Demographic Statistics

Methods And Measures In Demography

Nicholas N.N. Nsowah-Nuamah

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NICHOLAS N.N. NSOWAH-NUAMAH

DEMOGRAPHIC STATISTICS

METHODS AND MEASURES IN DEMOGRAPHY

Demographic Statistics: Methods And Measures In Demography
2n edition

ISBN 978-87-403-1693-3

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In Memory of my beloved Godson, the late
Wisdom Ato Kwamina Otoo, alias Leftie

FOREWORD

In terms of demographic training, what has occurred at the University of Ghana is the most important development in tropical Africa. Teaching was developed by the university, at first within its Sociology Department and latter on, at the United Nations Regional Institute for Population Studies (RIPS). With the exception of Dr. Barclay's book – "Techniques for Population Analysis" – most of the accepted texts in demographic methods available at that time were designed for the purpose of training actuaries. Dr. Barclay's book was, on the other hand, intended for more general purpose of population analysis. It was not until the late 1980s that the late Professor G.M.K. Kpedekpo's book – Essentials of Demographic Analysis for Africa – provided a course in techniques of demographic analysis with special emphasis on the use of these methods for analysing African population data. Nevertheless, there was still the need for a text that would be of immense value to students with limited background in mathematics or statistics. Professor Nsowah-Nuamah has now filled this need. This new book is intended both as a classroom text for courses in techniques of demographic analysis, aimed at instructing students in how to use African population data for analytical studies, and as reference for researchers who have the occasion to use population data.

The book provides an examination of the principles and assumptions underlying the specific methods that are needed for a formal analysis of population data of greatly varying quality and content. In particular, Professor Nsowah-Nuamah has gone out of his way to draw illustrations from the problems and statistics of the African countries.

Professor Nsowah-Nuamah takes nothing for granted in the previous training of his readers. As he points out, the techniques of demographic analysis are less formidable than they often seem to the uninitiated once the underlying logic of the method is understood. In the hope of making the book as widely useful as possible, he has presented the methodological material in the simplest mathematical form. With the exception of the chapter on the life tables the book is written to be comprehensible to readers without training in mathematics or statistics. In fact, students and researchers with diverse backgrounds, some of whom may have scarcely more than an elementary knowledge of algebra, will find the book extremely useful.

University of Ghana
April 2007

Professor Samuel Kwesi Gaisie

PREFACE TO THE SECOND EDITION

I have been encouraged by the widespread use of the first edition and the positive comments received from students, lecturers and researchers. The need to update the book with current data and approaches had been identified for some time now but my status as the Rector of the Kumasi Polytechnic and then the Vice Chancellor of Kumasi Technical University coupled with the fact that I wanted to complete a book on Regression Analysis made me put off the writing of this edition for far too long.

With humility, I am presenting this second edition, thoroughly revised and expanded taking into consideration the feedback from readers. One such feedback is the absence of a topic on "Nuptiality" and this have been included as Chapter 10. Most of the old chapters have been restructured to allow for easy comprehension.

I wish to thank all those who in any way have given me assistance in completing this edition. This should include Dr John Agyei, a lecturer at Kumasi Technical University, who went through this edition and also Joshua Appiah at the Publication Unit of the University who drew and inserted all the diagrams in the main body of the book and also designed the cover.

I also show appreciate to my wife Gladys and my children, Nicholas Jnr, Nicolina and Nicole who denied the family good to enable me complete this edition.

Kumasi *N.N.N. Nsowah-Nuamah*
December, 2016

PREFACE TO THE FIRST EDITION

This text aims at providing students and researchers with an insight into basic demographic techniques which will provide the quantitative background knowledge required in the modern environment.

The book is designed primarily for students whose major interest lies in applying statistical methods to problems in demography. These include graduate students who need a rapid and fairly intensive overview of the subject. It also has in mind students who are specialising in demography and need a basic introduction to the discipline. It is directed at researchers who are interested in the methods of calculation used by demographers. Finally it will be helpful to health workers and researchers in health who may require calculation and interpretation of health indicators.

