necessary to ensure that risk averse managers choose a desired high-risk strategy can be considered to be a cost of risk-taking.

5.3.2 Market Discipline

It is rational for debtholders to anticipate not only agency costs, but also bankruptcy and liquidation costs and the behavior of other debtholders. This leads to risk management motives, which are referred to as market discipline. The term market discipline actually includes the two aspects market monitoring and market influence.[12] Market monitoring describes the fact that investors observe changes inside a firm, in particular those changes that are relevant to the riskiness of their claims, and the information gathered has an effect on their behavior. Market influence suggests that the resulting behavior of investors influences the firm's risk-taking, thus causing a risk management motive.

In the following, two risk management motives resulting from market monitoring are presented. The first one is based on the threat of a bank run and is thus caused by the disciplining by depositors. The second one is based on the disciplining by uninsured debtholders.

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[9] See Smith and Stulz (1985), p. 399.

[10] See Smith and Stulz (1985), p. 402, and Stulz (1996), pp. 17-19.

[11] See DeMarzo and Duffie (1995).

[12] See Bliss and Flannery (2002), pp. 362-363.

Threat of a Bank Run

The disciplining by depositors goes along with the danger of a bank run triggered by financial distress, the occurrence of which is more likely if a bank is exposed to high risk. A bank run is a scenario in which depositors all withdraw their capital at the same time because they fear that the unrestricted repayment of their deposits is not guaranteed anymore.[13] They thus anticipate that depositors who are waiting too long might miss out because of the sequential service constraint on deposits.

Since the simultaneous payout of the depositors is linked with high liquidation costs, it follows that the threshold of the asset value from which a bank run is rational is above the value of deposits. So a bank run causes not only liquidation costs making the payoff of shareholders a concave function of the asset value and leading to costs of risk-taking. The bank might even be forced into bankruptcy, which is costly particularly in the case of high charter value, although the bank's asset value is above the debt value.[14]

However, this reasoning has a major drawback: If deposits are insured to a high degree - as holds true for most countries - and there are no significant costs depositors have to bear if a bank goes bankrupt, it cannot be argued anymore that bank runs are rationally triggered by information about the value of the bank's assets. Instead, bank runs become highly irrational. So in real life, the linkage between financial distress and a bank run is weaker than this theory predicts.

Disciplining by Uninsured Debtholders

Especially uninsured subordinated debtholders such as bondholders discipline the risk-taking of banks by worsening their refinancing possibilities and conditions.[15] The precondition for this behavior is that debtholders are sufficiently informed about the bank's risk-taking. The disciplining can be traced back to the chance of a bank going bankrupt as well as to direct and indirect costs in connection with a bank's

[13] See Diamond and Dybvig (1983) and Jacklin and Bhattacharya (1988).

[14] See Ryser (2003), pp. 70-92, and Bauer and Ryser (2004).

[15] See Smith and Stulz (1985), pp. 395-398, Shapiro and Titman (1986), pp. 219-220, Stulz (1996), pp. 16-17.

bankruptcy, which are anticipated by debtholders. Direct costs that debtholders face in case of a bankruptcy are legal, administration, and reorganization costs. Indirect costs in case of insolvency are induced by the necessity of selling illiquid assets.[16] With equity being a scarce resource that makes debt-financing necessary, the costs of risk-taking occur in terms of higher financing costs as well as opportunity costs if a bank cannot finance all profitable projects. Furthermore, lower debt is disadvantageous for banks if it goes along with a smaller tax shield.[17]

The fact that equity is a scarce resource can be explained by the pecking order theory.[18] Assuming costly state verification, the issuance of new shares from outsiders might be costly if investors believe that the firm will raise new capital only if the market price of equity is too high. This leads to a reduction in share price, when issuing new shares. The reaction of shareholders entails the pecking order that corporations generally prefer internal financing over debt-financing and debt-financing over equity-financing.

While banks experience the disciplining by depositors permanently as a threat, they notice the disciplining by uninsured debtholders only at times when they are issuing new debt. Thus, it could be concluded that the disciplining by uninsured debtholders only temporarily leads to a risk management motive. However, in real life, the disciplining by uninsured debtholders is nearly continuously present over time since banks place bonds several times a year.

There is a solid empirical support for the disciplining by uninsured debtholders. Studies find a positive relation between bank risk and spreads on uninsured deposits,[19] a positive relation between bank risk and spreads on subordinated uninsured debt,[20] and a negative relation between bank risk and uninsured deposits.[21]

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[16] See Smith and Stulz (1985), pp. 395-396, Stulz (1996), p. 12.

[17] See Leland (1994).

[18] See Myers and Majluf (1984).

[19] See Ellis and Flannery (1992), Cook and Spellmann (1994), Berger (1995), Park and Peristiani (1998), Billet, Garfinkel, and O'Neal (1998), and Jordan (2000).

[20] See Flannery and Sorescu (1996), DeYoung, Flannery, Lang, and Sorescu (2001), Morgan and Stiroh (2001), Jagtiani, Kaufman, and Lemieux (2002), Covitz, Hancock, and Kwast (2004).

[21] See Goldberg and Hudgins (1996), Park and Peristiani (1998), Jordan (2000), Goldberg and Hudgins (2002), McDill and Maechler (2003).

5.3.3 Capital Market Imperfections

In the discussion on the disciplining by uninsured debtholders, it was already argued that equity-financing is costly due to the adverse selection problem of new shareholders. A further related risk management motive can be derived from the capital market imperfection. Consistent with the pecking order theory, it is assumed that costs of external funding rise convexly in the amount raised.[22] With increasing marginal financing costs, a risk management motive can be evolved that arises from a possible higher financing cost and missed business opportunities provoked by a lack of internal financing. Thus, while the risk-management motive caused by the disciplining by uninsured debtholders stems from debtholders anticipating the default probability and the bankruptcy costs, this risk-management motive arises from the firm itself anticipating the harm that risk-taking can provoke.

The risk management motive can be illustrated in a two-period model.[23] At date one, the firm can decide on activities such as hedging that influence the volatility of internal wealth w at date two. At date two, internal wealth w is needed to invest an amount I in a profitable project with payoff $f(I)$ with $f'(I) > 0$ and $f''(I) < 0$ in date three. The project can also be financed through outside investors raising an amount e:

$$I = e + w. \tag{5.3}$$

However, raising external funds is costly because there are financing costs $C(e)$ that increase in the amount raised ($C'(e) > 0$). The discount rate is assumed to be zero.

The focus of the following calculations is assessing whether volatile wealth w at date two leads to costs of risk-taking. As previously described, this is the case if the payoff of shareholders at date three is a concave function of wealth w.

With given wealth w at date two, it is assumed that maximizing the shareholders' payoff is equivalent to maximizing the profit P, which is

$$P(I) = \max_{I}[f(I) - I - C(e(I))]. \tag{5.4}$$

Its first-order condition with respect to I is

[22] See Myers and Majluf (1984).

[23] See Froot, Scharfstein, and Stein (1993).

$$f_I(I) - 1 - C_e(e(I)) = 0, \tag{5.5}$$

using $\frac{de}{dI} = 1$, since $e = I - w$. The first-order condition is solved for $I = I^*$, and it is assumed that the second-order condition for a maximum holds.

Now P is evaluated for any wealth w. In this case, the profit is

$$P(w, I^*(w)) = f(I^*) - I^* - C(e^*) \tag{5.6}$$

with

$$e^* = I^*(w) - w. \tag{5.7}$$

To assess whether P is a concave function of w, the second derivative of P with respect to w at $I = I^*$ needs to be calculated. It turns out (see appendix A.4) that

$$P_{ww}(w, I^*(w)) = \frac{dI^*}{dw} f_{II}(I^*) \tag{5.8}$$

with

$$\frac{dI^*}{dw} = \frac{-C_{ee}(e^*)}{f_{II}(I^*) - C_{ee}(e^*)}. \tag{5.9}$$

Since the marginal return on an investment $f_{II}(I^*)$ is negative by assumption, P_{ww} is negative if internal wealth has a positive impact on the optimal level of the investment ($\frac{dI^*}{dw} > 0$). As shown by equation (5.9), this holds for $C_{ee}(e^*)$ being positive, which implies a convex cost of external financing. Therefore, P is a concave function of internal wealth and costs of risk-taking exist under the assumption of an increasing marginal financing cost. Thus, the model shows that with decreasing marginal returns on investments and an increasing marginal financing cost, costs of risk-taking arise since a financial distress of the firm leads to a higher financing cost and missed business opportunities.

5.3.4 Taxes

Another risk management motive arises from a convex tax rate.[24] The tax rate must not be continuously progressive here. It is satisfactory if the tax rate is convex in a certain interval, which holds in case of limited loss carry forward. With a convex tax rate, income volatility increases the expected tax payment, therefore decreasing both the profit after

[24] See Smith and Stulz (1985), p. 392.

taxes and the payoff of shareholders. This leads to a risk management motive and to a cost of risk-taking.

To show this, assume that a firm decides between two portfolios based on the profit after taxes. The first portfolio pays out either 60 or 140 with the same probability. The second portfolio has a risk-free return of 100. Furthermore, assume that the firm has fully deductible expenses of 80 and the corporate tax rate is 30%. However, assume that a loss carry forward is not possible. So if the firm chooses the first portfolio, it can expect a profit after taxes of 11. But choosing the risk-free portfolio leads to a profit after taxes of 14, although in both cases the profit before taxes is 20. So the firm chooses the risk-free portfolio.

However, although costs of risk-taking caused by a convex tax rate are very plausible, the significance of this risk management motive should not be overestimated since firms usually have the option of earnings management and this risk management motive is not empirically significant.[25]

5.3.5 Regulatory Capital Requirements

Regulations affect the risk-taking of banks in several ways. Regulations raise the entrance barrier by defining the requirements for establishing banks. This increases the charter value and imposes a bankruptcy cost for shareholders. Therefore, banks try to avoid bankruptcy and fear bank runs, as already discussed. Furthermore, regulations directly intervene in asset-liability management to prohibit excessive risk-taking. However, the most important instruments for influencing a bank's risk-taking are regulatory capital requirements targeting the solvency of banks. Whether or not they are effective in limiting the risk-taking of banks is widely discussed in the literature.[26]

Koehn and Santomero (1980) and Kim and Santomero (1988) discuss capital requirements in a framework adapted from Pyle (1971) and Hart and Jaffee (1972) assuming that the bank has a given utility function and optimizes a portfolio containing both the assets and liabilities

[25] For international empirical evidence on earnings management, see Leuz, Nanda, and Wysocki (2003). Graham and Rogers (2002) find empirical evidence that firms hedge to increase debt capacity and interest rate deduction, but no evidence that firms hedge to reduce expected tax liability when their tax functions are convex.

[26] For an overview, see Palia and Porter (2003).

of the bank, as described in Section 4.4. They find that using a simple equity-to-assets ratio constraint leads to an ineffective regulation in the sense that banks with a high risk aversion become safer, while banks with a low risk aversion increase risk. However, it is possible to derive optimal weights on assets eliminating those adverse effects of this constraint. Using these weights, the capital requirement is effective by limiting the insolvency probability of the bank.

Rochet (1992) implements the limited liability of shareholders and constant equity in the bank model above. He finds that even with optimal weights on assets a capital ratio does not prevent banks from excessive risk-taking and does not induce the bank to make an efficient portfolio choice. He also discusses banks that maximize the market value of equity, assuming a complete capital market, fully insured depositors and a deposit insurance that charges a premium depending only on the volume of deposits. He finds that capital requirements are a very inefficient tool for limiting the risk-taking of banks in this case, too. However, he shows that a fair insurance premium can solve this problem.

Finally, assuming a value-maximizing bank financed through insured deposits and investments with positive net present value, John, Saunders, and Senbet (2000) show that the effectiveness of capital requirements depends on the opportunity set of asset investments.

Thus, it can be stated that regulatory capital requirements are not always effective. Furthermore, it has to be noted that the weights on assets given by the regulations are far from the theoretically optimal ones. The Basel Capital Accord of 1988 (Basel I), on which current capital requirements are based, defines weights on assets as primarily depending on asset classes and not on risk.[27] Thus, a lower portfolio risk does not necessarily lead to a less restrictive capital requirement. In the near future, regulatory capital requirements will be based on the New Basel Capital Accord (Basel II).[28] Basel II allows different approaches to be applied to determine the weights on assets. The most sophisticated one is the internal ratings-based approach. It defines weights for different asset classes as a function of single asset risk. However, the effect of these weights has not yet been sufficiently analyzed.

[27] See Basel Commitee on Banking Supervision (1988).
[28] See Basel Commitee on Banking Supervision (2004).

It is difficult to assess whether the capital requirements of Basel I and Basel II lead to costs of risk-taking. At the least when focusing on one single asset class, it can be stated that Basel I does not lead to a cost of risk-taking because capital requirements do not change. However, since Basel I regulates the asset volume of banks, it still can affect the risk-taking of banks by influencing the economic costs of risk-taking. In contrast, the internal ratings-based approach of Basel II fulfills the condition for directly causing costs of risk-taking since the capital requirement is closely related to the bank's risk-taking. However, whether it leads to costs of risk-taking depends on whether the capital requirements are restrictive, which has not yet been sufficiently assessed.

6

Risk-Return Trade-Offs for Commercial Banks

In this and the following chapter, optimal risk-return trade-offs of commercial banks are derived based on a selection of effects of risk-taking on shareholder value. Later on, the optimal risk-return trade-offs are compared to the risk-return trade-offs of the reward-to-risk ratios presented in Chapter 4 to analyze their suitability for loan portfolios.

6.1 Approach

To derive optimal risk-return trade-offs of commercial banks, first of all, the effects of risk-taking on shareholder value need to be selected. The analysis sticks to the risk management motives which the New Basel Capital Accord already trusts: The presented models rest primarily upon costs of risk-taking caused by market discipline in the form of the disciplining by uninsured debtholders. This is the third pillar of Basel II. Furthermore, they consider the costs of risk-taking caused by capital regulation corresponding to the first pillar of Basel II. In addition, the required risk premium of shareholders is taken into account. Benefits of risk-taking are considered by assuming that shareholders have limited liability.

To model market discipline, it is necessary to abandon the disconnected view of a commercial bank's lending and borrowing activities. The models assume instead that the volume of a bank's loan portfolio is determined by the debtor side of the balance sheet. Here the models relax the premise of the current literature on economic capital that a bank should preserve a certain solvency level. They assume instead that debtholders only invest in a bank if this investment is at least as profitable as alternative investments.[1] So with symmetric information about the credit standing of a bank, a risk increase of the bank's loan portfolio has the following effects: It increases the insolvency probability of the bank and thereby worsens its credit standing. In an equilibrium, this leads to lower debt or a higher interest rate on debt.

The models do not assume that the bank trades assets on the capital market in the role of a price-taker and optimizes its portfolios based on a given utility function, as in Hart and Jaffee (1972). They assume instead that the bank optimizes its market price of equity. The main difference to Rochet (1992) and John, Saunders, and Senbet (2000) is that costs of risk-taking are explicitly considered.

The models are built upon two capital market imperfections. First, they assume that equity-financing is too costly to be profitable. The second one is that insolvency causes bankruptcy costs. Furthermore, the models assume that debtholders are completely informed about the bank's credit standing and the bank does not undertake any risk-shifting activities. This should not be understood that it is possible for debtholders to observe and discipline all actions of the bank. A plausible explanation is rather that it is in the interest of the bank to signal its credit standing and to commit to not increasing risk after the bank's financing conditions have been fixed to avoid expensive agency conflicts.

6.2 Assumptions

Consider a commercial bank that starts without assets and liabilities, but with a certain amount of equity. The management acts in the interest of its shareholders having limited liability. The bank can raise debt and engage in the mortgage loan business. It can have a loan portfolio with a positive net present value. The bank is only exposed to credit risk. Operating costs are neglected. The bank management can also pay back the equity to shareholders if this is in their best interest. However, equity-financing is assumed to be too costly to be profitable.

[1] A similar market equilibrium is used in Gründl and Schmeiser (2002), who discuss the allocation of risk capital in insurance companies.

Furthermore, the bank management commits to not increasing risk after financing conditions have been fixed.

The model assumes that debtholders are uninsured and competitive. Furthermore, debtholders have complete information about the bank's credit standing. Both shareholders and debtholders have the possibility of investing in a risk-free alternative. Transaction costs for investing in the bank are not considered.

If the bank is insolvent, it goes bankrupt causing bankruptcy costs. Bankruptcy costs are assumed to be proportional to the remaining assets, which are equal to the payoff of the loan portfolio.

The model has two dates: At time $t = 0$, the bank management specifies the risk-return profile of the loan portfolio and sets the interest rate on debt r_D. Debtholders have the possibility of investing in the bank afterwards. Subsequently, the total capital of the bank TC consisting of equity E and debt D is invested in a loan portfolio with the specified risk-return profile. Here the bank does not have any limitations concerning the portfolio volume.

At time $t = 1$, the portfolio yields a stochastic return r, and the following stochastic free cash flow Y results, with τ being the linear corporate tax rate:

$$\begin{aligned} Y &= (E+D)(1+r) - \tau((E+D)r - Dr_D) - D(1+r_D) \\ &= (E+D)(1+r(1-\tau)) + \tau D r_D - D(1+r_D). \end{aligned} \tag{6.1}$$

Debtholders obtain their invested capital and the interest rate on it afterwards, in case the bank is solvent. Otherwise they obtain the remaining assets after bankruptcy costs determined by the percentage rate of bankruptcy costs b, following a pro rata allocation. Therefore, the stochastic payoff of debtholders X_D is

$$X_D = \begin{cases} D(1+r_D) & \text{if } Y \geq 0; \\ (1-b)(Y + D(1+r_D)) & \text{else.} \end{cases} \tag{6.2}$$

If the bank is solvent, shareholders have a claim to a pro rata distribution of the bank's free cash flow. If the bank defaults, their return is zero. The stochastic payoff of shareholders X_E is thus

$$X_E = \max\{Y, 0\}. \tag{6.3}$$

The density function of the portfolio return r is determined by the properties of the lending business. The portfolio return is bounded by the (average) interest rate on loans r_L. The volatility of the portfolio return stems from the stochastic default rate d defined on a value basis. So the stochastic portfolio return is

$$r = (1 + r_L)(1 - d) - 1. \tag{6.4}$$

For the default rate d, the two-parametric beta-distribution is assumed. Consequently, the density function of the default rate d is completely specified by the expected default rate μ_d and the standard deviation of the default rate σ_d. Therefore, the density function of the portfolio return r is completely specified by the following three parameters: the interest rate on loans r_L, the expected default rate μ_d, and the standard deviation of the default rate σ_d:

$$f(r) = f(r; r_L, \mu_d, \sigma_d). \tag{6.5}$$

In the following, a loan portfolio with a large number of loans of the same size having equal probabilities of default PD and equal and constant default correlations $\rho > 0$ is considered. Loans are either paid back completely or paid back with an equal and constant loss given default LGD. Then, as shown in appendix B.1, the interest rate on loans r_L, the expected default rate μ_d, and the standard deviation of the default rate σ_d are completely specified by the first two statistic moments of the portfolio return. So when the bank management decides on the risk-return profile of the loan portfolio, it is sufficient to determine μ_r and σ_r, since

$$f(r) = f(r; r_L(\mu_r, \sigma_r), \mu_d(\mu_r, \sigma_r), \sigma_d(\mu_r, \sigma_r)) = f(r; \mu_r, \sigma_r). \tag{6.6}$$

The density function of the portfolio return is specified in appendix B.2 and presented in Figure 6.1. Appendix B.2 also specifies the density function of the bank's free cash flow.

Both shareholders and debtholders valuate their stochastic investment payoff X based on the following valuation function:

$$P(X) = \frac{1}{1 + r_f}(\mathbb{E}(X) - \delta\sigma(X)). \tag{6.7}$$

$\mathbb{E}(X)$ denotes the expected payoff X, $\sigma(X)$ denotes the standard deviation of the payoff X, and δ is a non-negative parameter describing

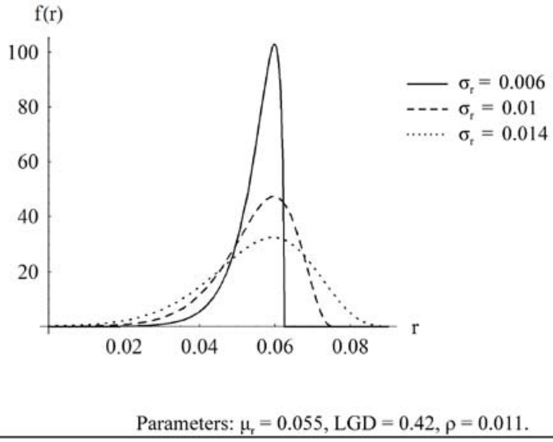

Figure 6.1. Density functions of the loan portfolio return r for a constant expected portfolio return μ_r and different standard deviations of the portfolio return σ_r.

the risk aversion of investors. r_f is the yield of the risk-free investment opportunity.