The text is set up to serve either as terminal discussion or as an introduction to advanced demographic techniques. The emphasis is not on the statistical methods themselves, but their proper use in demography. While it has little to say of the art of the tool-maker (the mathematician or the statistician who invents the method for performing a certain type of demographic job), it takes great pains to make clear the use and limitations of each tool. In order to help researchers and students acquire a "feeling" of usefulness and to make them translate the concepts and formulas in demography into practice terms, the basic ideas of demographic techniques are presented clearly with worked examples illustrating their application. As much as possible, data for countries in Africa have been used in the examples. Much emphasis has been placed on demographic data for Mauritius because they are comprehensive and easily available. It must, however, be emphasised that the data used here are just for illustrating the application of techniques and not an attempt to analyse countries' data. This explains why some data used are very old, sometimes modified to fit the illustration and sometimes hypothetical. This text is, therefore, not a good source of obtaining a country's demographic data.

Most texts on demography ignore the interpretation of results. A distinctive feature of this text, therefore, is the proper demographic interpretation given to results obtained from calculations. What the text does not do always is describing trends and patterns depicted in tables.

With the exception of a few results, the book does not claim originality for its contents but for its style of presentation. It is user-friendly and the student may not need the help of any teacher to understand the content. No prior knowledge of demography and statistics is assumed. The approach is non-mathematical although some familiarity with the use of symbolic notation and elementary algebra is helpful in mastering the material presented. So far as mathematical and statistical prerequisites are concerned, this text is self-contained and self-explanatory.

A brief tour through the book. Chapter 1 gives the history of demography and introduces basic concepts in this field. Chapter 2 discusses the sources and uses of and the basic errors in demographic data. The basic analytical tools for measuring demographic phenomena and processes, such as the ratio and the rate are described in Chapter 3. Chapter 4 deals with the statistical methods of determining population size and growth. Measures of population composition by sex, age and by both sex and age are treated in Chapter 5. The remaining chapters discuss the measures of factors of population change: Chapters 6 and 7 on mortality and life tables, respectively; Chapters 8 and 9 on fertility and reproductivity, respectively; and Chapter 10 on migration.

Although the text is designed principally for students of demography and statistics, the brevity and relative simplicity of this text and most importantly the wide coverage of methods and measures of demography make it a useful reference work for other researchers including sociologists, geographers, economists, planners, census and statistical officers and even the general public.

Accra *N.N.N.Nsowah-Nuamah*
February, 2007

ACKNOWLEDGEMENT

First and foremost, I express my sincere thanks to the Almighty God both for the gift of writing He has endowed me with and for the stamina I always have to complete the books I write.

I would like to thank all those who had time to go through the text and made valuable comments and suggestions. Space will allow me to mention the names of only David Kombat, Victor Boadu and Anthony Pharin Amuzu. While the first two pointed out a number of typographical errors, the latter went through all the calculations and made corrections.

In spite of careful checking by me and others some of whom I have acknowledged, it is possible that some occasional errors might have crept through. All such errors are, of course, my responsibility, including any flaws and inadequacies. I would appreciate if information about such mistakes or comments would be sent directly to me, (my email address is n3n_nuamah.yahoo.com) or via ACADEC Press (acadecpress@gmail.com).

1 INTRODUCTION

1.1 HISTORY OF DEMOGRAPHY

Interest in demography can be traced back to ancient times. In its early stages, demography was considered primarily as an enumeration of a population, and was documented as early as 3800 BC in Babylon. Oriental legends and biblical references indicate that an enumeration or census of a population by age and by locality was not uncommon. The Persian Empire's military, for example, conducted a census in the year 500–499 BC for the purpose of issuing land grants and taxation. We also know of the directives by God to Moses to conduct censuses as captured in the Bible: *Take a census of the whole Israelite community by their clans and families, listing every man by name, one by one. You and Aaron are to number by their divisions all the men in Israel twenty years old or more who are able to serve in the army* (Numbers 1:1 in the Old Testament). A number of such counts abound in Chapter 4 of Numbers. In the New Testament of the Bible also, the counting of people around the birth of Jesus Christ is well documented.