The simplifying valuation function P is abstracted from portfolio considerations on investor level and has the comfortable property of being positively homogeneous of the first degree in the payoffs.[2] Thus, P can also be interpreted as the pricing function of the capital market, which is consistent with the law of one price. Furthermore, under the simplifying assumption that the coefficient of correlation ρ_{X,r_M} between the payoff X and the return of the market r_M is constant and the same for shareholders and debtholders, consistency with the pricing function of the CAPM can even be established.[3] The risk aversion parameter δ can then be described as the Sharpe ratio of the capital market SR_M multiplied by the coefficient of correlation ρ_{X,r_M}:

$$\delta = \rho_{X,r_M} SR_M \tag{6.8}$$

with

[2] For a constant λ, it holds that $P(\lambda X) = \frac{1}{1+r_f}(\mathbb{E}(\lambda X) - \delta\sigma(\lambda X)) = \frac{1}{1+r_f}(\lambda\mathbb{E}(X) - \delta\lambda\sigma(X)) = \lambda\frac{1}{1+r_f}(\mathbb{E}(X) - \delta\sigma(X)) = \lambda P(X)$.

[3] The pricing function of the CAPM is $P(X) = \frac{1}{1+r_f}(\mathbb{E}(X) - \frac{\mathbb{E}(r_M)-r_f}{\sigma^2(r_M)} Cov(X, r_M))$. Assuming that $\rho_{X,r_M} = \frac{Cov(X,r_M)}{\sigma(r_M)\sigma(X)} = const.$, it results: $P(X) = \frac{1}{1+r_f}(\mathbb{E}(X) - \delta\sigma(X))$ with $\delta = \frac{\mathbb{E}(r_M)-r_f}{\sigma(r_M)}\rho_{X,r_M}$.

$$SR_M = \frac{\mathbb{E}(r_M) - r_f}{\sigma(r_M)}.$$

From the valuation function and the alternative investment opportunity, the following participation conditions result (see appendix B.3): For shareholders, the participation condition is that the net present value of equity, called equity value EV, is positive:

$$EV = P(X_E(\mu_r, \sigma_r, r_D, E, D)) - E \geq 0. \tag{6.9}$$

Similarly, for debtholders, the net present value of debt, called debt value DV, needs to be positive:

$$DV = P(X_D(\mu_r, \sigma_r, r_D, E, D)) - D \geq 0. \tag{6.10}$$

Equity value and debt value are specified in appendix B.4. In the following, the equity value EV is used as a substitute for shareholder value.

6.3 Solving Method and Parameter Values

The model tries to consider the most relevant effects of a bank's risk-return profile on its shareholder value. This goes along with a complex framework that does not always allow solutions to be derived by just using algebra alone. Therefore, in most cases, numerical solving methods are employed. The following parameter values are used here:

The percentage rate of bankruptcy costs b is set to 20%. This is lower than the bankruptcy costs of 30.5% found in an early study of US bank failures occurring from 1985 to 1988.[4] However, the study also considers a discount for the time to final resolution, which is subsumed in the parameter LGD. Furthermore, it can be assumed that losses from early liquidation have decreased because the secondary loan market has become more developed.

The parameter loss given default LGD for loans granted by the bank is set to 42%. It is based on the economic loss given default presented in a study for JPMorgan Chase's real estate sector considering losses between 1982 and 1999.[5] The economic loss given default is the present

[4] See James (1991).

[5] See Araten, Jacobs, Jr., and Varshney (2004), p. 30.

value of cash losses with respect to the initial book value of a defaulted loan. Loans are also considered to be defaulted after they are 90 days past due principal or interest payment.

The default correlation between single loans ρ is set to 1.1%. This value is based on a study using data of 30-year fixed-rate residential loans that are 90 or more days delinquent from a US subprime lender between 1995 and 2001.[6] Data from the subprime lender are relevant to this model because its portfolio consists of loans with both low and high credit scores.

Determining the capital requirements of Basel II requires the nominal loss given default LGD_N, which is set to 30%.[7] Furthermore, the three year average of positive annual gross income of the business line retail banking is needed, which is assumed to be 15% of the bank's equity.

The risk aversion parameter δ is fixed by looking at the empirical correlation between the banking sector and the market portfolio (around 70%) and the market's historical Sharpe ratio (around 42%).[8] Given these figures, a risk aversion parameter δ of 29% seems adequate. However, in the first calculations this parameter is set to zero to show that the findings also hold for risk neutral investors.

The corporate tax rate τ is set to 35%. Without loss of generality, the yield of the risk-free bond r_f is set to 5% and equity E is fixed at 100.

In the following, several programs are solved which analyze the impact of the loan portfolio's risk-return profile in the range of $\mu_r \in [0.05, 0.065]$ and $\sigma_r \in [0.004, 0.02]$. Such a choice seems reasonable since it implies that the expected excess returns over the risk-free rate ranges from 0% to 1.5% and the expected loss of the loan portfolio ranges from 0.31% to 8.14% for $\mu_r = 5.5\%$.[9]

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[6] See Cowan and Cowan (2004), p. 759.

[7] See Araten, Jacobs, Jr., and Varshney (2004), p. 30.

[8] See Hodges, Taylor, and Yoder (1997), p. 79.

[9] The expected loss of the loan portfolio EL_P is
$EL_P = PD\ LGD = \frac{LGD\sqrt{\rho} - \sqrt{LGD^2\rho - 4\sigma_d^2(\mu_r, \sigma_r)}}{2\sqrt{\rho}}$ (see appendix B.1).

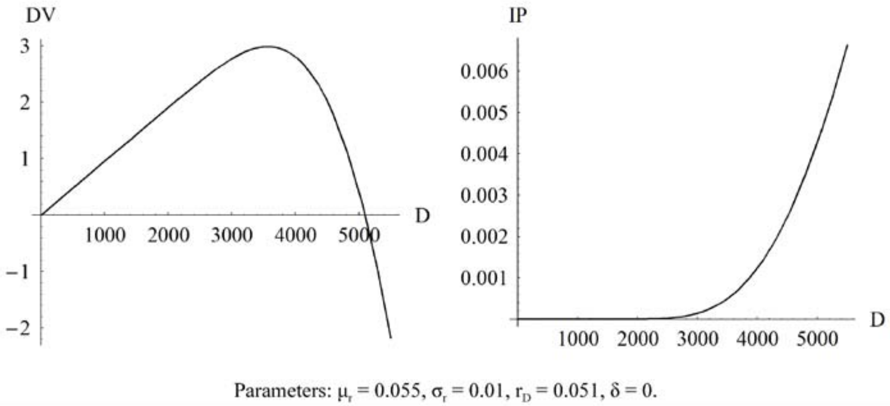


Figure 6.2. On the left: Debt value DV (net present value of debt) as a function of debt D for a given risk-return profile of the loan portfolio and a constant interest rate on debt, assuming risk neutral investors. On the right: The corresponding insolvency probability IP of the bank as a function of debt D.

6.4 Disciplining by Debtholders

Since the primary costs of risk-taking stem from the disciplining by uninsured debtholders in this model, it is first analyzed how the decision-making of the bank management influences the possibilities and conditions of debt-financing.

The interest rate on debt plays a peculiar role here: It can be shown that with a positive volatility of the portfolio return implying a positive insolvency probability of the bank, an interest rate on debt exceeding the risk-free rate is a necessary participation condition of debtholders (see appendix B.5). This is apparent since the return on debt is bounded by the interest rate on debt, but it is also negatively influenced by the insolvency probability of the bank.

First of all, the equilibrium that determines the investment volume of debtholders is identified. For this purpose, the debt value (see equation (6.10)) is drawn against debt with a given equity, interest rate on debt, and risk-return profile of the loan portfolio, assuming risk neutral investors (see Figure 6.2, on the left).

The debt value increases in debt at first due to the interest rate on debt being above the risk-free rate. However, the insolvency probability of the bank

$$IP = \int_{-\infty}^{0} f(Y)dY \tag{6.11}$$

also rises in debt as it has a negative effect on the debt value (see Figure 6.2, on the right). So the debt value descends from a certain debt level.

At equilibrium, debtholders realize a debt value of zero. This results from the competitive behavior of single debtholders investing in the bank as long as the debt value is non-negative. So the bank's debt D solves

$$P(X_D(\mu_r, \sigma_r, r_D, E, D)) - D = 0. \tag{6.12}$$

Equation (6.12) describes the debtholders' reaction to the bank's decision-making and equity. This reaction function is analyzed in the following.

First, the relation of debt to equity is evaluated. Under the given assumptions, it can be shown that the bank's debt is proportional to its equity (see appendix B.6). Consequently, a linear relation of equity to the bank's total capital also exists, i.e., with a given decision-making by the bank management, the equity-to-assets ratio is constant. This can be interpreted as a capital requirement of debtholders. Hence, according to this model, not only regulatory capital requirements, but also market discipline in the form of a required equity-to-assets ratio may restrict debt.

The required equity-to-assets ratio rises in the portfolio risk (see Figure 6.3). This is intuitive since a higher volatility of the portfolio return raises the bank's insolvency probability, thus reducing the debt value. For the participation condition of debtholders to still hold, the effect of an increased insolvency probability has to be balanced out by a higher proportion of equity serving as a risk buffer. Thus, debtholders require a higher equity-to-assets ratio for a higher portfolio risk.

In contrast, a higher expected portfolio return reduces the insolvency probability of the bank. Thus, an increase of the expected portfolio return reduces the required equity-to-assets ratio (see Figure 6.4).

A higher insolvency probability of the bank cannot only be compensated by a higher equity serving as a risk buffer, but also by raising the interest rate on debt. Thus, the bank's equity and the interest rate on debt are substitutes. Therefore, the equity-to-assets ratio decreases in the interest rates on debt (see Figure 6.5).

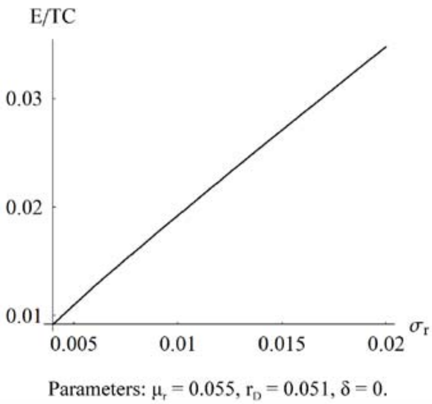


Figure 6.3. Endogenous capital requirement E/TC of debtholders as a function of the standard deviation of the portfolio return σ_r for a given expected portfolio return and a constant interest rate on debt, assuming risk neutral investors.

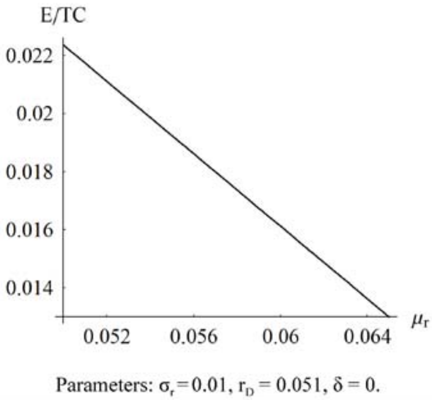


Figure 6.4. Endogenous capital requirement E/TC of debtholders as a function of the expected portfolio return μ_r for a given standard deviation of the portfolio return and a constant interest rate on debt, assuming risk neutral investors.

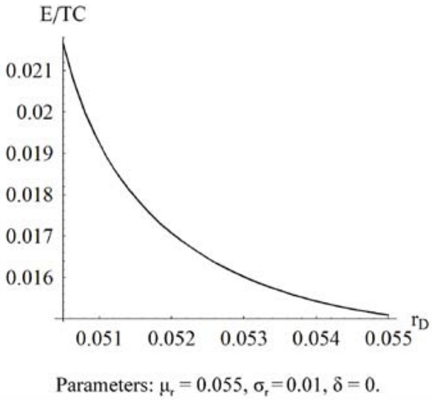


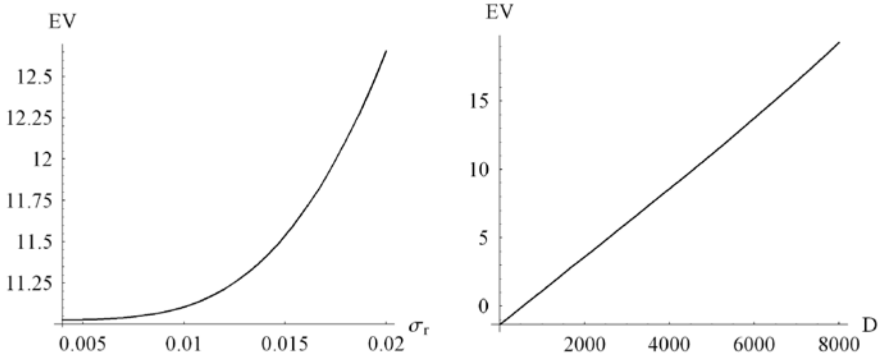
Figure 6.5. Endogenous capital requirement E/TC of debtholders as a function of the interest rate on debt r_D for a given risk-return profile of the loan portfolio, assuming risk neutral investors.

If equity is constant, which is assumed in this model, it follows from the required equity-to-assets ratio that debtholders react to the decision-making of the bank management by changing their investment volume. So the bank's debt decreases in the portfolio risk, increases in the expected portfolio return, and increases in the interest rate on debt.

6.5 Optimal Risk-Return Trade-Offs

After discussing the disciplining by debtholders, the objective is now to derive optimal risk-return trade-offs of the bank to deciding on the risk-return profile of the loan portfolio. Risk-return trade-offs are optimal if they indicate how much additional expected portfolio return is necessary to compensate the negative effect of marginally higher portfolio risk on shareholder value. So to determine optimal risk-return trade-offs, the equity value described by equation (6.9) needs to be evaluated as a function of the risk-return profile of the loan portfolio.

Analyzing the impact of a change in the portfolio's risk-return profile on the equity value is not straightforward because the equity value is also a function of debt, which itself depends on the decision-making of the bank management, as shown in the previous section. Thus, a change of the risk-return profile entails several effects:



Parameters: $\mu_r = 0.055$, $r_D = 0.051$, $\delta = 0$, on the left D/E = 50, on the right $\sigma_r = 0.01$.

Figure 6.6. On the left: Equity value EV (net present value of equity) as a function of the standard deviation of the portfolio return σ_r for a given expected portfolio return, constant debt, and constant interest rate on debt, assuming risk neutral investors. On the right: Equity value EV as a function of debt D for a given risk-return profile of the loan portfolio and constant interest rate on debt, assuming risk neutral investors.

With constant debt, the equity value increases in the volatility of the portfolio return because the resulting higher risk of a low performance is overcompensated by the new chance of a high performance for shareholders with limited liability (see Figure 6.6, on the left). This is called the direct effect of a volatility increase. However, there is also an indirect effect caused by the disciplining by debtholders: A volatility increase in the portfolio return leads to lower debt, which reduces the equity value since debt leverages the payoff of shareholders (see Figure 6.6, on the right). The bank has the possibility of counterbalancing the impact of higher portfolio risk on debt by raising the interest rate on debt. However, a higher interest rate on debt raises the financing cost, also reducing the equity value. The resulting negative effect of a volatility increase can be interpreted as an impact of market discipline on the equity value. For determining the overall impact of a volatility increase on the equity value, it has to be evaluated which effect dominates. In addition, the risk premium of shareholders has to be considered.

An increase in the expected return of the loan portfolio also has a direct and an indirect effect on the equity value. However, in contrast to the effects of a volatility increase, these effects enforce each other.

A higher expected portfolio return leads to a higher expected payoff of shareholders, which is valuated positively. In addition to this direct effect, an indirect effect exists because a higher expected portfolio return raises the debt capacity, as shown in the previous section, thus raising the equity value due to higher leverage, too.

Optimal risk-return trade-offs are determined using different model setups. The baseline model is a model without constraints. It is discussed in subsection 6.5.1. The baseline model is first evaluated with a constant interest rate on debt using program P-I. Optimal interest rates on debt are then analyzed using program P-II. The baseline model with an optimized interest rate on debt is evaluated using program P-1a.

Subsection 6.5.2 adds to the baseline program P-1a by introducing different regulatory constraints that are evaluated using program P-2a and P-3a. Finally, in subsection 6.5.3 the case of fixed portfolio volume is analyzed using program P-4a.

To continue concentrating on market discipline and to assure that the findings do not depend on the risk premium of shareholders, the calculations are made for risk neutral investors in this section. Effects of investors being risk averse are considered starting in the following section, which also presents the baseline model.

6.5.1 Baseline Model

Model with a Constant Interest Rate on Debt

Without constraints, the portfolio volume is restricted only by the amount that debtholders are willing to invest in the bank. With a constant interest rate on debt r_D and a constant equity E, the following program determines the equity value for different expected returns μ_r and standard deviations of the return σ_r subject to the debtholders' reaction function $\mathbf{R}$:

$$\textbf{P-I:} \begin{cases} EV(\mu_r, \sigma_r) = P(X_E(\mu_r, \sigma_r, r_D, E, D^*)) - E \\ \text{subject to:} \\ \mathbf{R} : P(X_D(\mu_r, \sigma_r, r_D, E, D^*)) - D^* = 0. \end{cases}$$

Figure 6.7 shows the resulting equity value and debt as functions of the portfolio's risk-return profile as well as isoquants of equity value, assuming risk neutral investors. Here and in the following, resulting

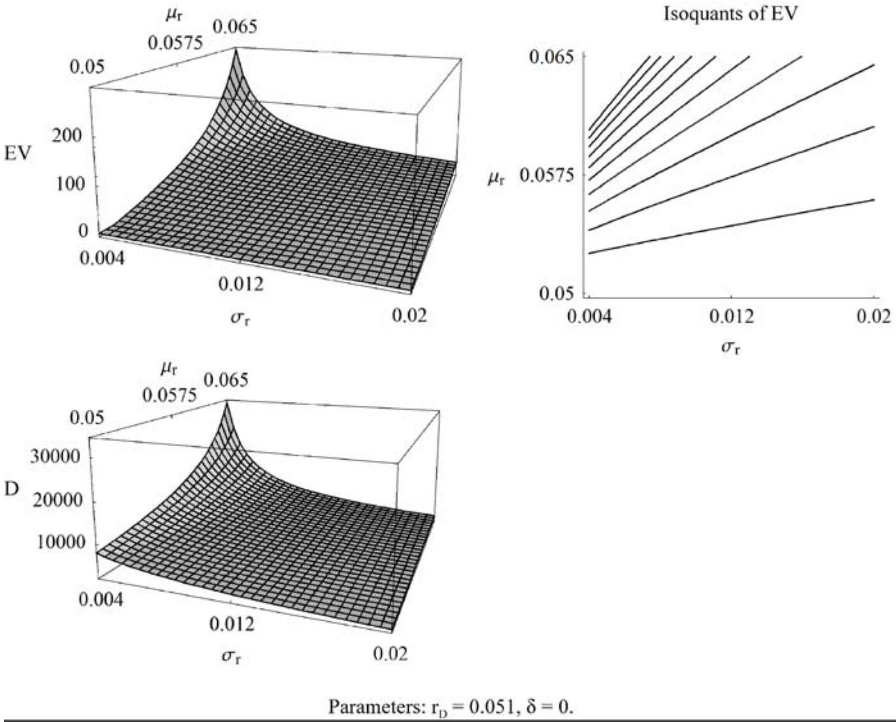


Figure 6.7. Evaluation of the baseline model for a constant interest rate on debt (program P-I), assuming risk neutral investors. The figure shows the equity value EV and debt D as functions of the risk-return profile of the loan portfolio and isoquants of equity value.

negative equity values are set to zero. This is done because with negative equity values the participation condition of shareholders is not fulfilled and the bank management pays back the equity to shareholders.

The isoquants of equity value represent risk-return profiles of the loan portfolio leading to the same equity value. The following applies here: the higher the vertical axis intercept of the isoquants, the higher the represented equity value. The isoquants can be used to compare portfolios with different risk-return profiles. The slopes of the isoquants represent the optimal risk-return trade-off of the bank. Another interpretation for the slopes of the isoquants is the bank's optimal pricing of risk.

The figure shows that the bank's debt decreases in the portfolio risk, as pointed out in the previous section. This entails that the equity value also decreases in the portfolio risk even though investors are risk neutral. Therefore, the indirect negative effect of a volatility increase based on market discipline dominates the direct effect based on the limited liability of shareholders in the considered range of values. The figure also shows that an increase in the expected portfolio return leads to a higher equity value. This stems directly from the higher expected portfolio return and from higher debt.

Since the overall effect of a volatility increase is negative, an increase in the expected portfolio return at the expense of higher portfolio risk does not necessarily lead to a higher equity value. Therefore, the bank has to take into account a risk-return trade-off.