The main objective of earliest census exercises was primarily for military records and manpower, as well as for taxation. It was, therefore, not surprising that the focus was on male adults only.

Besides enumeration, mortality was also a concern in ancient times. One of the earliest accountings of mortality can be traced back to the Stone Age where, from stone etchings it was determined that only 54 percent of the population reached the age of 5; and only 3 percent of the population reached the age of 70. In the Bronze Age, a similar pattern was demonstrated with the peak of mortality occurring at 15 years of age.

Demography started with the Life table. The first mathematical formula in demography can be attributed to Girolamo Cardano (Cardan) in 1570 but the first substantive work in demography was published in 1662 by John Graunt who is often called the Father of Demography. The accomplishments of Graunt and Edmund Halley (the first to construct modern mortality table) had marked a cornerstone in the field of demography at the end of the seventeenth century. Modern censuses were started by the United States of America in 1790 and the United Kingdom in 1801.

1.2 DEFINITION OF CONCEPTS

Demography is an interdisciplinary subject cutting across many academic disciplines, such as statistics, mathematics, medicine, biology, geography, sociology, economics, history, ecology, environment, health. For example, the study of mortality is the crossing over between formal demography and health sciences, the analysis of fertility is crossing over between formal demography on one side and biology and sociology on the other side. Again, demographers rely heavily on economics and geography in migration studies. Hence many demographers, have received training in other disciplines before specialising in demography.

Demography, as an interdisciplinary subject, has its own key terms, concepts and phrases and these are now clarified.

1.2.1 POPULATION

Population, in a wider sense, is the totality of all persons living at a certain time within a territory demarcated by natural, cultural or political boundaries. The territory may be distinct social communities, whether a village, a nation, or the entire world. From the point of view of demographic studies, a greater importance is attached to the population of a country.

In ***demo-geography***, population is all human generations in terms of their settlement and migration within a given territory. In ***philosophic-sociological*** terms, population is the subject and at the same time the object of social production. In ***political economy***, it is the basic source of labour force, on one hand, and the subject of consumption, on the other.

In ***demography***, population is all human generations distinguished by their size, age and sex and other compositional variations.

1.2.2 DEMOGRAPHY

The term "demography" was introduced by a Belgian, Achille Guillard in 1855. The word was derived from two Greek words "demos" the people and "graphy" to draw or write.

Demography may, therefore, be defined simply as the science of human population. The population is viewed as an aggregate of persons, represented by certain types of statistics; and demography is concerned with the behaviour of the aggregate (or some of its parts), and not with the behaviour of individuals. According to the United Nations *Multi-lingual Demographic Dictionary*, demography is the scientific study of human populations, primarily with respect to their size, their structure, and their development.

In a narrower sense, demography is concerned with the size, distribution, structure, and change of human populations. In a broader sense, demography is a science that studies the size, territorial distribution, structure and composition of human populations, and of changes over time in these aspects, the causes and consequences of such changes, and the interrelationship of socio-economic factors and changes in the population.

Size of Population

The size of a population is simply the number of persons in a well defined area and at a particular point in time. It increases through births and immigration and decreases through deaths and emigration.

Distribution of Population

The distribution of a population refers to the arrangement of the population in space (that is, geographically or among various types of residential areas) at a given time.

Structure of Population

The structure of a population in its narrowest sense, is the distribution of the population among its sex and age groupings.

Change of Population

The change of a population is the growth or decline of the total population or one of its structural units. The components of change in total population are births, deaths, and net migration.

1.2.3 DEMOGRAPHIC STATISTICS

Demographic statistics, sometimes referred to as ***population statistics***, actually has a dual meaning. It may be used in a singular or plural sense.

In the plural sense, the term "demographic statistics" is used to indicate the collections of numerical data on population. Here, demographic statistics are the population figures themselves, suitably classified and tabulated according to the size and territorial distribution of population of an area, distribution by sex, age, social class, kind of employment, nationality, level of education, marital status, etc. of the population.