Now focus on the isoquants of equity value. Their positive slopes restate that it is optimal for the bank to price risk positively and to trade off risk and expected return. Furthermore, it can be concluded from the positive slopes that the bank should price risk above the market price of risk. This holds because the assumed risk neutrality of investors implies a market price of risk of zero, while the bank's optimal price of risk is always positive. Consequently, it is optimal for the bank to behave risk aversely, although shareholders valuate their investment only based on the expected payoff. Furthermore, the slopes of the isoquants show that the optimal price of risk increases in the expected portfolio return and decreases in the portfolio risk. Moreover, the optimal price of risk tends to increase in the profitability of the bank.

Model with Optimized Interest Rate on Debt

At the same time as the decision on the risk-return profile of the loan portfolio, the bank management also chooses the interest rate on debt. The interest rate on debt influences the equity value directly (see equation (6.9)) and indirectly by influencing the bank's debt, as shown in the previous section.

However, before integrating the management's optimal decision on the interest rate on debt into the baseline model, it has to first be evaluated whether or not optimal interest rates on debt exist or if it is always optimal for the bank to raise the interest rate on debt to benefit from higher debt due to the leverage effect. So the equity value

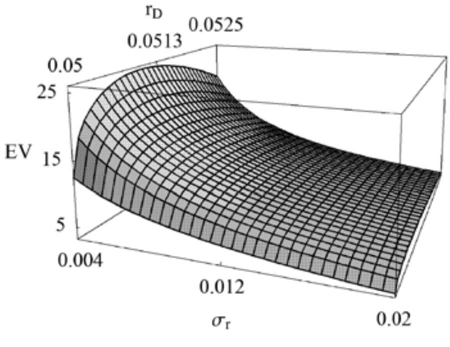


Figure 6.8. Evaluation whether optimal interest rates on debt exist in the baseline model using program P-II. The figure shows the equity value EV as a function of the standard deviation of the portfolio return σ_r and the interest rate on debt r_D with a given expected portfolio return, assuming risk neutral investors.

is determined as a function of the standard deviation of the return σ_r and the interest rate on debt r_D for a constant expected return μ_r and a constant equity E subject to the debtholders' reaction function $\mathbf{R}$ using the program

$$\textbf{P-II:} \begin{cases} EV(\sigma_r, r_D) = P(X_E(\mu_r, \sigma_r, r_D, E, D^*)) - E \\ \text{subject to:} \\ \mathbf{R} : P(X_D(\mu_r, \sigma_r, r_D, E, D^*)) - D^* = 0. \end{cases}$$

Figure 6.8 shows the resulting equity values, assuming risk neutral investors. It illustrates that an interest rates on debt exists which maximizes the equity value for a given risk-return profile of the loan portfolio. So although an increase in the interest rate on debt leads to a higher leverage, it is not always optimal for the bank to increase the interest rate on debt since the benefit of additional debt is smaller than its cost from a certain debt level.

Therefore, the equity value can be determined as a function of the portfolio's risk-return profile using the interest rates on debt optimizing the equity value. This is done by evaluating the following program for different expected returns μ_r and standard deviations σ_r for a constant equity E subject to the debtholders' reaction function $\mathbf{R}$ and the management's optimal decision on the interest rate on debt $\mathbf{O}$:

$$
\textbf{P-1a} \begin{cases} EV(\mu_r, \sigma_r) = P(X_E(\mu_r, \sigma_r, r_D^{opt}, E, D^*)) - E \\ \text{subject to:} \\ \mathbf{R} : P(X_D(\mu_r, \sigma_r, r_D, E, D^*)) - D^* = 0, \\ \mathbf{O} : r_D^{opt} = \arg\max_{r_D}[P(X_E(\mu_r, \sigma_r, r_D, E, D^*)) - E]. \end{cases}
$$

Figure 6.9 shows the resulting equity values, debt volumes, and interest rates on debt as well as isoquants of equity value, assuming risk neutral investors.

The figure shows that it is optimal for the bank to raise the interest rate on debt with a higher expected portfolio return and to reduce it with a higher portfolio risk. Since a change of the interest rates on debt implies a change in the solvency level of the bank, this can be interpreted such that it is optimal for the bank to adjust its solvency level to the risk-return profile of the loan portfolio.

The optimized interest rate on debt leads to debt increasing in the expected portfolio return and decreasing in the portfolio risk even more strongly than in the baseline model with a constant interest rate on debt. So due to the positive leverage effect of debt, the optimization of the interest rate on debt raises the equity value significantly.

Analogous to the case of a constant interest rate on debt, the equity value increases in the expected portfolio return and decreases in the portfolio risk even though investors are assumed to be risk neutral. Thus, the indirect effect of a volatility increase also dominates the direct effect in the baseline model with optimized interest rate on debt. Therefore, market discipline causes costs of risk-taking and makes risk averse behavior optimal for banks even if shareholders are risk neutral and have limited liability.

Furthermore, the slopes of the isoquants of equity value show that the optimal price of risk increases in the expected portfolio return and decreases in the portfolio risk, similar to the model with a constant interest rate debt. Moreover, the previous finding that the optimal price of risk tends to increase in the profitability of the bank holds in the model with an optimized interest rate on debt, too.

6.5.2 Modeling Regulatory Constraints

In reality, a bank not only faces market discipline, but also has to comply with regulatory capital requirements. The purpose of the fol-

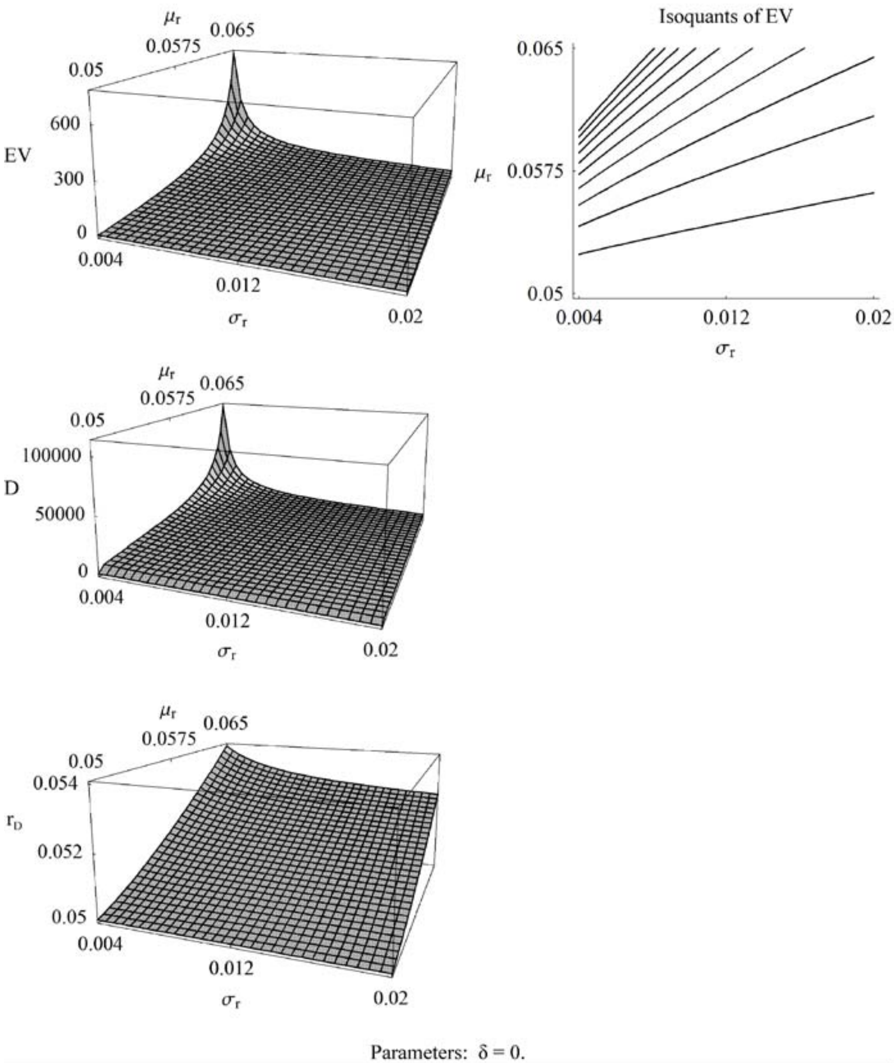


Figure 6.9. Evaluation of the baseline model with an optimized interest rate on debt (program P-1a), assuming risk neutral investors. The figure shows the equity value EV, debt D, and the optimal interest on debt r_D as functions of the risk-return profile of the loan portfolio as well as isoquants of equity value.

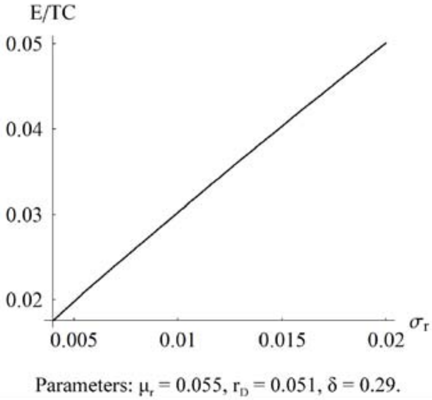


Figure 6.10. New endogenous capital requirement E/TC of debtholders as a function of the standard deviation of the portfolio return σ_r for a given expected portfolio return and a constant interest rate on debt, assuming risk averse investors.

lowing analysis is to determine to what extent such regulations restrict decision-making and how such capital requirements affect the optimal risk-return trade-off of the modeled bank.

Two different regulatory capital requirements are analyzed. The first one regulates the volume of the loan portfolio by requiring a certain equity-to-assets ratio independently of the portfolio risk. Such a regulation is imposed by Basel I, which links capital requirements only to asset classes and not to asset risk. The second one is based on Basel II. Its internal ratings-based approach for determining capital requirements for credit risk defines capital requirements for different asset classes as a function of the risk profile of single assets.

In order to make the model more realistic, the required risk premiums of shareholders and debtholders are considered. The risk premiums intensify both the participation condition of shareholders and the capital requirement of debtholders. The new intensified capital requirement of debtholders is presented in Figure 6.10.

Basel I

Basel I requires a minimum ratio of regulatory capital to risk weighted assets of 8% and a minimum ratio of core capital to risk weighted assets of 4%.

Regulatory capital is defined as core capital plus supplementary capital. Core capital consists of equity and published reserves from post-tax retained earnings less deductions, e.g. for goodwill. Supplementary capital is comprised, amongst other things, of undisclosed reserves as well as, up to a maximum of 50% of the core capital, of subordinated debt with a minimum original term to maturity of more than five years. Supplementary capital is restricted to an amount equal to the core capital.

Risk weights are related to different categories of assets or off-balance-sheet exposures. Loans secured on residential property are weighted with 50%.[10]

Assuming that the core capital is equal to equity E and the supplementary capital is $0.5E$, the regulatory capital RC amounts $1.5E$. Therefore, the regulatory capital constraint can be written as

$$\frac{RC}{RWA} = \frac{1.5E}{0.5(E+D)} \geq 0.08. \tag{6.13}$$

$$\Leftrightarrow D \leq 36.5E. \tag{6.14}$$

Having derived the capital constraint of Basel I **BI**, the following program needs to be solved to determine the equity value for different expected returns μ_r and standard deviations σ_r with a constant equity E:

$$\textbf{P-2a} \begin{cases} EV(\mu_r, \sigma_r) = P(X_E(\mu_r, \sigma_r, r_D^{opt}, E, D^*)) - E \\ \text{subject to:} \\ \mathbf{R}: P(X_D(\mu_r, \sigma_r, r_D, E, D^*)) - D^* = 0, \\ \mathbf{O}: r_D^{opt} = \arg\max_{r_D} [P(X_E(\mu_r, \sigma_r, r_D, E, D^*)) - E], \\ \mathbf{BI}: D^* \leq 36.5E. \end{cases}$$

Figure 6.11 compares the unrestricted baseline model evaluated on the basis of program P-1a to a bank regulated by Basel I evaluated on the basis of program P-2a, assuming risk averse investors.

Comparing the debt volumes with and without regulation, it can be observed that Basel I is only restrictive if the bank is very profitable, i.e., it has a high expected excess return-to-risk ratio. In this case, Basel I limits the debt that can be invested in the loan portfolio. To attract less debt, the bank lowers its interest rate on debt. So if

[10] See Basel Commitee on Banking Supervision (1988), §§ 14-27, § 41, and § 44.

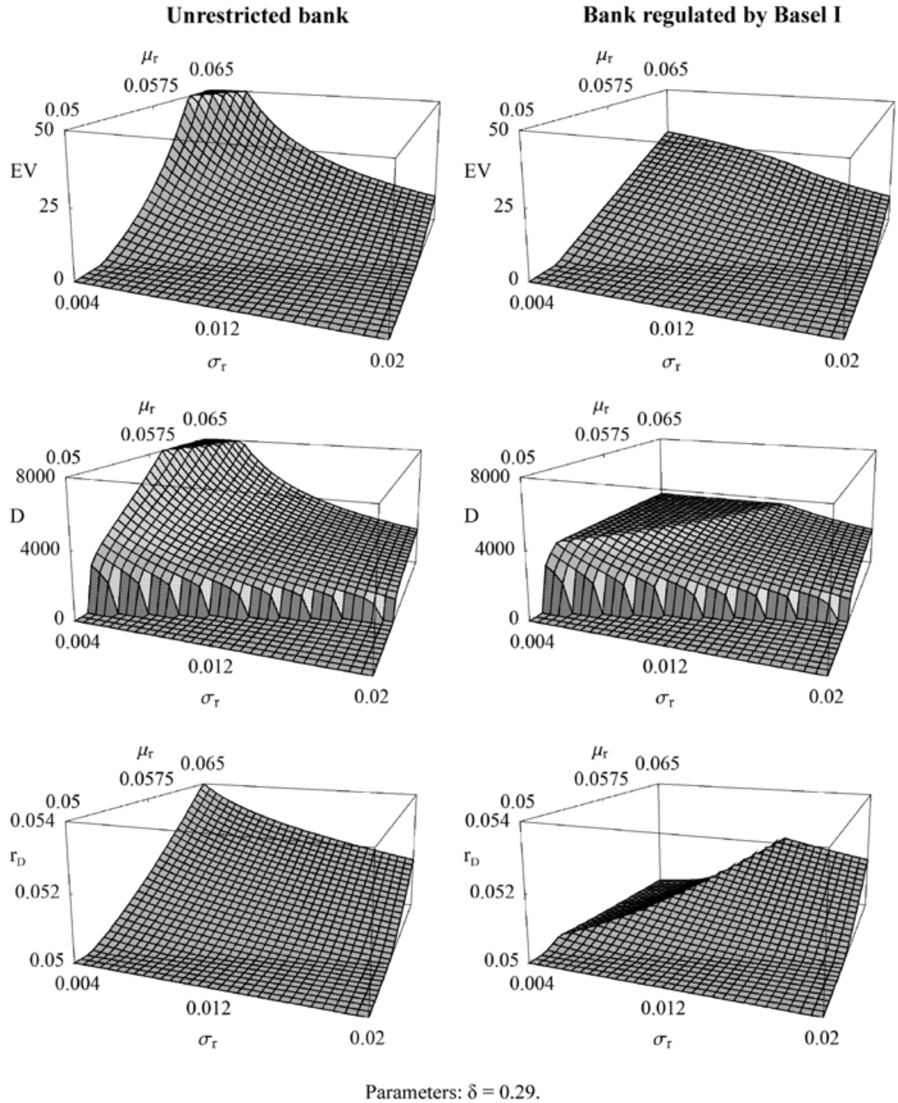


Figure 6.11. Impact of the Basel I regulation on the baseline model. On the left: Evaluation of the unrestricted baseline model (program P-1a). On the right: Evaluation of the Basel I model (program P-2a). The figure compares the equity value EV, debt D, and the optimal interest on debt r_D depending on the risk-return profile of the loan portfolio, assuming risk averse investors.

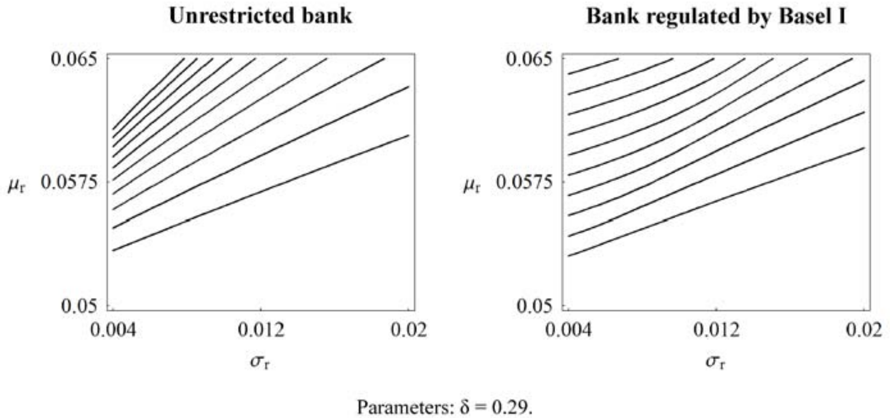


Figure 6.12. Impact of the Basel I regulation on the optimal risk-return trade-off. The figure presents isoquants of equity value corresponding to the models presented in Figure 6.11.

Basel I is restrictive, the increase of the equity value in the expected excess return-to-risk ratio is only due to higher portfolio profitability and cheaper financing and not due to higher volume of the loan portfolio as in the case of the unrestricted baseline model. Therefore, if Basel I is restrictive, the equity value is significantly lower compared to the case of an unrestricted bank.

Figure 6.12 compares the corresponding isoquants of equity value from the unrestricted baseline model with those of a bank regulated by Basel I.

The figure shows that if Basel I is restrictive, the slopes of the isoquants are lower compared to those in the case of an unrestricted bank, indicating that it is optimal for the bank to price risk lower and to behave less risk aversely in the presence of Basel I. This is because market discipline only affects the interest rate on debt but no longer the bank's debt since debt is already restricted by the capital regulation.

For risk-return profiles in which Basel I is not restrictive, the bank is disciplined by debtholders only. Thus, the bank's optimal risk-return trade-off is the same as in the case of an unrestricted bank.

Basel II

Basel II requires a minimum total capital ratio of 8%. The total capital ratio TCR is defined as

$$TCR = \frac{RC}{12.5(CRM + CRO) + RWAC} \tag{6.15}$$

with the regulatory capital RC, the capital requirement for market risk CRM, the capital requirement for operational risk CRO, and the risk weighted assets for credit risk $RWAC$.[11]

Regulatory capital is defined similarly to Basel I. Therefore, regulatory capital is also set to $1.5E$. Since the model assumes that the bank is only exposed to credit risk, the capital requirement for market risk CRM is set to zero. What remains to be considered are the capital requirement for operational risk CRO and risk weighted assets for credit risk $RWAC$.

The standardized approach for determining the capital requirement for operational risk CRO, which is used in the following, divides the bank's activities into eight business lines. The bank's total capital requirement for operational risk is defined as the three-year average of positive summations of regulatory capital charges that are assigned to these business lines. Since it is assumed that the bank only has mortgage loans, its only business line is retail banking, the capital charge of which is 12% of the annual gross income.[12] Therefore, assuming that the three-year average of positive annual gross income is 15% of the bank's equity E, the capital requirement for operational risk CRO is

$$CRO = 0.018E. \tag{6.16}$$

The internal ratings-based approach for determining risk weighted assets for credit risk $RWAC$, which is applied in the following, divides the banking-book exposures of credit risk into several asset classes. For each class, formulas to calculate the risk weights are defined on an asset basis depending on the probability of default PD, the nominal loss given default LGD_N, and the exposure at default EAD, which is the outstanding amount at the time of default. For mortgage exposures, risk weighted assets are calculated as

[11] See Basel Commitee on Banking Supervision (2004), §§ 40-44.

[12] See Basel Commitee on Banking Supervision (2004), § 231 and § 654.

$$RWAC = 12.5\ K\ EAD \tag{6.17}$$

with

$$K = LGD_N\ \Phi[\frac{1}{\sqrt{1-\widehat{\rho}}}\Phi^{-1}(PD) + \sqrt{\frac{\widehat{\rho}}{1-\widehat{\rho}}}\Phi^{-1}(0.999)] - PD\ LGD_N.$$

Here Φ denotes the cumulative distribution function of a standard normal random variable and $\widehat{\rho}$ denotes the asset correlation. For mortgage exposures, Basel II fixes the asset correlation to 15%.[13]

Since the bank invests its total capital in loans, the exposure at default EAD is

$$EAD = (E + D)(1 + r_L). \tag{6.18}$$

Using equation (6.15) to (6.18), the following capital constraint results:

$$D \leq (\frac{1.482}{K(1 + r_L)} - 1)E. \tag{6.19}$$

Having derived the capital constraint of Basel II **BII**, the equity value can be determined for a constant equity E using the following program:

$$\textbf{P-3a}\ \begin{cases} EV(\mu_r, \sigma_r) = P(X_E(\mu_r, \sigma_r, r_D^{opt}, E, D^*)) - E \\ \text{subject to:} \\ \mathbf{R} : P(X_D(\mu_r, \sigma_r, r_D, E, D^*)) - D^* = 0, \\ \mathbf{O} : r_D^{opt} = \arg\max_{r_D}[P(X_E(\mu_r, \sigma_r, r_D, E, D^*)) - E], \\ \mathbf{BII} : D^* \leq (\frac{1.482}{K(\mu_r,\sigma_r)(1+r_L(\mu_r,\sigma_r))} - 1)E \end{cases}$$

with

$$K(\mu_r, \sigma_r) = LGD_N\ \Phi[\sqrt{\frac{20}{17}}\Phi^{-1}(PD(\mu_r, \sigma_r)) + \sqrt{\frac{3}{17}}\Phi^{-1}(0.999)] \\ -PD(\mu_r, \sigma_r)\ LGD_N$$

and

$$PD(\mu_r, \sigma_r) = \frac{LGD\sqrt{\rho} - \sqrt{LGD^2\ \rho - 4\sigma_d^2(\mu_r, \sigma_r)}}{2\ LGD\sqrt{\rho}}$$

(see appendix B.1).

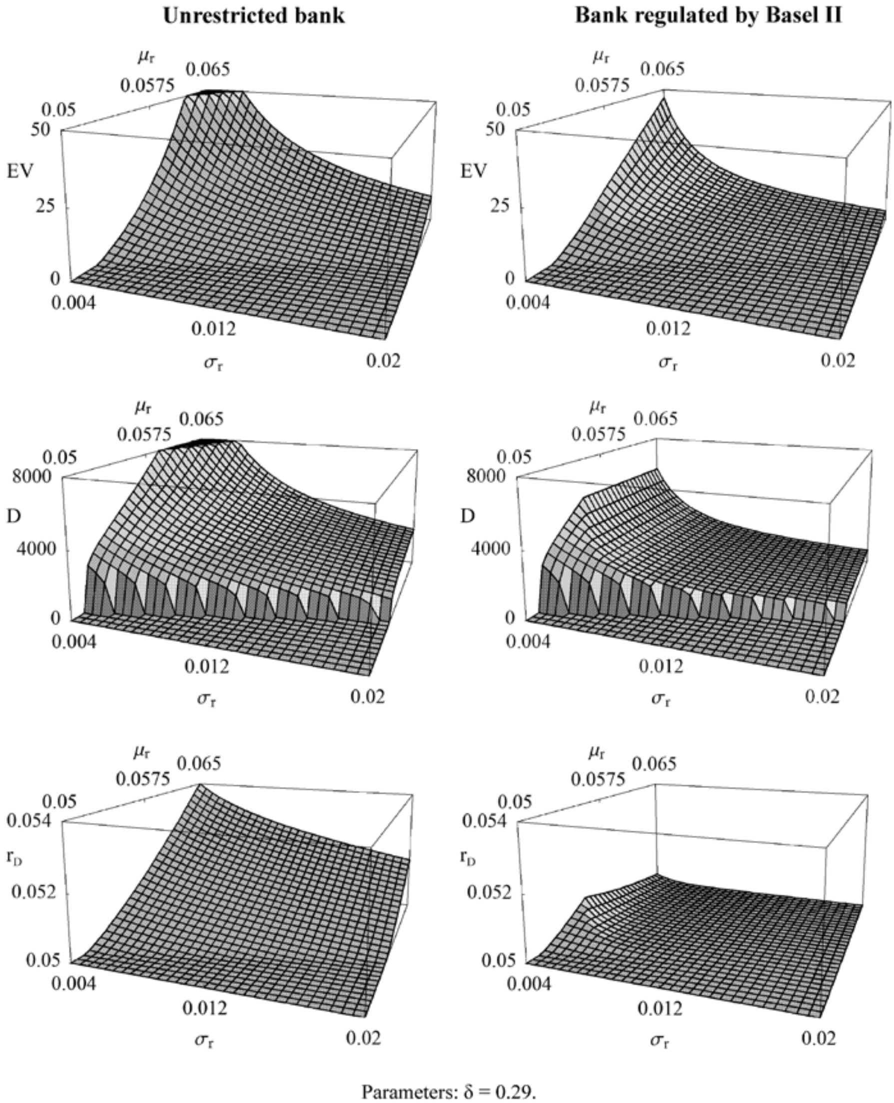


Figure 6.13. Impact of the Basel II regulation on the baseline model. On the left: Evaluation of the unrestricted baseline model (program P-1a). On the right: Evaluation of the Basel II model (program P-3a). The figure compares the equity value EV, debt D, and the optimal interest on debt r_D depending on the risk-return profile of the loan portfolio, assuming risk averse investors.

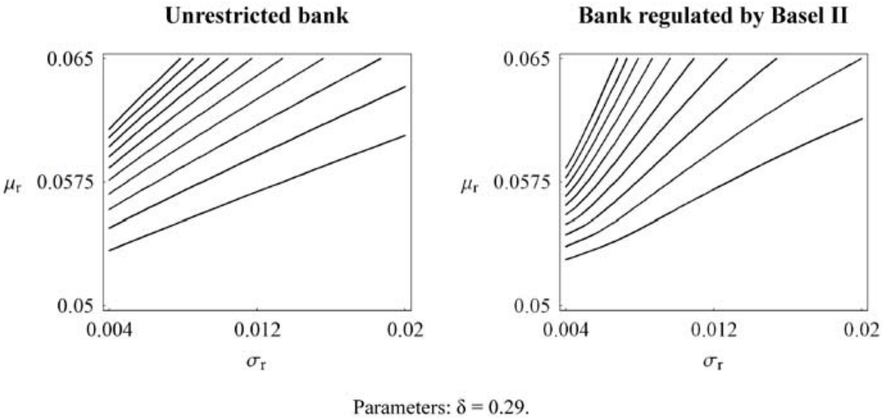


Figure 6.14. Impact of the Basel II regulation on the optimal risk-return trade-off. The figure presents isoquants of equity value corresponding to the models presented in Figure 6.13.

Figure 6.13 compares the unrestricted baseline model evaluated on the basis of program P-1a with a bank regulated by Basel II evaluated on the basis of program P-3a, assuming risk averse investors.

The figure shows that the considered capital requirements of Basel II influence the management's decision-making. The bank under the Basel II regulation is mostly financed through significantly less debt than in the unrestricted baseline model, and the interest rates on debt are clearly lower for the most part, too. Therefore, the equity value is usually also significantly lower in the presence of Basel II than without regulation.

However, the capital requirements of Basel II work very differently from the ones of Basel I. While Basel I only restricts in case of loan portfolios with a high expected excess return-to-risk ratio, Basel II restricts for a wider range of risk-return profiles. Furthermore, the considered capital requirements of Basel II regulates more strongly than the capital requirement of Basel I for high portfolio risk, while it regulates less for low portfolio risk.

Figure 6.14 compares the corresponding isoquants of equity value from the unrestricted baseline model with those of a bank regulated by Basel II.

[13] See Basel Commitee on Banking Supervision (2004), § 328.

The figure shows that the considered capital requirements of Basel II determine the bank's optimal risk-return trade-off to a great extent. In contrast, market discipline is hardly significant for the most part in the presence of Basel II since the portfolio volume is highly determined by the capital requirement, and the interest rates on debt are too low to have a significant influence on the equity value of the bank, except for very low portfolio risk and low expected portfolio return.

Furthermore, the higher slopes of the isoquants indicate that it is optimal for the bank to price risk significantly higher and to behave more risk aversely when regulated by the considered capital requirements of Basel II. So the considered capital requirements of Basel II work in the opposite direction of the capital requirement of Basel I.

6.5.3 Modeling a Fixed Portfolio Volume

While it can be expected that a bank can adjust the volume of its loan portfolio in the long term, in the short term, the portfolio volume might be fixed. But even in this situation, a bank has the possibility of changing the risk-return profile of its portfolio, e.g., by engaging in credit derivatives. With symmetric information, this influences the interest rate on debt a bank has to pay.

To model a bank with a fixed portfolio volume, assume that the return of the portfolio remains beta-distributed when its risk-return profile is changed. The fixed portfolio volume is modeled by imposing the constraint **V** requiring a constant debt-to-equity ratio. The following program then determines the impact of the risk-return profile on the equity value for a constant equity E:

$$\textbf{P-4a}\quad \begin{cases} EV(\mu_r, \sigma_r) = P(X_E(\mu_r, \sigma_r, r_D^*, E, D)) - E \\ \text{subject to:} \\ \mathbf{R} : P(X_D(\mu_r, \sigma_r, r_D^*, E, D)) - D = 0. \\ \mathbf{V} : D/E = const. \end{cases}$$

Figure 6.15 represents the resulting equity value and interest rate on debt as functions of the portfolio's risk-return profile as well as isoquants of equity value, assuming risk averse investors. It shows that the interest rate on debt that is necessary to raise the required debt increases in the portfolio risk and decreases in the expected portfolio return. This is because the insolvency probability of the bank, for which

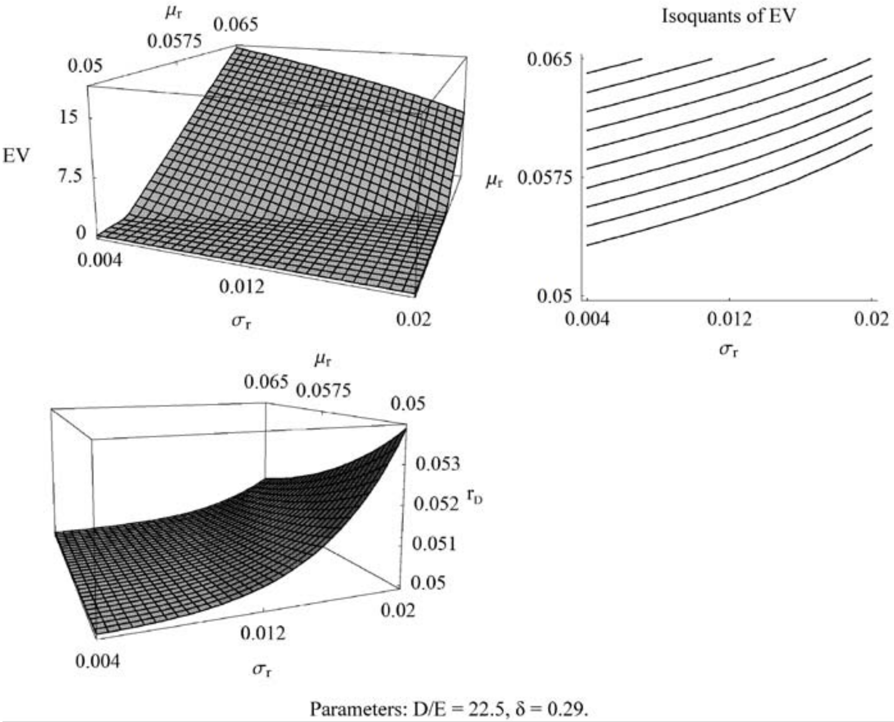


Figure 6.15. Evaluation of the model with a fixed portfolio volume (program P-4a), assuming risk averse investors. The figure shows the equity value EV and the interest on debt r_D as functions of the risk-return profile of the loan portfolio as well as isoquants of equity value.

the interest rate on debt has to compensate, increases in the portfolio risk and decreases in the expected portfolio return, too. So a cost of risk-taking again arises from market discipline, making higher interest rates on debt necessary if the bank's insolvency probability increases.

Furthermore, the figure shows that the equity value increases in the expected portfolio return and decreases in the volatility of the portfolio return. Thus, it is again optimal for the bank to trade off risk and expected return and to behave risk aversely.

However, for low portfolio risk and low portfolio volume, the cost of risk-taking caused by market discipline is only marginal. This is shown by the displayed isoquants of equity value, the slopes of which are nearly exclusively determined by the risk aversion parameter of shareholders.

So the bank's optimal risk-return trade-off is not significantly influenced by market discipline.

In contrast, for high portfolio risk, costs of risk-taking caused by market discipline are significant. This is shown again by the isoquants of equity value rising convexly in the portfolio risk, indicating that the optimal price of risk increases in the portfolio risk. Therefore, banks with the same equity value but distinct risk levels should price risk differently.

6.6 Findings

In this model, a commercial bank is considered that has completely informed uninsured debtholders and does not undertake risk-shifting activities. The model assumes that the bank can have a loan portfolio with a positive net present value. Furthermore, it is assumed that equity-funding is unprofitable and insolvency causes bankruptcy costs.

It is first shown that debtholders require a certain equity-to-assets ratio depending on the insolvency probability of the bank and the interest rate on debt, which is interpreted as market discipline. So with given equity of the bank, debtholders react to the decision-making of the bank management by changing their investment volume. Thus, the bank's debt decreases in the portfolio risk, increases in the expected portfolio return, and increases in the interest rate on debt.

Since debt leverages the profitability of the bank, in addition to the limited liability and the risk premium of shareholders, the reaction of debtholders is taken into account when assessing the optimal risk-return trade-off. While the limited liability of shareholders leads to a benefit of risk-taking, the market discipline and the risk premium of shareholders lead to costs of risk-taking.

For realistic parameter values, it is shown that the costs of risk-taking caused by the reaction of debtholders dominate the benefit of risk-taking caused by the limited liability of shareholders. Thus, it follows that even if investors are risk neutral, it is optimal for the bank to behave risk aversely and to take into account a risk-return trade-off. Here banks should price risk above the market price of risk and higher with increasing expected portfolio return and decreasing portfolio risk. Furthermore, the bank should adjust its solvency level to the risk-return profile of the loan portfolio.

Effects of capital requirements of Basel I and Basel II on the optimal risk-return trade-off are analyzed afterwards. It is shown that Basel I is only restrictive if the bank is very profitable. Otherwise, the disciplining of debtholders restricts more strongly. If Basel I is restrictive, it is optimal for the bank to behave less risk aversely than in the case of an unrestricted bank since market discipline is less effective.

Basel II is restrictive for most risk-return profiles of the loan portfolio using the internal ratings-based approach for determining risk weighted assets for credit risk. In contrast to Basel I, the considered capital requirements of Basel II make it optimal for the bank to behave more risk aversely than in the case of an unrestricted bank. Here the risk-return trade-off is nearly exclusively determined by the considered capital requirements, whereas market discipline is not significant for the most part.

Furthermore, the case of a fixed portfolio volume is analyzed. It is found that it is optimal for the bank to behave risk aversely in this case, too. However, for low portfolio risk and a low portfolio volume, costs of risk-taking primarily stem from the risk premium of shareholders. However, costs of risk-taking caused by market discipline significantly increase in the portfolio risk, making it optimal for banks to behave more risk aversely.

In Figure 6.16, the results on optimal risk-return trade-offs are graphically summarized by plotting a characteristic isoquant of equity value for each case, the slopes of which represent optimal risk-return trade-offs.

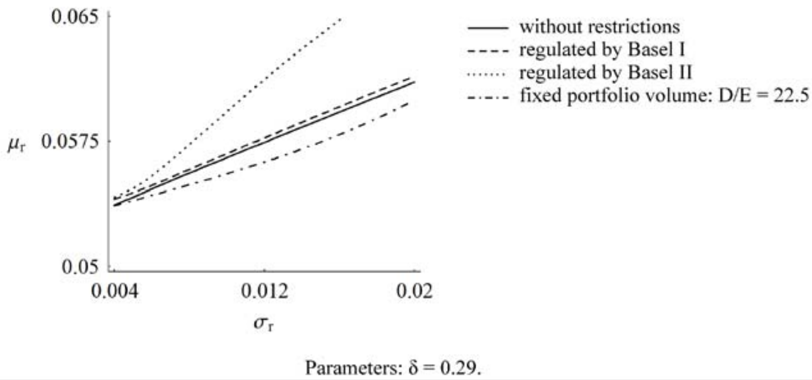


Figure 6.16. Characteristic isoquants of equity value representing optimal risk-return trade-offs for the unrestricted baseline model, the Basel I model, the Basel II model, and the fixed portfolio volume model.

7

Deposits and the Risk-Return Trade-Off

The models presented in the previous chapter assume that the bank is only financed through equity and uninsured debt. This is not very realistic, since, besides uninsured debt, deposits are a major funding source of banks. For the protection of private depositors and to prohibit bank runs, which can disequilibrate an economy, deposits are insured in most countries and for the most part, they are senior to other debt.

For example, in the United States the *Federal Deposit Insurance Corporation* FDIC insures bank accounts with a maximum amount of USD 100,000, and all bank-issued bonds are subordinated in liquidation to the claims of the FDIC.[1] In Germany, deposits are secured up to an amount of EUR 20,000 by law.[2] However, insurance funds usually secure deposits to a nearly unlimited amount. For example, the association *Bundesverband deutscher Banken*, of which most banks are members, secures single deposits with a maximum amount of 30% of the bank's liable equity. In Switzerland, deposits are senior up to an amount of CHF 30,000 and are paid out in advance by the *Schweizerische Bankiervereinigung*.[3]

However, insured depositors do not exert market discipline on the bank. In contrast, it is shown that increasing the portfolio risk is profitable for a bank completely financed through insured deposits since

[1] See *Federal Deposit Insurance Corporation Improvement Act* of 1991 and *Depositor Preference Act* of 1993.

[2] See *Einlagensicherungs- und Anlegerentschädigungsgesetz* of 1998.

[3] See Art. 37a *Bundesgesetz über die Banken und Sparkassen* of 1934 and *Vereinbarung der Schweizerischen Bankiervereinigung über den Einlegerschutz bei zwangsvollstreckungsrechtlicher Liquidation einer Bank* of 1993.

this leads to a transfer of value from the deposit insurance to shareholders.[4] Therefore, it is important to study optimal risk-return trade-offs of banks in the presence of insured senior deposits as well.

7.1 Extending the Models

The models from the previous chapter are extended by assuming that in addition to the uninsured debt D, the bank is financed through insured deposits S being senior to the uninsured debt. So the bank's total debt TD is

$$TD = D + S. \tag{7.1}$$

Furthermore, it is assumed that deposits are constant and the bank sets the interest rate on deposits r_S equal to the yield of the risk-free bond r_f. The uninsured subordinated debtholders have complete information about the bank's deposits and the interest rate on deposits.

The bank's free cash flow $\overline{Y}$ in $t = 1$ is the payoff of the newly financed loan portfolio less the liabilities to both the subordinated debtholders and the senior depositors. Therefore, compared to equation (6.1), the bank's free cash flow $\overline{Y}$ changes to

$$\overline{Y} = (E+D+S)(1+r(1-\tau))+\tau(Dr_D+Sr_S)-D(1+r_D)-S(1+r_S). \tag{7.2}$$

The density function of the bank's free cash flow is specified in appendix B.7.

If the bank is solvent, the subordinated debtholders obtain their invested capital and the interest on it. However, if the bank is insolvent and defaults, the bank's residual value after bankruptcy costs is first used to repay deposits and the interest on them. What is left is used to settle the claims of the subordinated debtholders. Therefore, similarly to equation (6.2), the payoff of subordinated debtholders $\overline{X_D}$ equals

$$\overline{X_D} = \begin{cases} D(1+r_D) & \text{if } \overline{Y} \geq 0; \\ (1-b)(\overline{Y} + D(1+r_D)) & \text{otherwise.} \end{cases} \tag{7.3}$$

Since the claims of shareholders are subordinated to all debt claims, the payoff of shareholders $\overline{X_E}$ is

[4] See Merton (1977).

$$\overline{X_E} = \max\{\overline{Y}, 0\}. \tag{7.4}$$

In the following, the same parameter values are used as in the previous chapter. The influence of insured senior deposits is discussed for an unrestricted bank, for banks having to comply with regulatory capital requirements, and for a bank with a fixed portfolio volume, assuming risk averse investors.

7.2 Influence of Deposits

7.2.1 Baseline Model with Deposits

The analysis starts with the case of an unrestricted bank optimizing its interest rate on subordinated debt. Since it is assumed that deposits are unaffected by the decision-making of the bank management, only the reaction function of the subordinated debtholders has to be considered. Adapting the same equilibrium argument as in the previous models, the subordinated debtholders realize a debt value of zero. Therefore, the following program needs to be evaluated for a constant equity E:

$$\textbf{P-1b} \quad \begin{cases} EV(\mu_r, \sigma_r, S) = P(\overline{X_E}(\mu_r, \sigma_r, r_D^{opt}, E, D^*, S)) - E \\ \text{subject to:} \\ \mathbf{R}: P(\overline{X_D}(\mu_r, \sigma_r, r_D^{opt}, E, D^*, S)) - D^* = 0, \\ \mathbf{O}: r_D^{opt} = \arg\max_{r_D} [P(\overline{X_E}(\mu_r, \sigma_r, r_D, E, D^*, S)) - E]. \end{cases}$$

First of all, the influence of deposits on the equity value, the total debt, and the interest rate on debt is analyzed. For this purpose, Figure 7.1 shows the equity value, the total debt, and the optimal interest rate on subordinated debt as functions of deposits for two different levels of portfolio risk with a constant expected portfolio return, assuming risk averse investors.

It can be observed that the optimal interest rate on subordinated debt rises convexly in deposits. The interest rate on subordinated debt has to be raised to compensate for the increased risk-sensitivity of the subordinated debtholders. The higher risk-sensitivity of the subordinated debtholders results from the subordinated debt serving as a risk buffer for the deposits.