In a singular sense, the term "demographic statistics" is used to refer to a subject field of study concerned with methods and procedures for collecting, organising, presenting, analysing and interpreting demographic data relating to:

1) the size and composition of population;
2) factors of natural change (increase or decrease) of population (births and deaths);
3) factors of mechanical change of population (that is, migration and naturalisation).

Population statistics is also meant to study the dissemination of these phenomena, and their changes in given economic and social conditions.

In this book we will not use the word "population statistics" in the plural sense. In reference to facts and figures about population the term population data or demographic data will be used.

1.2.4 DEMOGRAPHIC EVENTS

The factors of natural and mechanical change of population are all sequence of events, called demographic events. Thus, by ***demographic events***, we mean such events that cause natural change (births and deaths) and mechanical change (migration and naturalisation). Events such as entry into school and entry into the labour market are of interest in demographic analysis because of their impact on population change, yet they are not categorised as demographic events.

1.2.5 VITAL EVENTS

Vital events deal with the individual's entrance into life or exit from life and the changes in social and civil status that may occur during a specified duration of time among the members of a population residing within a country or any delimited territory during the same period. Thus, by ***vital events***, we mean such events of human life as births, deaths[1] and changes in civil status throughout lifetime (marriages, divorces, adoptions, legitimations, recognitions, annulments, and legal separations)[2]. Other demographic events, such as migration and naturalisation, are not generally considered as vital events.

1.2.6 VITAL STATISTICS

The term ***vital statistics*** has dual meaning. In a singular sense, it refers to the numerical data pertaining to vital events occurring in a given section of a population. It is synonymous to vital data. In a singular sense, it is the methods and procedures that deal with the registration, compilation, analysis and reporting of aggregates of vital events that occur in a given section of a population during a specified period of time.

1.2.7 VITAL STATISTICS SYSTEM

Related to vital events and vital statistics is vital statistics system. A ***vital statistics system*** can be defined as the total process of legal registration, statistical recording, analysis, presentation and reporting of statistics pertaining to vital events.

1.2.8 DEMOGRAPHIC PROCESS

The demographic process is the sequence of changes that take place in a population as a result of the interplay of demographic events. It differs for the individual and for the group.

Demographic Process with respect to Individual

The demographic process with respect to an individual can be considered as the continuous formation and re-formation of sub-groups in a population due to the occurrence of several events that can take place only in a definite natural sequence. It has the characteristics that:

1) Not all individuals will experience similar sequences;
2) Certain events cannot take place before others. Births, marriages and deaths occur in a definite sequence: a birth marks the beginning of life and death the end of life. Such events as entry to school, labour force, marriage and pregnancy, occur in-between births and deaths in a definite order.

Demographic Process with respect to Group

With respect to a group, if members from other groups are not being permitted to join it at any stage, then the size of the group will steadily reduce to zero or to values close to zero after a period of time. For example, a group of unmarried persons will slowly reduce in size if its members get married and, therefore, leave the group of unmarried persons. However, this phenomenon need not happen. This is because a group in a population could increase in size with every re-formation when during the period of observation, the group itself takes on new characteristics and become a new group, which again is exposed to a new set of events. In the process of group re-formation, the new group draws members from other identified groups as well. What, therefore, determine the basic unit for measuring the process of demographic change of a group are:

1) the information on the exposed group;
2) the number of demographic events occurring in the group during a unit of time.

1.2.9 COHORT

A cohort is a group of individuals who share together a particular event in their life histories during a particular time span. Examples are birth cohort and marriage cohort. A birth cohort or simply the cohort of a given year consist of all those born during that year. For example, those aged 10 years in 2000 are the survivors of the birth cohort of 1990. A marriage cohort means all those who were married during a given period.

Different cohorts are different in their characteristics because they would have passed through different social, economic and demographic conditions. For this reason, cohort analysis of demographic data seeks to understand the process of change over time.

1.2.10 CHARACTERISTICS OF POPULATION

A characteristic of a population is any piece of personal information that can be ascertained in a consistent manner for everyone. It serves to classify each person in a statistical table. Examples of characteristics of a population that have been found to be generally satisfactory include: age, sex, marital status, occupation, residence, religion, race, ethnicity, place of birth, and perhaps number of children ever born. The distribution of most characteristics in a population is related to age and sex.