However, the figure shows that the higher interest rates on subordinated debt do not lead to a constant volume of subordinated debt:

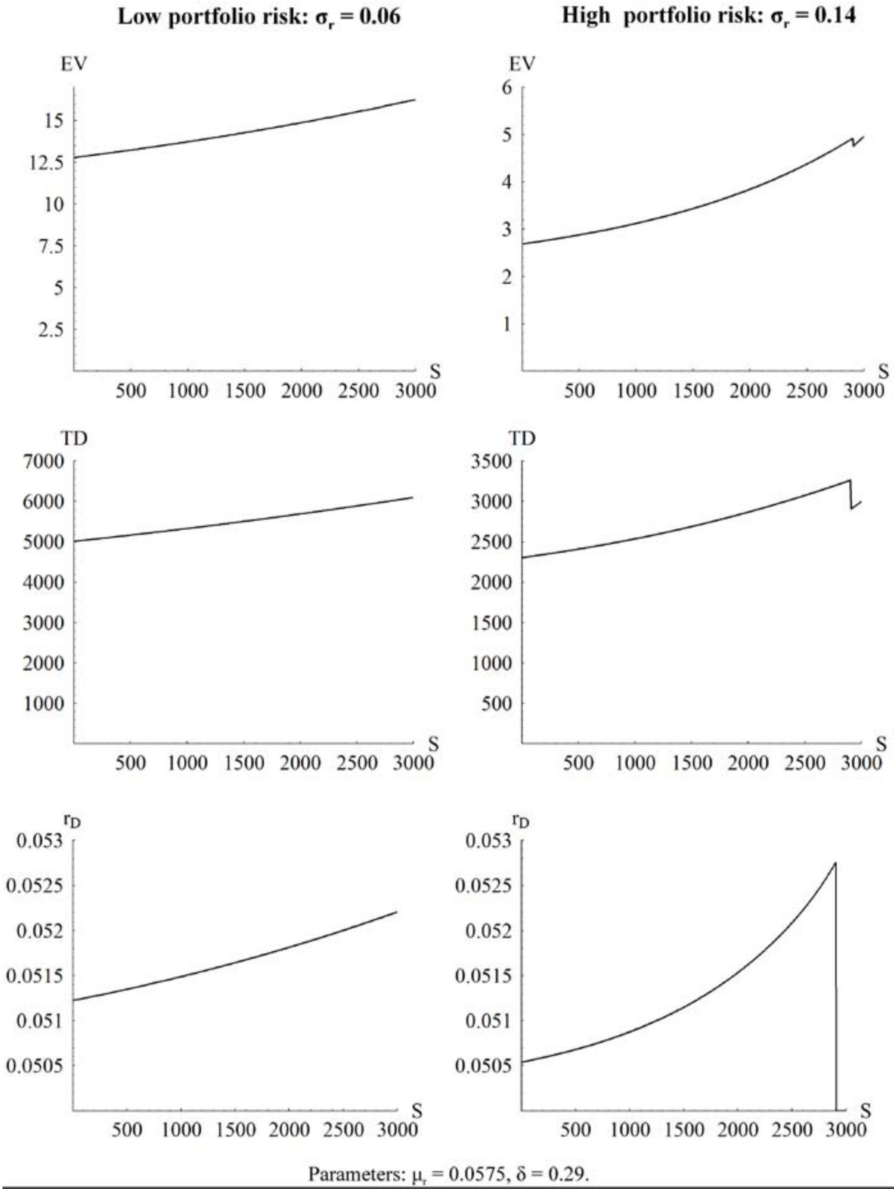


Figure 7.1. Influence of insured senior deposits on the unrestricted baseline model. The figure presents the evaluation of program P-1b and shows the equity value EV, the total debt TD, and the optimal interest rate on subordinated debt r_D as functions of deposits S for two different levels of portfolio risk with a constant expected portfolio return, assuming risk averse investors.

Total debt increases only at a rate smaller than one. Thus, it is optimal for the bank to have less subordinated debt when deposits increase.

Nevertheless, because of the increase of the total debt and the bank's new possibility of cheap funding, the equity value increases in deposits.

The figure further shows for the case of a high portfolio risk that the optimal interest rate on subordinated debt drops to the risk-free rate at a very high level of deposits and remains at this level for higher deposits. If the interest rate on debt is at the risk-free rate, the necessary participation condition of uninsured debtholders is not fulfilled (see appendix B.5). So from this deposit level, it is optimal for the bank to not have any uninsured subordinated debt at all, since the risk premium which the bank would have to pay to the subordinated debtholders becomes too large.

With increasing portfolio risk, this complete crowding-out effect happens for lower deposits. It leads to a jump in the bank's total debt. Thus, the total debt can decrease in deposits. The complete crowding-out effect also leads to a slightly lower equity value.

Having discussed the influence of deposits on the bank's profitability, total debt, and interest rate on debt, changes to the bank's optimal risk-return trade-offs in the baseline model are analyzed. Figure 7.2 compares isoquants of equity value from a bank only financed through uninsured debt to those of a bank also financed through deposits, assuming risk averse investors. Here the assumed deposits-to-equity ratio of 10 is typical for commercial banks.

The figure shows that in the case of a deposit-financed bank, isoquants of equity value only change slightly by becoming more flat. This minor change is surprising since the proportion of debt that is uninsured and thus sensitive to the bank's risk profile decreases in deposits. Nevertheless, the optimal risk-return trade-off is nearly unaffected since the bank is not only financed through more insured debt, but the uninsured subordinated debtholders also become more sensitive to the portfolio risk.

Therefore, it is optimal for the bank to behave risk aversely in the presence of deposits as well. So the costs of risk-taking caused by market discipline dominate the benefit of risk-taking caused by the limited liability of shareholders in the baseline model with deposits as long as it is optimal for a bank to have subordinated uninsured debt.

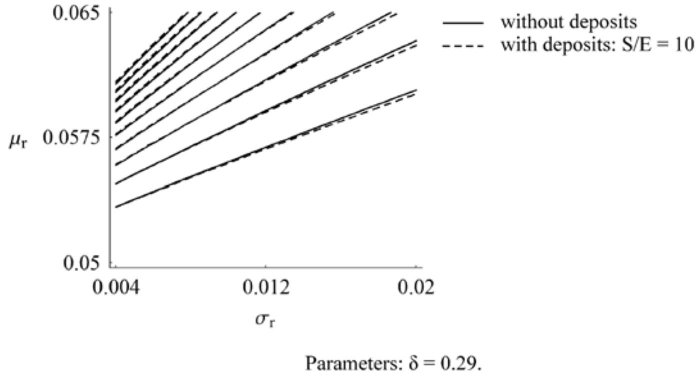


Figure 7.2. Impact of deposits on the optimal risk-return trade-off in the baseline model. The figure compares isoquants of equity value from the baseline model with and without deposits resulting from the evaluation of program 1-b for two different levels of deposits, assuming risk averse investors.

7.2.2 Modeling Regulatory Constraints with Deposits

Having discussed the baseline model with deposits, this subsection analyzes the influence of regulatory capital requirements on the optimal risk-return trade-off in the presence of insured senior deposits. As before, the capital requirement of Basel I is considered first. The case of a bank having to comply with capital requirements of Basel II is analyzed afterwards.

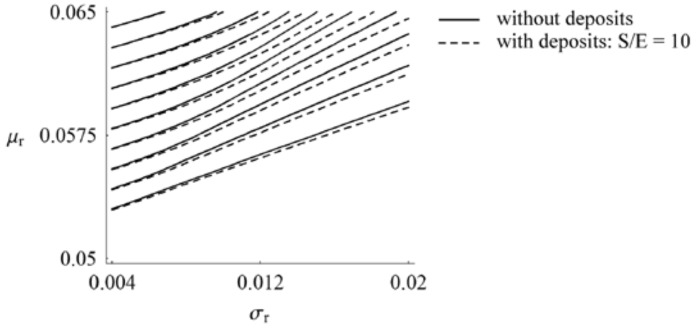
Basel I

The capital requirement of Basel I is quantified analogously to equation (6.14) with the inclusion of deposits S in the banks total debt TD, i.e.,

$$TD = D + S \leq 36.5E. \tag{7.5}$$

So the equity value can be evaluated using the following program with deposits S satisfying $S \leq 36.5E$ and constant equity E:

$$\textbf{P-2b} \begin{cases} EV(\mu_r, \sigma_r, S) = P(\overline{X_E}(\mu_r, \sigma_r, r_D^{opt}, E, D^*, S)) - E \\ \text{subject to:} \\ \mathbf{R} : P(\overline{X_D}(\mu_r, \sigma_r, r_D^{opt}, E, D^*, S)) - D^* = 0, \\ \mathbf{O} : r_D^{opt} = \arg\max_{r_D} [P(\overline{X_E}(\mu_r, \sigma_r, r_D, E, D^*, S)) - E], \\ \mathbf{BI} : D^* \leq 36.5E - S. \end{cases}$$



Parameters: δ = 0.29.

Figure 7.3. Impact of deposits on the optimal risk-return trade-off in the Basel I model. The figure compares isoquants of equity value from the baseline model with and without deposits resulting from the evaluation of program 2-b for two different levels of deposits, assuming risk averse investors.

Figure 7.3 shows isoquants of equity value for a bank regulated by Basel I with and without deposits, assuming risk averse investors.

For low expected excess return-to-risk ratios, Basel I is not restrictive. So isoquants of equity value only become slightly flatter in the presence of deposits analogous to the unrestricted baseline model.

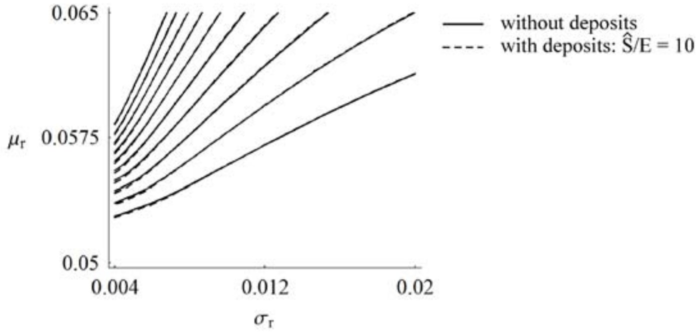
However, if Basel I is restrictive, the influence of deposits on the optimal pricing of risk is stronger. Isoquants of equity value become significantly flatter, implying that it is optimal for the bank to price risk lower. This is because market discipline is not only weakened by the capital requirement, but additionally by deposits since the proportion of uninsured debt decreases in deposits while the cost of subordinated debt does not increase to the same degree. Nevertheless, it is still optimal for the bank to behave risk aversely, even if Basel I is restrictive.

Basel II

The capital requirement of Basel II is quantified analogously to equation (6.19), substituting total debt TD for debt D:

$$TD = D + S \leq (\frac{1.482}{K(\mu_r, \sigma_r)(1 + r_L(\mu_r, \sigma_r))} - 1)E. \tag{7.6}$$

Since the allowed total debt depends on the risk-return profile of the loan portfolio, it could happen that the allowed total debt is smaller



Parameters: δ = 0.29.

Figure 7.4. Impact of deposits on the optimal risk-return trade-off in the Basel II model. The figure compares isoquants of equity value from the baseline model with and without deposits resulting from the evaluation of program 3-b for two different levels of deposits, assuming risk averse investors.

than deposits, assuming that deposits are constant, as has been done so far. This would violate the capital constraint (7.6). Therefore, not constant deposits, but a maximum amount of deposits $\widehat{S}$ is assumed. Should the allowed total debt be smaller than the maximum amount of deposits, deposits are reduced to comply with the capital requirement.

So with this constraint on deposits **S**, the program for determining the equity value reads

$$\textbf{P-3b} \quad \begin{cases} EV(\mu_r, \sigma_r, \widehat{S}) = P(\overline{X_E}(\mu_r, \sigma_r, r_D^{opt}, E, D^*, S)) - E \\ \text{subject to:} \\ \mathbf{R}: P(\overline{X_D}(\mu_r, \sigma_r, r_D^{opt}, E, D^*, S)) - D^* = 0, \\ \mathbf{O}: r_D^{opt} = \arg\max\limits_{r_D}[P(\overline{X_E}(\mu_r, \sigma_r, r_D, E, D^*, S)) - E], \\ \mathbf{BII}: D^* \leq (\frac{1.482}{K(\mu_r,\sigma_r)(1+r_L(\mu_r,\sigma_r))} - 1)E - S, \\ \mathbf{S}: S = \min(\widehat{S}, (\frac{1.482}{K(\mu_r,\sigma_r)(1+r_L(\mu_r,\sigma_r))} - 1)E). \end{cases}$$

Here $K(\mu_r, \sigma_r)$ and $PD(\mu_r, \sigma_r)$ are the same as in program P-3a and equity E is assumed to be constant.

Figure 7.4 compares isoquants of equity value from a bank only financed through uninsured debt with isoquants of equity value from a bank with deposits, assuming risk averse investors.

It is found that the isoquants are hardly distinguishable. This is not really surprising because, as already discussed, the optimal risk-return trade-off is nearly exclusively determined by the considered capital requirements of Basel II, whereas the effect of market discipline is not significant for the most part. Therefore, the bank's optimal risk-return trade-off hardly changes.

7.2.3 Modeling a Fixed Portfolio Volume with Deposits

This subsection analyzes the influence of deposits in the case of a fixed portfolio volume. The fixed portfolio volume limits the total debt a bank raises. So in the presence of constant deposits, the bank has to adjust the interest rate on subordinated debt to raise subordinated debt in the amount of the difference between the required total debt and the existent deposits. So the following program needs to be evaluated with a constant equity E:

$$\textbf{P-4b} \begin{cases} EV(\mu_r, \sigma_r, S) = P(\overline{X_E}(\mu_r, \sigma_r, r_D^*, E, D, S)) - E \\ \text{subject to:} \\ \mathbf{R} : P(\overline{X_D}(\mu_r, \sigma_r, r_D^*, E, D, S)) - D = 0, \\ \mathbf{V} : (D + S)/E = const. \end{cases}$$

Figure 7.5 represents the resulting equity value and interest rate on debt as functions of deposits for two different levels of portfolio risk with a constant expected portfolio return, assuming risk averse investors.

The figure shows that the interest rate on subordinated debt remains nearly constant for a wide range of deposits. This is because, on the one hand, the bank should pay a higher risk premium for subordinated debt since it has to buffer senior deposits. But on the other hand, this effect is counterbalanced by the fact that the bank has to raise a smaller amount of uninsured debt. However, the risk premium effect dominates for very high deposits, as can be seen from the rising interest rate on subordinated debt.

The figure also shows that the equity value increases in deposits. The increase of the equity value can be traced back to the reduced total financing cost. It can also be observed that primarily the high-risk bank profits from cheap deposit-financing. This is intuitive since high-risk banks have to pay a higher risk premium to uninsured subordinated debtholders than low-risk banks.

Finally, the influence of deposits on the optimal risk-return trade-off of a bank having a fixed portfolio volume is analyzed. Figure 7.6

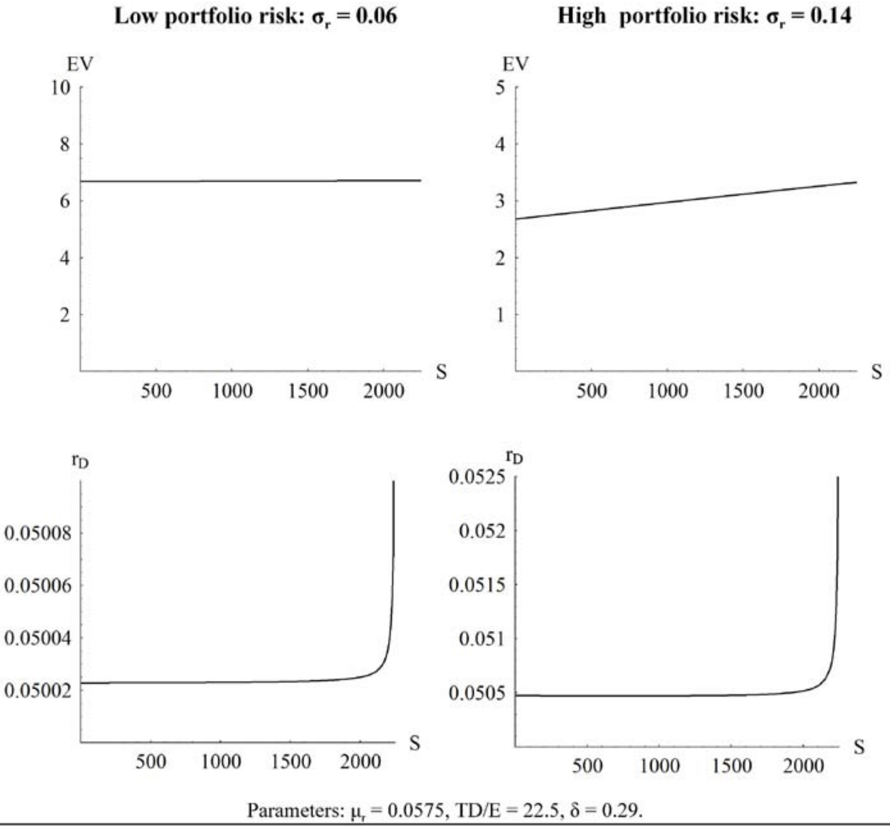


Figure 7.5. Influence of insured senior deposits on the fixed portfolio volume model. The figure presents the evaluation of program P-4b and shows the equity value EV and the interest rate on subordinated debt r_D as functions of deposits S for two different levels of portfolio risk with a constant expected portfolio return, assuming risk averse investors.

shows isoquants of equity value with and without deposits, assuming risk averse investors.

For low portfolio risk, isoquants are nearly unaffected by deposits. This is because in the fixed portfolio volume model, the optimal risk-return trade-off of a bank without deposits is not significantly affected by market discipline either.

However, for high portfolio risk, costs of risk-taking caused by market discipline are significant for a bank only financed through uninsured deposits in the fixed portfolio volume model. But market discipline is

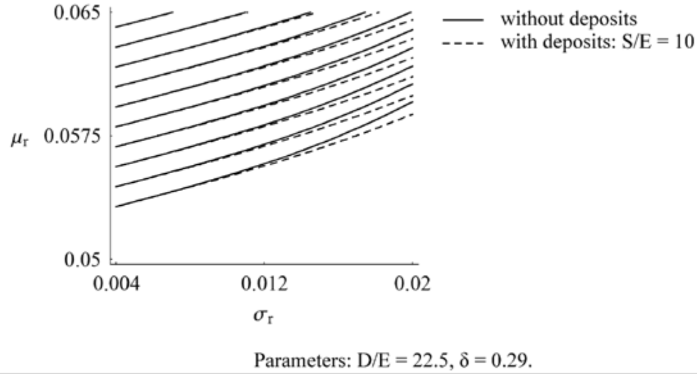


Figure 7.6. Impact of deposits on the optimal risk-return trade-off in the fixed portfolio volume model. The figure compares isoquants of equity value from the baseline model with and without deposits resulting from the evaluation of program 4-b for two different levels of deposits, assuming risk averse investors.

weakened by deposits since the proportion of uninsured debt decreases in higher deposits, while the interest rate on subordinated debt necessary to raise the required subordinated debt nearly stays constant. Therefore, the optimal price of risk is lower for a bank with deposits compared to a bank without deposits. Nevertheless, the risk premium of shareholders still makes it optimal for the bank to behave risk aversely, as can be seen from the positive slopes of the isoquants.

7.3 Findings

In this chapter, the impact of deposits on the profitability and financing of banks as well as on their optimal risk-return trade-off is analyzed by extending the models from the previous chapter. The extended models assume that, in addition to the uninsured debt, banks are financed through insured deposits. The uninsured debtholders care about deposits, since deposits are assumed to be senior to their claims.

It is shown for the baseline model that it is optimal for the bank to raise the interest rate on subordinated debt convexly in deposits. However, the bank will not entirely compensate for the increased risk of the subordinated debtholders, by increasing the interest rate on subordinated debt. It is optimal for the bank to instead have less subordinated debt when deposits increase. Furthermore, it is shown that it is even optimal for the bank to have not any subordinated debt at all from a very high level of deposits.

Although the profitability of the unrestricted bank rises in deposits, its optimal risk-return trade-off is only slightly affected by deposits since the uninsured subordinated debtholders become more sensitive to the risk of the loan portfolio. So it is optimal for the bank just to price risk marginally lower.

However, in the case of a bank having to comply with the capital requirement of Basel I, risk should be priced lower compared to a bank without deposits if Basel I is restrictive. This is because market discipline is weakened by deposits. In contrast, in the case of a bank having to comply with the considered capital requirements of Basel II, the optimal risk-return trade-off does not change significantly since the trade-off is determined nearly exclusively by the capital requirements even for a bank without deposits and not by market discipline.

In the model of a bank with a fixed portfolio volume, the interest rate on subordinated debt nearly stays constant for a wide range of deposits. This is because the effect on the interest rate on debt that the subordinated debtholders become more risk averse and the effect that the bank is financed with less uninsured debt counterbalance each other. However, the bank becomes more profitable with higher deposits.

For low portfolio risk, the optimal risk-return trade-off does not significantly change with deposits in the model of a bank having a fixed portfolio volume since the optimal risk-return trade-off is not significantly influenced by market discipline in the case of a bank without deposits, either. However, for high portfolio risk, the optimal price of risk is lower in the case of a bank with deposits since market discipline causes a significant cost of risk-taking for high-risk banks without deposits and is weakened by deposits.

Summarizing, it can be stated that although insured senior deposits can weaken market discipline, it is found in all models that banks having uninsured subordinated debt should price risk positively. So it is optimal for them to behave risk aversely in the presence of insured senior deposits as well.

8

Profitability Measures for Loan Portfolios

This chapter examines whether reward-to-risk ratios derived from capital market models, which are presented in Chapter 4, are suitable for loan portfolios. This is done by comparing the risk-return trade-offs of those profitability measures to the endogenously derived optimal risk-return trade-offs of commercial banks.

Profitability measures considered are the Sharpe ratio and the reward-to-VaR ratio since these measures are explicitly proposed for loan portfolios.[1] Additionally, further plausible profitability measures derived from other capital market models, namely the reward-to-shortfall ratio and the reward-to-ES ratio, are assessed.

Since the optimal risk-return trade-offs of commercial banks are only slightly affected by insured senior deposits, except for high portfolio risk, risk-return trade-offs calculated for banks only financed through uninsured debt are used for the following comparison. However, it has to be kept in mind that for high-risk banks having insured senior deposits, the optimal price of risk is lower than the plotted isoquants of equity value suggest, except for banks having to comply with the considered capital requirements of Basel II. Thus, the slopes of the displayed isoquants of equity value represent the upper limit of the optimal price of risk.

[1] See Altman and Saunders (1998), pp. 1728-1740, Campbell, Huisman, and Koedijk (2001), and Alexander and Baptista (2003).

8.1 Comparison of the Risk-Return Trade-Offs

The considered profitability measures are defined for portfolio return r with density function f and distribution function F as

1. Sharpe ratio
$$SR = \frac{\mu_r - r_f}{\sigma_r}, \tag{8.1}$$
2. reward-to-shortfall ratio
$$RSR = \frac{\mu_r - r_f}{\theta_r} \tag{8.2}$$
$$\text{with } \theta_r = (\int_{-\infty}^{r_f} (r - \mu_r)^2 f(r) dr)^{\frac{1}{2}},$$
3. reward-to-VaR ratio with confidence level α
$$RVaR = \frac{\mu_r - r_f}{VaR_{ex}} \tag{8.3}$$
$$\text{with } VaR_{ex} = VaR(r) + r_f = -F^{-1}(1-\alpha) + r_f,$$
4. reward-to-ES ratio with confidence level α
$$RES = \frac{\mu_r - r_f}{ES_{ex}} \tag{8.4}$$
$$\text{with } ES_{ex} = ES(r) + r_f = -\mathbb{E}(r | r \leq F^{-1}(1-\alpha)) + r_f.$$

The risk measures used in these ratios are all based on the density function of the portfolio return, which is completely specified by the first two statistic moments of the portfolio return. Therefore each pair of variates (μ_r, σ_r) can be transformed into a pair of variates of expected portfolio return and one of the risk measures above, i.e., in $(\mu_r,\ \theta_r)$, $(\mu_r,\ VaR_{ex})$, and $(\mu_r,\ ES_{ex})$. Thus, isoquants of equity value can also be represented using the semi-standard deviation θ_r, the Value at Risk based on the excess return VaR_{ex}, and the Expected Shortfall based on the excess return ES_{ex}. In the following, the isoquants of equity value are plotted for ranges of portfolio risk corresponding to the considered range of σ_r at an expected excess return of zero.

Figure 8.1 compares valuation isoquants of the reward-to-risk ratios above with isoquants of equity value derived from the unrestricted

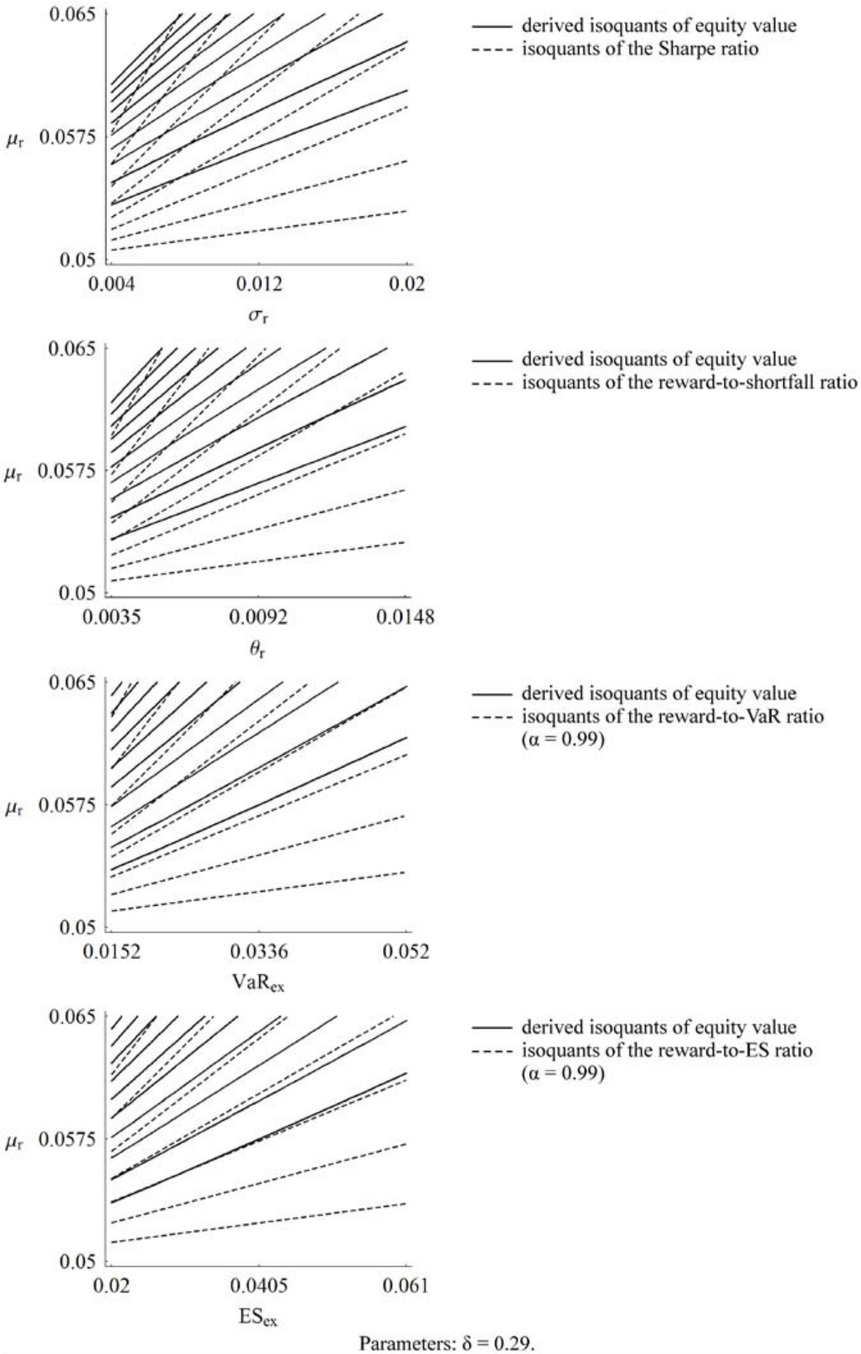


Figure 8.1. Comparison of isoquants of equity value from the unrestricted baseline model, assuming risk averse investors, with isoquants of reward-to-risk ratios.

baseline model, assuming risk averse investors. The risk-return trade-offs of the reward-to-risk ratios are represented by the slopes of the valuation isoquants.

Obviously, both the Sharpe ratio and the reward-to-shortfall ratio have risk-return trade-offs that highly deviate from the derived optimal trade-offs. The risk adjustment of those ratios lead to valuation isoquants that are too steep to approximate the derived isoquants of equity value. In contrast, the risk-return trade-offs of the reward-to-VaR ratio and the reward-to-ES ratio come close to the derived risk-return trade-offs, at least for moderate values. In particular, the reward-to-ES ratio can be used as a good approximation of the profitability of loan portfolios for not too high values.

Looking for an economic interpretation of these findings is difficult since the reward-to-risk ratios are developed to optimize an utility function in the presence of a perfect capital market. Therefore, reward-to-risk ratios are suited for measuring the profitability of loan portfolios only by chance, since for this task they do not need to represent the risk-return trade-off of capital market investors, but rather the optimal risk-return trade-off of a bank. As already discussed in detail, these trade-offs differ.

So the findings can only be interpreted such that, in the case of an unrestricted bank, the costs of risk-taking caused by market discipline and the risk premium of shareholders are not high enough for the Sharpe ratio and the reward-to-shortfall ratio to represent optimal risk-return trade-offs under the given assumption. However, how can it be explained that the reward-to-VaR ratio and the reward-to-ES still represent risk-return trade-offs that fit well with the derived risk-return trade-offs? To answer this question, a technical argument is employed. While σ_r and θ_r can be considered to be scale parameters of the return's density function, which are independent of the density function's location parameter μ_r, the risk measures VaR_{ex} and ES_{ex} strongly depend on μ_r; namely VaR_{ex} and ES_{ex} decrease in μ_r. So when plotting isoquants of equity value, their slopes are higher under the risk measures VaR_{ex} and ES_{ex} compared to the slopes under the risk measures σ_r and θ_r. Thus the isoquants of equity value based on the risk measures VaR_{ex} and ES_{ex} come closer to the valuation isoquants of the reward-to-risk ratios than the isoquants of equity value based on the risk measures σ_r and θ_r. So using a risk measure that takes into account the location parameter of the distribution can help measure the suitability of loan portfolios in the case of an unrestricted bank. However, it does not result from the findings that risk measures based on the concept of economic capital are more suited in general for making risk adjustments in the loan portfolio context, as will be shown in the following.

Figure 8.2 compares valuation isoquants of the reward-to-risk ratios with isoquants of equity value from the Basel I model, assuming risk averse investors.

Since Basel I does not restrict in case of low expected excess return-to-risk ratios, the reward-to-VaR ratio and the reward-to-ES ratio are again good proxies for moderate values, while the Sharpe ratio and the reward-to-shortfall ratio are not suited for approximating the profitability of loan portfolios analogous to the baseline model.

For high expected excess return-to-risk ratios, however, Basel I is restrictive and the isoquants of equity value become flatter than in the baseline model, implying significantly lower optimal prices of risk. Since even in baseline model, all considered reward-to-risk ratios make a too strong risk adjustment for high expected excess return-to-risk ratios, in this range none of the above profitability measures is suited for approximating the profitability of loan portfolios of a bank regulated by Basel I, either.

Figure 8.3 presents valuation isoquants of the reward-to-risk ratios and isoquants of equity value from the Basel II model, assuming risk averse investors.

In contrast to the unrestricted baseline model and the Basel I model, the comparison of the risk-return trade-offs shows that the reward-to-VaR ratio and the reward-to-ES ratio are not at all suitable as profitability measures for loan portfolios for banks having to comply with the considered capital requirements of Basel II. The optimal prices of risk derived from the Basel II model are much higher than the risk-return trade-offs of the reward-to-VaR ratio and the reward-to-ES ratio suggest.

Compared to the valuation isoquants of the reward-to-VaR ratio and the reward-to-ES ratio, the valuation isoquants of the reward-to-shortfall ratio come closer to the derived isoquants of equity value. Surprisingly, the valuation isoquants of the Sharpe ratio approximate the derived isoquants of equity value even better than the reward-

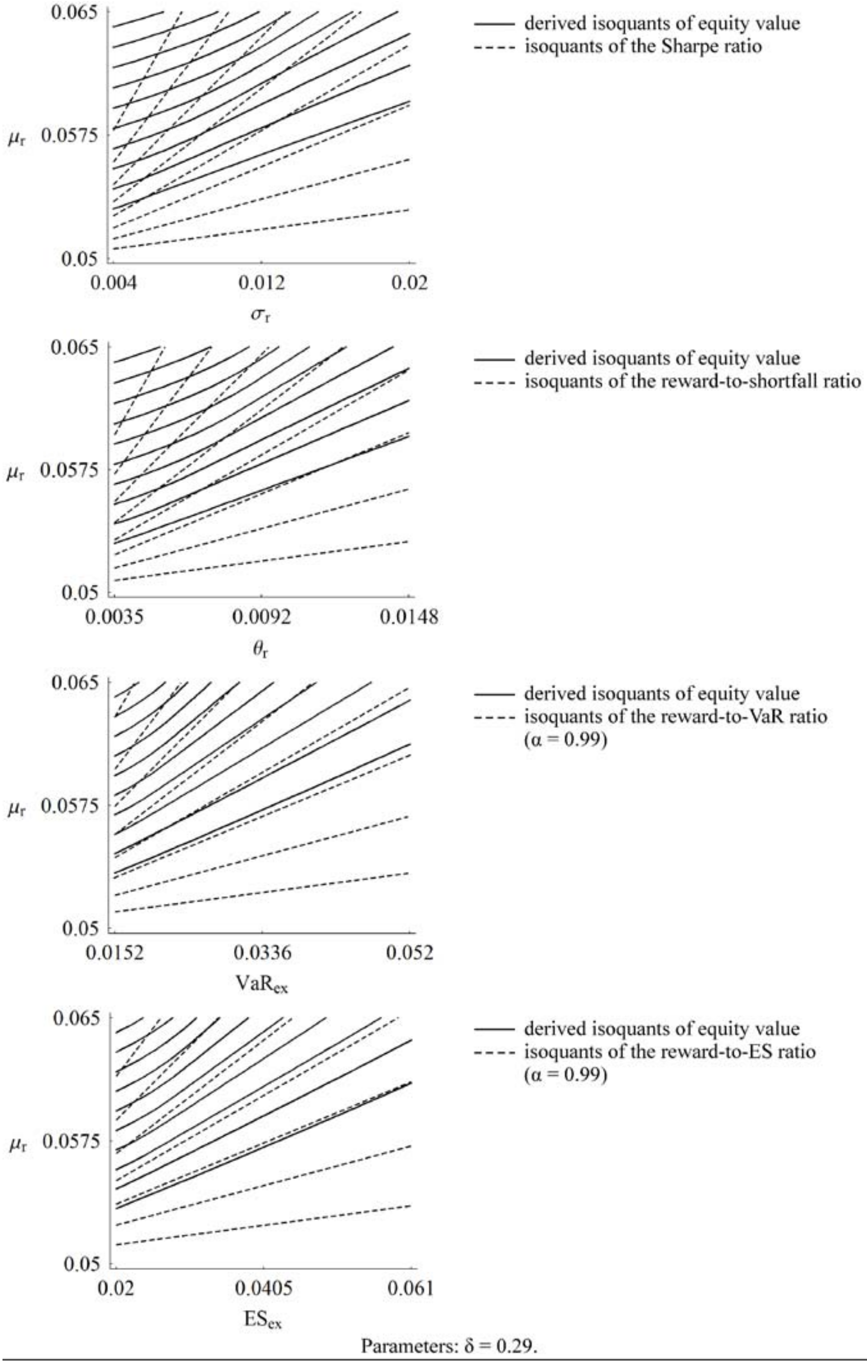


Figure 8.2. Comparison of isoquants of equity value from the Basel I model, assuming risk averse investors, with isoquants of reward-to-risk ratios.

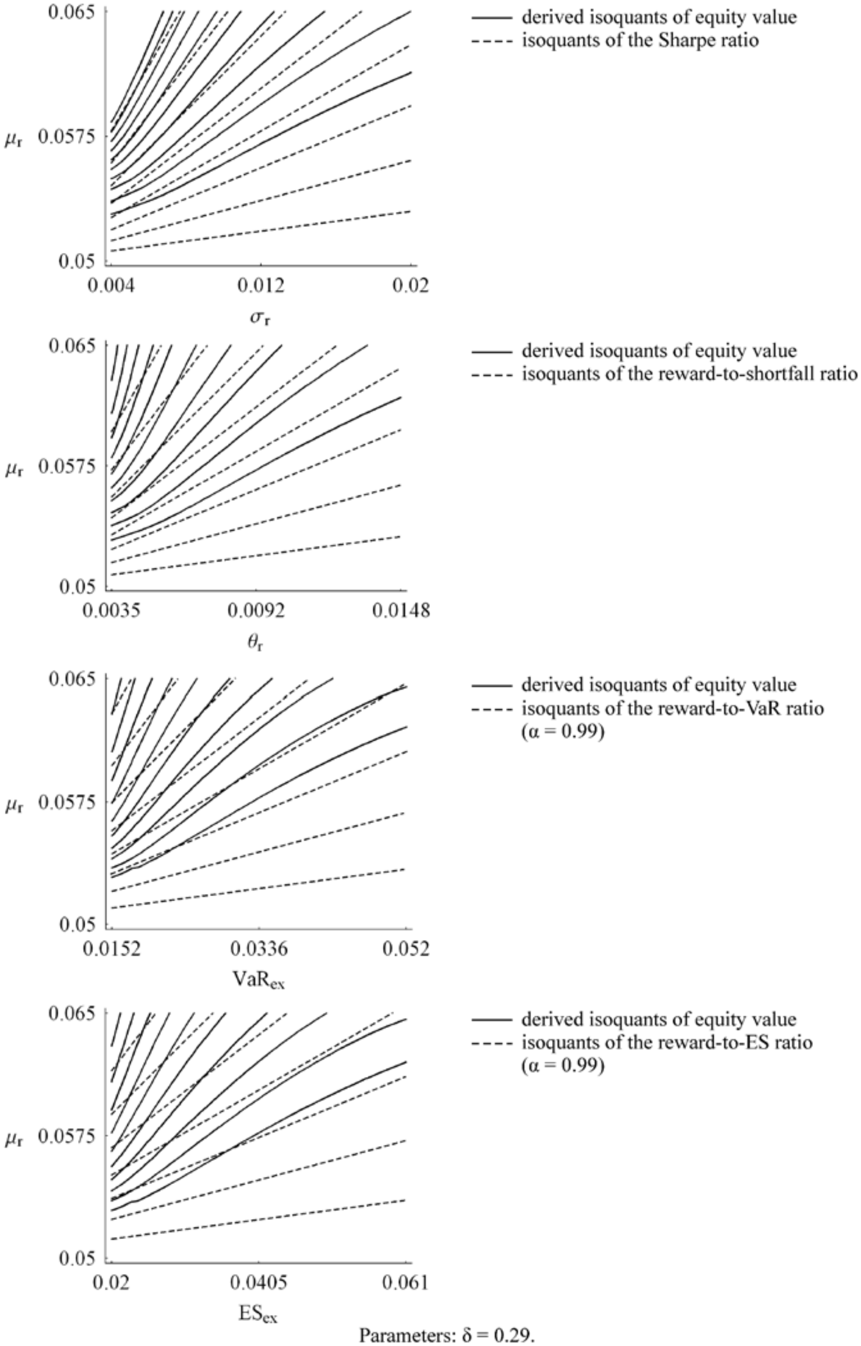


Figure 8.3. Comparison of isoquants of equity value from the Basel II model, assuming risk averse investors, with isoquants of reward-to-risk ratios.

to-shortfall ratio since the Sharpe ratio approximates isoquants more closely for high excess return-to-risk ratios than the reward-to-shortfall ratio. However, even the Sharpe ratio is not a good measure for very low and high portfolio risk.

Thus, although it is quite appealing that downside risk measures such as semi-standard deviation, Value at Risk and Expected Shortfall are more closely related to costs of risk-taking, it is more appropriate to use the volatility measure standard deviation than the downside risk measures to valuate risk-return profiles of loan portfolios based on reward-to-risk ratios under the given assumptions. However, this finding does not prove that the volatility measure standard deviation is more appropriate for use in reward-to-risk ratios in general in the context of the considered capital requirements of Basel II since this finding might not hold for other return distributions. It just shows again that it is important to be very careful with economic intuitions when dealing with risk measures. This is because they do not directly measure costs of risk-taking, but rather only specify the distribution of risk factors.

Figure 8.4 compares valuation isoquants of the reward-to-risk ratios to the derived isoquants of equity value from the fixed portfolio volume model, assuming risk averse investors.

It is obvious that none of the profitability measures approximates the profitability of loan portfolios well. The courses of the derived isoquants of equity value based on the standard deviation and the semi-standard deviation imply rather that different measures pricing risk constantly might be more suitable for this task, especially for low portfolio risk. This could have been expected since in the model of a bank with a fixed portfolio volume having low portfolio risk the costs of risk-taking primarily stem from the shareholders' risk premium, and risk should thus be priced closely to the constant market price of risk. So the profitability measures

$$PM_A = \mu_r - \gamma\sigma_r \tag{8.5}$$

and

$$PM_B = \mu_r - \gamma\theta_r \tag{8.6}$$

with $\gamma > 0$, which are very similar to the assumed valuation function of investors, can be used as profitability measures for low portfolio risk,

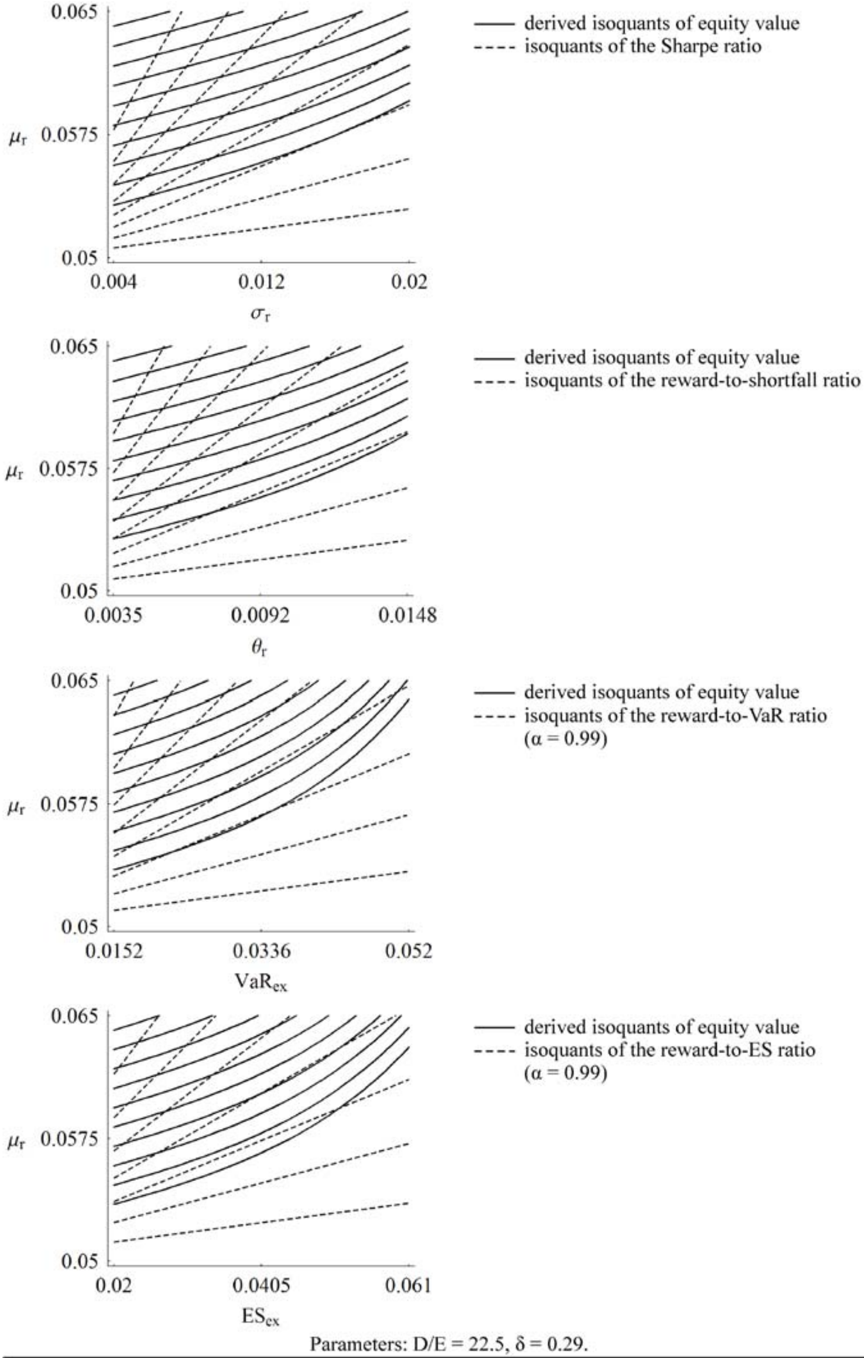


Figure 8.4. Comparison of isoquants of equity value from the fixed portfolio volume model, assuming risk averse investors, with isoquants of reward-to-risk ratios.

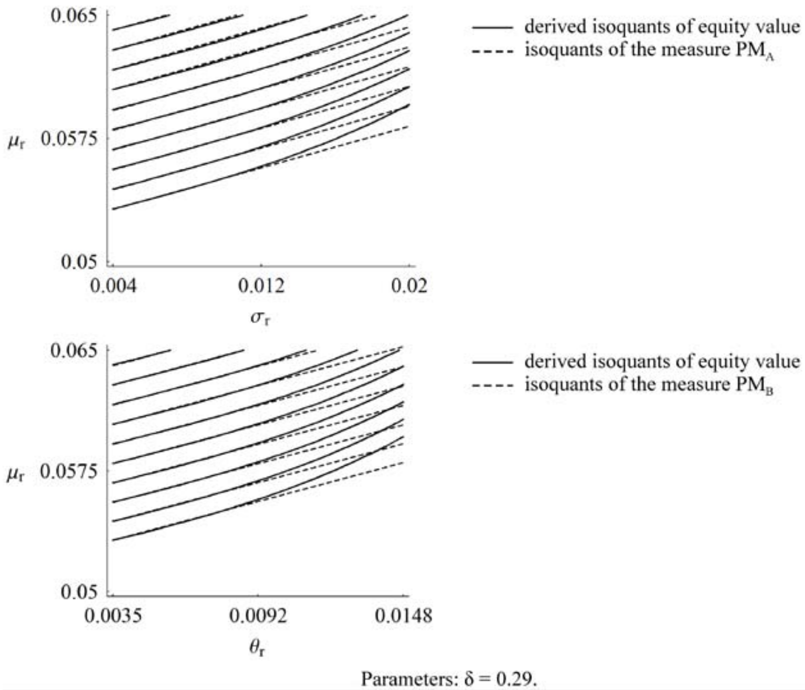


Figure 8.5. Comparison of isoquants of equity value from the fixed portfolio volume model, assuming risk averse investors, with isoquants of the measures PM_A and PM_B pricing risk constantly.

as shown in Figure 8.5. However, for high portfolio risk, costs of risk-taking caused by market discipline are weighted more heavily. So costs of risk-taking are not constant anymore, and the profitability measures PM_A and PM_B become misleading.

As Figure 8.2 suggests, the profitability measures PM_A and PM_B are also suitable for loan portfolios in the case of a bank regulated by Basel I when considering low portfolio risk. This is shown in Figure 8.6. However, in this case, risk should be priced above the market price of risk. Thus, the appropriate parameter γ is higher compared to the case of a bank with a fixed portfolio volume.

8.2 Findings

In this chapter, the endogenously derived optimal risk-return trade-offs of commercial banks financed through uninsured debt are compared

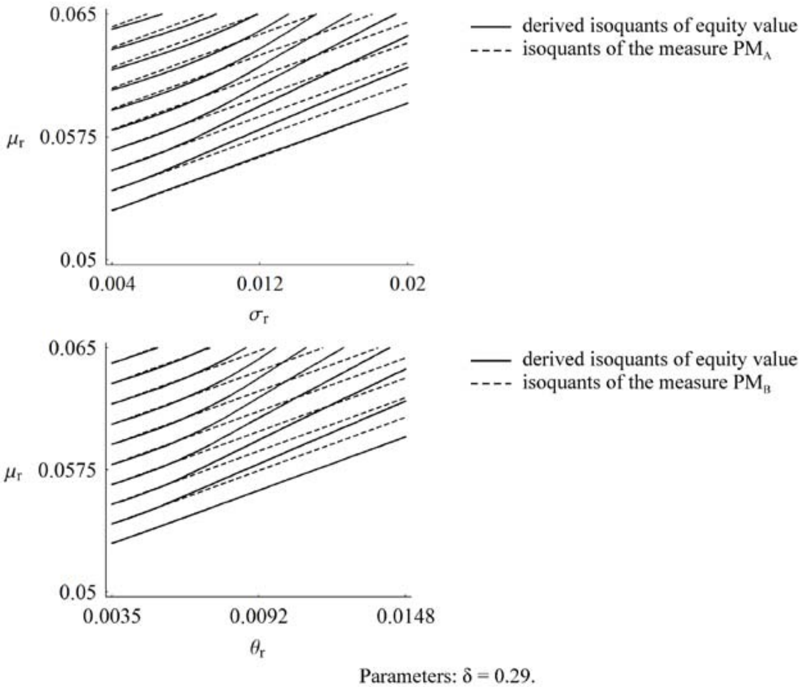


Figure 8.6. Comparison of isoquants of equity value from the Basel I model, assuming risk averse investors, with isoquants of the measures PM_A and PM_B pricing risk constantly.

to the risk-return trade-offs of profitability measures to assess their suitability for loan portfolios.

It is found that the reward-to-VaR ratio and the reward-to-ES ratio approximate the profitability of loan portfolios well for moderate values in the case of an unrestricted bank and in the case of a bank regulated by Basel I. However, these ratios are highly misleading for banks restricted by the considered capital requirements of Basel II and for banks with a fixed portfolio volume.

The Sharpe ratio and the reward-to-shortfall ratio approximate the profitability of loan portfolios well for the most part in the case of a bank regulated by Basel II for not too high values. But those ratios are highly misleading in all other cases.

Furthermore, it is shown for banks with a fixed portfolio volume and for banks regulated by Basel I that profitability measures that are similar to the investor's valuation function approximate the profitability of loan portfolios well for low portfolio risk.

In summary, it can be stated that none of the assessed profitability measures is suited for loan portfolios in all considered cases. This also holds for considering banks with insured senior deposits. As shown in the previous chapter, risk should be priced lower for high portfolio risk in the presence of insured senior deposits, except for the case of a bank regulated by Basel II. This makes the reward-to-VaR ratio and the reward-to-ES ratio somewhat less suited in the cases of an unrestricted bank and a bank regulated by Basel I. However, the suitability of the Sharpe ratio and the reward-to-shortfall ratio in the case of a bank regulated by Basel II is unaffected by deposits. Moreover, in the case of a bank with a fixed portfolio volume, profitability measures that are similar to the investors' valuation function are suitable even for a wider range of portfolio risk in the presence of deposits.

The findings further illustrate that risk measures only specify the distribution of risk factors and do not directly specify costs of risk-taking. Thus the economic intuition can be misleading when arguing about risk measures.

9

Conclusion

9.1 Summary

The dissertation criticizes the fact that profitability measures such as the Sharpe ratio and the reward-to-VaR ratio are proposed for loan portfolios although it is not assessed whether their risk-return trade-offs are optimal for banks. The profitability measures are based on capital market models instead and represent the optimal risk-return trade-off of individual capital market investors. However, the optimal risk-return trade-off of individual capital market investors and banks do not need to be the same since they have different targets and banks are confronted with specific costs of risk-taking. Therefore, it can be stated that profitability measures for loan portfolios are at the very least not well founded.

The dissertation intends to fill this gap and answers the question of whether or not profitability measures derived from capital market models are suitable for loan portfolios. It is found that none of the assessed reward-to-risk ratios is suited for loan portfolios in all considered cases. Even the reward-to-VaR ratio, which is explicitly developed for the purpose of valuating loan portfolios, can be highly misleading.

The approach of the dissertation is to endogenously derive optimal risk-return trade-offs of commercial banks and to compare them with the risk-return trade-offs of reward-to-risk ratios derived from capital market models. For this purpose, models of commercial banks are developed which are based on two capital market imperfections. They assume that equity-funding is too costly to be profitable, and that insolvency causes bankruptcy costs. In addition, it is assumed that uninsured debtholders have complete information about the riskiness of their claims and that the bank management commits to a certain risk-return profile of the loan portfolio.

The models derive optimal risk-return trade-offs by quantifying the effects of the loan portfolio's risk-return profile on the market price of equity. The models consider the limited liability of shareholders, the risk premium of shareholders, and market discipline in the form of the disciplining by uninsured debtholders. Furthermore, it is calculated how the optimal risk-return trade-off changes in the presence of regulatory capital requirements and the restriction of a fixed portfolio volume. Finally by extending the models, the influence of insured senior deposits on the optimal risk-return trade-off is analyzed.

However, the models not only contribute to the answering of the research question. They also help in understanding risk management motives of banks, in particular, how market discipline and capital requirements of Basel I and Basel II affect the optimal decision-making of banks with and without the presence of deposits.

9.2 Limitations

Although the dissertation gives important insights into optimal risk-return trade-offs of commercial banks and the suitability of profitability measures for loan portfolios, it has several limitations, which are discussed in the following.

One basic limitation is that the models developed cannot determine the suitability of profitability measures for loan portfolios in general since the assumptions of the models are quite specific. It is important to remark that the findings only hold as long as it is realistic to assume that the bank's portfolio return is beta-distributed. In the case of credit derivatives being used to change the portfolio's risk-return profile, this might not hold. Furthermore, the embedded loan portfolio model is very simplifying. This holds in particular for the assumption that loans have the same size and equal probabilities of default as well as that both the losses given default and the default correlations between loans are constant and equal.

A further limitation of the models is that they only consider a limited number of effects of risk-taking on shareholder value. As discussed, there are further risk management motives which have the potential to significantly influence the optimal risk-return trade-off of commercial banks. However, some of them require dynamic multi-period models and not static one-period models such as the ones presented.

Another limitation of the models is that they neglect the fact that banks are not only exposed to credit risk, but that they also have to deal with other sources of risk such as market and operational risk. This could increase the risk-sensitivity and with it the required equity-to-assets ratio of debtholders, leading to higher costs of risk-taking. Furthermore, for banks not only involved in the lending business, besides the risk-return profile of the loan portfolio, the correlations between the loan portfolio return and the return of other business lines are also relevant. In this case, optimal risk-return trade-offs can only be determined for the aggregate business portfolio if the derivation of optimal risk-return trade-offs is possible at all.

A limitation of the models is also that they do not endogenize the bank's decision to commit to a certain risk-return profile and to signal the correct credit standing. The assumption of this behavior might be plausible in reference to expensive agency conflicts. However, for models explaining optimal risk-return trade-offs, it would be beneficial to consider agency costs as well. To consider agency costs, it is necessary to adopt a framework with asymmetric information which also allows costs of equity-funding to be explained. Analyzing optimal risk-return trade-offs by considering asymmetric information and agency costs could be a very interesting future research opportunity.

Furthermore, it would have been desirable to derive solutions explicitly instead of using numerical solving methods. Numerical solving methods have the drawback that they only work with concrete parameter values. Therefore, the generality of the results cannot be assured. Alternatively, more restrictive assumptions could be made to allow algebra to solve the problems. But although the results become more general, they can lose their validity at the same time because simplified assumptions sacrifice the subtle modeling of benefits and costs of a bank's risk-taking. However, the models are tested using a wide range of parameter values. It is found here that the results only change quantitatively but not qualitatively for realistic parameter values. This even holds for very low bankruptcy costs. Therefore, the findings are very robust.

9.3 Implications

The findings of the models are relevant in several respects.

First, there are implications for the risk management of commercial banks: To optimize shareholder value, banks should act risk aversely even if shareholders have limited liability. Therefore, risk should be priced above the market price of risk, consistent with Froot and Stein (1998).

Second, the findings suggest a different approach for the loan portfolio optimization than suggested by the literature: Instead of relying on profitability measures based on capital market models, loan portfolio optimization should start by analyzing the optimal risk-return trade-off of the considered bank, which depends on its concrete situation. This even holds if the capital market models from which the profitability measures are derived are based on bank-specific risk measures such as the Value at Risk and the Expected Shortfall, as in Campbell, Huisman, and Koedijk (2001) and Alexander and Baptista (2003).

Third, there are implications for the theory of economic capital: In the models presented, it is optimal for banks to adjust their solvency level to the business opportunities. Therefore, risk adjusted profitability measures such as RORAC might be misleading if they are applied with a fixed solvency level.

Fourth, the findings are important for the banking regulation: It is found that regulatory capital requirements affect the optimal risk-return trade-offs of commercial banks. A fixed equity-to-assets ratio, as required by Basel I, can make it optimal for banks to behave less risk aversely. However, this result is not the same as in Koehn and Santomero (1980), who find that banks with a high risk aversion select a less risky portfolio if they are confronted with a fixed equity-to-assets ratio. This is because they assume that the bank has a fixed optimal risk-return trade-off given by the utility function of the bank. In contrast, the models presented show that the optimal risk-return trade-off of banks itself changes in the presence of capital requirements.

While the capital requirement of Basel I can lead to less risk averse behavior by banks, it is found that the considered capital requirements of Basel II based on the internal ratings-based approach influence a bank's risk-taking in the opposite way by making it optimal for banks to behave significantly more risk aversely. This gives reason to expect that Basel II will alter the lending decisions of banks with respect to debtors having a low credit standing.

Fifth, the findings are also relevant to deposit insurances: It is shown that a deposit insurance can charge an insurance premium only dependent on the volume of deposits without provoking banks to significantly increase risk at the expense of the deposit insurance, as argued by Merton (1977). This is not because the deposit insurance covers the risk of depositors only partly or pays late, as argued for insurance guarantee funds and property liability insurance companies by Cummins and Sommer (1996). Banks will not significantly increase risk since it is optimal for them to behave risk aversely in the presence of uninsured senior deposits as well.

A

Derivations for Chapters 2 to 5

A.1 Expected Shortfall

$$
\begin{aligned}
& VaR + \frac{LPM_{-VaR,1}(P)}{LPM_{-VaR,0}(P)} && \text{(A.1)}\\
&= VaR + \frac{\int\limits_{-\infty}^{-VaR} (-VaR - P) f(P) dP}{\int\limits_{-\infty}^{-VaR} f(P) dP} && \text{(A.2)}\\
&= VaR + \frac{-VaR \int\limits_{-\infty}^{-VaR} f(P) dP - \int\limits_{-\infty}^{-VaR} P f(P) dP}{\int\limits_{-\infty}^{-VaR} f(P) dP} && \text{(A.3)}\\
&= VaR - VaR - \frac{\int\limits_{-\infty}^{-VaR} P f(P) dP}{\int\limits_{-\infty}^{-VaR} f(P) dP} && \text{(A.4)}\\
&= -\frac{\int\limits_{-\infty}^{-VaR} P f(P) dP}{\int\limits_{-\infty}^{-VaR} f(P) dP} && \text{(A.5)}\\
&= ES. && \text{(A.6)}
\end{aligned}
$$

A.2 Covariance

Let X, Y, and Z be random variables and a, b, c, d be constant parameters. Then

$$Cov(X+Y,Z) = Cov(X,Z) + Cov(Y,Z) \tag{A.7}$$

and

$$Cov(a+bX, c+dY) = Cov(bX, dY) = bdCov(X,Y). \tag{A.8}$$

So

$$-Cov(m_{t+1}, r_{i,t+1}) = Cov(\sum_{k=1}^{K} b_{kt} S_{k,t+1}, r_{i,t+1}) \tag{A.9}$$

$$= \sum_{k=1}^{K} Cov(b_{kt} S_{k,t+1}, r_{i,t+1}) \tag{A.10}$$

$$= \sum_{k=1}^{K} b_{kt} Cov(S_{k,t+1}, r_{i,t+1}). \tag{A.11}$$

A.3 Efficient Frontier

Substituting r_P in equation (4.7)

$$\theta_T(r_P) = (\int_{-\infty}^{T} (r_P - T)^2 f(r_P) dr_P)^{\frac{1}{2}}$$

with the right-hand side of equation (4.6)

$$r_P = w r_M + (1-w) r_f,$$

the new boundaries for r_M, $b1$ and $b2$, result from

$$-\infty = w\ b1 + (1-w) r_f \ \Rightarrow\ b1 = -\infty \tag{A.12}$$

and

$$T = w\ b2 + (1-w) r_f \ \Rightarrow\ b2 = \frac{T-(1-w)r_f}{w}. \tag{A.13}$$

Furthermore, from

$$\frac{dr_P}{dr_M} = w, \tag{A.14}$$

it results that

$$dr_P = w dr_M. \tag{A.15}$$

Thus,

$$\theta_T(r_P) = (\int_{-\infty}^{\frac{T-(1-w)r_f}{w}} (w r_M + (1-w) r_f - T)^2 f(w r_M + (1-w) r_f) w dr_M)^{\frac{1}{2}} \tag{A.16}$$

$$= (\int_{-\infty}^{\frac{T-(1-w)r_f}{w}} (w r_M + (1-w) r_f - T)^2 g(r_M) dr_M)^{\frac{1}{2}} \tag{A.17}$$

with g being the density function of r_M.[1]

[1] From $1 = \int_{-\infty}^{\infty} f(r_P) dr_P = \int_{-\infty}^{\infty} f(w r_M + (1-w) r_f) w dr_M = \int_{-\infty}^{\infty} g(r_M) dr_M$, it follows that $g(r_M) = f(w r_M + (1-w) r_f) w dr_M$ is the density function of r_M.

Substituting w with the right-hand side of equation (4.8)

$$w = \frac{\mu_P - r_f}{\mu_M - r_f}$$

leads to

$$\theta_T(r_P) = \Big(\int_{-\infty}^{\frac{-T\mu_M + r_f(T+\mu_M-\mu_P)}{r_f-\mu_P}} (r_f - T + \frac{(r_M - r_f)(\mu_P - r_f)}{\mu_M - r_f})^2 g(r_M) dr_M \Big)^{\frac{1}{2}}. \tag{A.18}$$

A.4 Froot et al. (1993)

The profit P is given as

$$P(w, I^*(w)) = f(I^*) - I^* - C(e^*) \tag{A.19}$$

with

$$e^* = I^*(w) - w. \tag{A.20}$$

The first derivative of P with respect to w at $I = I^*$ is

$$\begin{aligned} P_w(w, I^*(w)) &= f_I(I^*)\frac{dI^*}{dw} - \frac{dI^*}{dw} - C_e(e^*)\frac{de^*}{dw} \\ &= f_I(I^*)\frac{dI^*}{dw} - \frac{dI^*}{dw} - C_e(e^*)(\frac{dI^*}{dw} - 1). \end{aligned} \tag{A.21}$$

The second derivative of P with respect to w at $I = I^*$ is

$$\begin{aligned} &P_{ww}(w, I^*(w)) \\ &= f_{II}(I^*)(\frac{dI^*}{dw})^2 + f_I(I^*)\frac{d^2I^*}{dw^2} - \frac{d^2I^*}{dw^2} \\ &\quad -C_{ee}(e^*)(\frac{dI^*}{dw} - 1)^2 - C_e(e^*)\frac{d^2I^*}{dw^2} && \text{(A.22)} \\ &= f_{II}(I^*)(\frac{dI^*}{dw})^2 - C_{ee}(e^*)(\frac{dI^*}{dw} - 1)^2 && \text{(A.23)} \\ &\quad +\frac{d^2I^*}{dw^2}[f_I(I^*) - 1 - C_e(e^*)]. && \text{(A.24)} \end{aligned}$$

Since the first-order condition of P with respect to I is

$$h(w, I^*(w)) = f_I(I^*) - 1 - C_e(e^*) = 0, \tag{A.25}$$

it holds that

$$P_{ww}(w, I^*(w)) = f_{II}(I^*)(\frac{dI^*}{dw})^2 - C_{ee}(e^*)(\frac{dI^*}{dw} - 1)^2. \tag{A.26}$$

Applying the implicit function theorem to the first-order condition of P with respect to I, it follows that

$$\begin{aligned} \frac{dI^*}{dw} &= -\frac{\frac{\partial h(w, I^*(w))}{\partial w}}{\frac{\partial h}{\partial I^*}} \\ &= -\frac{-C_{ee}(e^*)\frac{\partial e^*}{\partial w}}{f_{II}(I^*) - C_{ee}(e^*)\frac{\partial e^*}{\partial I}} \\ &= -\frac{C_{ee}(e^*)}{f_{II}(I^*) - C_{ee}(e^*)}. \end{aligned} \tag{A.27}$$

Solving this equation for $C_{ee}(e^*)$ leads to

$$C_{ee}(e^*) = \frac{\frac{dI^*}{dw} f_{II}(I^*)}{\frac{dI^*}{dw} - 1}. \tag{A.28}$$

Using this equation, the second derivative of P with respect to w at $I = I^*$ can be represented as

$$P_{ww}(w, I^*(w)) = \frac{dI^*}{dw} f_{II}(I^*). \tag{A.29}$$

B

Derivations for Chapters 6 and 7

B.1 Parameters as Functions of Return Moments

Assume that a loan is either paid back completely or, in case of a default, is partly paid back. Then the standard deviation of losses from a loan i, σ_i, is

$$\begin{aligned} \sigma_i &= \sqrt{PD_i \; (EAD_i \; LGD_i)^2 - EL_i^2} \\ &= \sqrt{PD_i - PD_i^2} \; EAD_i \; LGD_i \end{aligned} \tag{B.1}$$

with the loan's probability of default PD_i, the loan's exposure at default EAD_i, the loan's loss given default LGD_i, and the expected loss $EL_i = PD_i \; EAD_i \; LGD_i$.

The standard deviation of the losses of a loan portfolio P, σ_P, is therefore

$$\sigma_P = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{i,j} \sigma_i^2}. \tag{B.2}$$

It is assumed that the loan portfolio consists of N loans with equal PD as well as equal and constant EAD and LGD. Therefore it is set $\sigma_i = \sigma$. Furthermore, it is assumed that the default correlations between loans are constant, i.e., $\rho_{i,j} = \rho$, $i \neq j$. Then σ_P can be represented as

$$\begin{aligned}
\sigma_P &= \sqrt{\sum_{i=1}^{N} (\sigma^2 + \sum_{j=1, j \neq i}^{N} \rho \sigma^2)} \\
&= \sqrt{N(\sigma^2 + (N-1)\rho\sigma^2)} \\
&= \sqrt{N\sigma^2 + N(N-1)\rho\sigma^2}. \qquad \text{(B.3)}
\end{aligned}$$

Recall that $\sigma_d = \frac{\sigma_P}{N \; EAD}$ and assume that the portfolio consists of a large number of loans and the correlation ρ is strictly positive. Then σ_d can be approximated as

$$\begin{aligned}
\sigma_d &= \frac{\sqrt{N\sigma^2 + N(N-1)\rho\sigma^2}}{N \; EAD} \\
&\approx \frac{\sqrt{\rho}\sigma}{EAD} \\
&= \sqrt{\rho}\sqrt{PD - PD^2} \; LGD. \qquad \text{(B.4)}
\end{aligned}$$

Rearranging equation (B.4) gives

$$PD = \frac{LGD \sqrt{\rho} - \sqrt{LGD^2 \; \rho - 4\sigma_d^2}}{2 \; LGD \sqrt{\rho}}. \qquad \text{(B.5)}$$

Since the expected default rate μ_d is

$$\mu_d = PD \; LGD, \qquad \text{(B.6)}$$

it follows that

$$\mu_d = \frac{LGD \sqrt{\rho} - \sqrt{LGD^2 \; \rho - 4\sigma_d^2}}{2\sqrt{\rho}}. \qquad \text{(B.7)}$$

From equation (6.4), it results that

$$r_L = \frac{1 + \mu_r}{1 - \mu_d} - 1 \qquad \text{(B.8)}$$

and

$$\sigma_d = \frac{\sigma_r}{1 + r_L}. \qquad \text{(B.9)}$$

Inserting equations (B.7) and (B.9) into equation (B.8) and solving for r_L leads to:

$$r_L = ((LGD + 2\mu_r - LGD\ \mu_r)\sqrt{\rho} - \sqrt{LGD^2\ (1+\mu_r)^2\rho - 4(1-LGD)\sigma_r^2}) / (2\sqrt{\rho}(1-LGD)). \tag{B.10}$$

Using equation (B.9), it follows that

$$\sigma_d = \frac{2\sqrt{\rho}(LGD-1)\sigma_r}{(LGD-2)(1+\mu_r)\sqrt{\rho} + \sqrt{LGD^2\ (1+\mu_r)^2\rho - 4(1-LGD)\sigma_r^2}}. \tag{B.11}$$

Inserting equation (B.11) into equation (B.7) leads to

$$\mu_d = \frac{1}{2}(LGD - \sqrt{LGD^2 - \frac{16(LGD-1)^2\sigma_r^2}{((LGD-2)(1+\mu_r)\sqrt{\rho} + \sqrt{LGD^2(1+\mu_r)^2\rho - 4(1-LGD)\sigma_r^2})^2}}) \tag{B.12}$$

Thus,

$$r_L = r_L(\mu_r, \sigma_r), \tag{B.13}$$

$$\mu_d = \mu_d(\mu_r, \sigma_r), \tag{B.14}$$

and

$$\sigma_d = \sigma_d(\mu_r, \sigma_r) \tag{B.15}$$

with given LGD and ρ.

B.2 Density Functions

Density Function of the Default Rate

The density function of the default rate d is given as

$$g(d) = \begin{cases} \frac{1}{B_g(\alpha,\beta)} d^{\alpha-1}(1-d)^{\beta-1} & \text{if } 0 \leq d \leq 1; \\ 0 & \text{otherwise,} \end{cases} \tag{B.16}$$

with the standardization factor

$$B_g(\alpha, \beta) = \int_0^1 d^{\alpha-1}(1-d)^{\beta-1} dd$$

and the parameters

$$\alpha = \frac{\mu_d^2(1-\mu_d)}{\sigma_d^2} - \mu_d$$

and

$$\beta = \frac{\mu_d(1-\mu_d)^2}{\sigma_d^2} + (\mu_d - 1).$$

Density Function of the Portfolio Return

Using equation (6.4), the default rate d can be represented as

$$d = 1 - \frac{1+r}{1+r_L}. \tag{B.17}$$

In order to obtain the density function $h(r)$ of the portfolio return r, d is substituted in equation (B.16) by the the right-hand side of equation (B.17).

The new boundaries for r, $b1_h$ and $b2_h$, are derived as follows:

$$0 = d = 1 - \frac{1+b2_h}{1+r_L} \;\Rightarrow\; b2_h = r_L \tag{B.18}$$

and

$$1 = d = 1 - \frac{1+b1_h}{1+r_L} \;\Rightarrow\; b1_h = -1. \tag{B.19}$$

Furthermore, from

$$\frac{dd}{dr} = -\frac{1}{1+r_L}, \tag{B.20}$$

it follows that

$$dd = -\frac{1}{1+r_L}dr. \tag{B.21}$$

Thus, the standardization factor becomes

$$B_g(\alpha,\beta) = \int_{-1}^{r_L}(1-\frac{1+r}{1+r_L})^{\alpha-1}(\frac{1+r}{1+r_L})^{\beta-1}(-\frac{1}{1+r_L})dr \tag{B.22}$$

and the density function (B.16) can be expressed as

$$g(r) = \begin{cases} \frac{1}{B_g(\alpha,\beta)}(1-\frac{1+r}{1+r_L})^{\alpha-1}(\frac{1+r}{1+r_L})^{\beta-1} & \text{if } -1 \le r \le r_L; \\ 0 & \text{otherwise.} \end{cases} \tag{B.23}$$

Since

$$\int_0^1 g(d)dd = \int_{-1}^{r_L} g(r)(-\frac{1}{1+r_L})dr = 1 = \int_{-1}^{r_L} h(r)dr, \tag{B.24}$$

$$h(r) = g(r)(-\frac{1}{1+r_L}) \tag{B.25}$$

is the density function of portfolio return r. It can be represented as

$$h(r) = \begin{cases} \frac{1}{B_h(\alpha,\beta)}(1-\frac{1+r}{1+r_L})^{\alpha-1}(\frac{1+r}{1+r_L})^{\beta-1} & \text{if } -1 \le r \le r_L; \\ 0 & \text{otherwise,} \end{cases} \tag{B.26}$$

with

$$B_h(\alpha,\beta) = -(1+r_L)B_g(\alpha,\beta) = \int_{-1}^{r_L}(1-\frac{1+r}{1+r_L})^{\alpha-1}(\frac{1+r}{1+r_L})^{\beta-1}. \tag{B.27}$$

Density Function of the Bank's Free Cash Flow

Using equation (6.4), the bank's free cash flow Y can be represented as

$$Y = (E+D)(1+((1+r_L)(1-d)-1)(1-\tau))+\tau D r_D - D(1+r_D) \tag{B.28}$$

$$\Leftrightarrow d = 1 - \frac{Y + D(1+r_D)(1-\tau) - E\tau}{(E+D)(1+r_L)(1-\tau)}. \tag{B.29}$$

Using a substitution analogous to the one above, the density function of the free cash flow $f(Y)$ turns out to be

$$f(Y) = \begin{cases} \frac{1}{B_f(\alpha,\beta)}(1 - \frac{Y+D(1+r_D)(1-\tau)-E\tau}{(E+D)(1+r_L)(1-\tau)})^{\alpha-1} \\ (\frac{Y+D(1+r_D)(1-\tau)-E\tau}{(E+D)(1+r_L)(1-\tau)})^{\beta-1} & \text{for } b1_f \leq Y \leq b2_f; \\ 0 & \text{otherwise,} \end{cases} \tag{B.30}$$

with

$$B_f(\alpha,\beta) = \int_{b1_f}^{b2_f} (1 - \frac{Y + D(1+r_D)(1-\tau) - E\tau}{(E+D)(1+r_L)(1-\tau)})^{\alpha-1} (\frac{Y + D(1+r_D)(1-\tau) - E\tau}{(E+D)(1+r_L)(1-\tau)})^{\beta-1} dY,$$

$b1_f = \tau(E + D) + \tau D r_D - D(1 + r_D)$,
and $b2_f = (E + D)(1 + r_L(1 - \tau)) + \tau D r_D - D(1 + r_D)$.

B.3 Participation Conditions

If investors invest their total initial capital C_0 in the risk-free alternative, their capital in the second period is

$$C_1 = C_0(1 + r_f). \tag{B.31}$$

Whereas if a part I of the initial capital is invested in the bank, the risky capital in the second period is

$$\widetilde{C}_1 = (C_0 - I)(1 + r_f) + X. \tag{B.32}$$

Investors only invest in the bank if they valuate the risky capital $\widetilde{C}_1$ at least as high as the safe capital C_1:

$$P(\widetilde{C}_1) \geq P(C_1). \tag{B.33}$$

Using properties of the valuation function (6.7), it results:

$$C_0 - I + P(X) \geq C_0 \tag{B.34}$$

$$\Leftrightarrow P(X) - I \geq 0. \tag{B.35}$$

This participation condition can be interpreted as a demand for a non-negative net present value.

B.4 Specification of Equity Value and Debt Value

Equity Value

The equity value is

$$EV = \frac{1}{1+r_f}(\mathbb{E}(X_E) - \delta\sigma(X_E)) - E, \tag{B.36}$$

with the expected payoff of shareholders

$$\mathbb{E}(X_E) = \int_0^{b2_f} Y f(Y) dY \tag{B.37}$$

and the standard deviation of the payoff

$$\sigma(X_E) = \sqrt{\int_0^{b2_f} Y^2 f(Y) dY - (\mathbb{E}(X_E))^2} \tag{B.38}$$

with $b2_f = (E + D)(1 + r_L(1 - \tau)) + \tau D r_D - D(1 + r_D)$.

Debt Value

The debt value is

$$DV = \frac{1}{1+r_f}(\mathbb{E}(X_D) - \delta\sigma(X_D)) - D, \tag{B.39}$$

with the expected payoff of debtholders

$$\begin{aligned}\mathbb{E}(X_D) = &\int_{b1_f}^{0} (1-b)(Y + D(1+r_D)) f(Y) dY \\ &+ \int_0^{b2_f} f(Y) dY D(1+r_D).\end{aligned} \tag{B.40}$$

and the standard deviation of the payoff of debtholders

$$\sigma(X_D) = (\int_{b1_f}^{0} ((1-b)(Y + D(1+r_D)))^2 f(Y)dY + \int_{0}^{b2_f} f(Y)dY(D(1+r_D))^2 - (\mathbb{E}(X_D))^2)^{\frac{1}{2}} \qquad \text{(B.41)}$$

with $b1_f = \tau(E+D) + \tau Dr_D - D(1+r_D)$
and $b2_f = (E+D)(1+r_L(1-\tau)) + \tau Dr_D - D(1+r_D)$.

B.5 Debtholders' Necessary Participation Condition

According to equation (6.10), and using equation (6.1), (6.2), and (6.7), the participation condition of debtholders is

$$\begin{aligned}
&\mathbb{E}(\mathbb{1}_{\{Y\geq 0\}}[D(1+r_D)] \\
&+\mathbb{1}_{\{Y<0\}}[(1-b)((E+D)(1+r(1-\tau))+\tau D r_D]) \\
&-\delta\sigma(\mathbb{1}_{\{Y\geq 0\}}[D(1+r_D)] \\
&+\mathbb{1}_{\{Y<0\}}[(1-b)((E+D)(1+r(1-\tau))+\tau D r_D]) \\
&\geq (1+r_f)D.
\end{aligned} \tag{B.42}$$

Due to the homogeneity of first degree of the valuation function, this is equivalent to

$$\begin{aligned}
&\mathbb{E}(\mathbb{1}_{\{Y\geq 0\}}[1+r_D]+\mathbb{1}_{\{Y<0\}}[(1-b)((\frac{E}{D}+1)(1+r(1-\tau))+\tau r_D]) \\
&-\delta\sigma(\mathbb{1}_{\{Y\geq 0\}}[1+r_D]+\mathbb{1}_{\{Y<0\}}[(1-b)((\frac{E}{D}+1)(1+r(1-\tau))+\tau r_D]) \\
&\geq 1+r_f,
\end{aligned} \tag{B.43}$$

with

$$1+r_D > (1-b)((\frac{E}{D}+1)(1+r(1-\tau))+\tau r_D \quad \text{for } Y<0.$$

The standard deviation $\sigma(\cdot)$ and the risk aversion parameter δ is non-negative by definition. Furthermore, with a positive standard deviation of the portfolio return, the expected value $\mathbb{E}(\cdot)$ is lower than $1+r_D$. This is because a positive standard deviation of the portfolio return implies a positive probability for $Y<0$. Therefore, the inequality can only be fulfilled if the interest rate on debt r_D is higher than the risk-free rate r_f. Therefore, with a positive standard deviation of the portfolio return,

$$r_D > r_f \tag{B.44}$$

is a necessary participation condition of debtholders.

B.6 Relation of Debt to Equity

Due to the homogeneity of first degree of the valuation function and using equation (6.1) and (6.2), equation (6.12) can be rearranged as follows:

$$\begin{aligned} P(\mathbb{1}_{\{Y\geq 0\}}[D(1+r_D)] \\ + \mathbb{1}_{\{Y<0\}}[(1-b)((E+D)(1+r(1-\tau))+\tau D r_D]) = D \end{aligned} \quad \text{(B.45)}$$

$$\begin{aligned} \Leftrightarrow P(\mathbb{1}_{\{Y\geq 0\}}[1+r_D] \\ + \mathbb{1}_{\{Y<0\}}[(1-b)((\frac{E}{D}+1)(1+r(1-\tau))+\tau r_D]) = 1. \end{aligned} \quad \text{(B.46)}$$

In order to fulfill this equation for all values of E, the ratio $\frac{E}{D}$ has to be constant. This only holds if the equity-to-assets ratio $\frac{E}{TC}$ is constant, since $\frac{E}{TC} = \frac{E}{E+D} = (1+(\frac{E}{D})^{-1})^{-1}$.

B.7 Density Function of the Free Cash Flow with Deposits

Using equation (7.2), the bank's free cash flow $\overline{Y}$ can be represented as

$$\overline{Y} = (E+D+S)(1+((1+r_L)(1-d)-1)(1-\tau)) \\ +\tau(Dr_D+Sr_S)-D(1+r_D)-S(1+r_S) \tag{B.47}$$

$$\Leftrightarrow d = 1 - \frac{\overline{Y}+(D(1+r_D)+S(1+r_S))(1-\tau)-E\tau}{(E+D+S)(1+r_L)(1-\tau)}. \tag{B.48}$$

Using a substitution analogous to the one used to compute the density function $h(r)$ (see appendix B.2), the following density function of the free cash flow $\overline{f}(\overline{Y})$ results:

$$\overline{f}(\overline{Y}) = \begin{cases} \frac{1}{B_{\overline{f}}(\alpha,\beta)}\Big((1-\frac{\overline{Y}+(D(1+r_D)+S(1+r_S))(1-\tau)-E\tau}{(E+D+S)(1+r_L)(1-\tau)})^{\alpha-1} \\ (\frac{\overline{Y}+(D(1+r_D)+S(1+r_S))(1-\tau)-E\tau}{(E+D+S)(1+r_L)(1-\tau)})^{\beta-1}\Big) & \text{for } b1_{\overline{f}} \le \overline{Y} \le b2_{\overline{f}}; \\ 0 & \text{otherwise,} \end{cases} \tag{B.49}$$

with

$$B_{\overline{f}}(\alpha,\beta) = \int_{b1_{\overline{f}}}^{b2_{\overline{f}}} (1-\frac{\overline{Y}+(D(1+r_D)+S(1+r_S))(1-\tau)-E\tau}{(E+D+S)(1+r_L)(1-\tau)})^{\alpha-1} \\ (\frac{\overline{Y}+(D(1+r_D)+S(1+r_S))(1-\tau)-E\tau}{(E+D+S)(1+r_L)(1-\tau)})^{\beta-1} dY,$$

$b1_{\overline{f}} = \tau(E+D+S)+\tau(Dr_D+Sr_S)-D(1+r_D)-S(1+r_S)$,
and
$b2_{\overline{f}} = (E+D+S)(1+r_L(1-\tau))+\tau(Dr_D+Sr_S)-D(1+r_D)$
$-S(1+r_S)$.

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