1.2.11 HOUSEHOLD

A ***household*** may broadly be defined as a group of people living together under the headship of an individual who is mainly responsible for looking after the interest and daily needs of its members. The ***household head*** is a member of the household recognised as such by the other household members. The members may or may not be related to the head (by blood, marriage or adoption) because non-relatives (e.g. house helps) may form part of a household. In general, a household consists of a man, his wife, children and some other relatives or a house help who may be living with them. A household may also be a single member unit. In Ghana, a household is usually defined simply as a person or a group of persons, who live together in the same house or compound and share the same house-keeping arrangements.

The core of a household is a family. A ***family*** is usually defined as the group of people who are related by blood, marriage or adoption. Individual members of a family may live together as a single unit at a place or may be living separately at different places. A family is of two types, namely, the nuclear family and the extended family.

The ***nuclear family*** system consists of the father, mother and children (adopted and/or biological). The ***extended family*** consists of the nuclear family, and parents, siblings, and other relatives. In both a household and a family, the head may be a male or female.

1.3 TYPES OF DEMOGRAPHIC STUDIES

There are two main aspects of human population that need to be studied: ***population static*** – composition of the population at a point in time, and ***population dynamics*** – changes in the population that occur during some period of observation. These two aspects of study have given rise to two methods for the collection of population data, namely, census taking and vital registration of events. These are discussed in Chapter 2.

1.3.1 POPULATION STATIC

The composition of the aggregate (or some of its parts) is described by the distribution of people among certain, more or less, standard categories, such as geographical, sex, age, racial, ethnic and economic. Here, the demographer deals with the study of demographic structure which is the status of persons at particular point in time. This aspect of demography, as stated earlier, is referred to as population static. To deal with population static, the demographer depends so much on general descriptive statistics with special ratios (such as, masculinity proportion and ratio, age dependency ratio and age ratio) and graphical devices (such as the population pyramid).

1.3.2 POPULATION DYNAMICS

Population is subject to constant changes. For example, the population present at the beginning of the year will not quite be the same as the population that will be present at the middle of the year.

Population changes are the result of "events", which add to or take away members of the population. It is for this reason that births and deaths are called vital events. The demographic events that produce population changes do not happen all at once; they are spread throughout a period of time, and the length of the period must be specified.

In this type of studies, demographers deal with the study of demographic processes. The focus is dynamics and occurrence of events over time. This aspect of demography is known as population dynamics and it includes, mortality, fertility, reproductivity, nuptiality and migration. For purposes of dealing with population dynamics, demographers have developed comprehensive and elaborate set of "rates" designed to measure demographic events or components of population dynamics. Finally, because population is changing, population composition must be reckoned with reference to some particular time which is usually taken as one year.

1.4 BRANCHES OF DEMOGRAPHY

In the broader sense, demography may be conceived as encompassing both "demographic analysis" and "population studies".

1.4.1 DEMOGRAPHIC ANALYSIS

Demographic analysis studies the components of variation and change in demographic variables[3] and the relationships between them. It is also known as ***formal demography*** or ***demographic methods*** and deals with the methods, techniques and measures used to study the mechanisms of population change in demography. This branch of demography uses simple or modest mathematics and statistics in dealing with fertility, mortality, migration and marriages/union formation. The focus is on relations within the demographic system.

The study of population, predominantly human population, achieved its mathematical character in the twentieth century and this has become known as ***mathematical demography*** or ***technical demography***. It is the study of population and its analysis through sophisticated mathematics and mathematical models, dealing only with demographic variables in a mathematical way. The subject of mathematical demography can be viewed from either a deterministic viewpoint or from a stochastic viewpoint. Mathematical or technical demographers are, therefore, usually mathematicians or statisticians and they are interested in a question such as: What are the possible effects on the birth rate in the future if the number of women of child-bearing age is changing?

1.4.2 POPULATION STUDIES

Population studies is the systematic study of population trends and phenomena in relation to their social setting. It deals with the relationship between demographic and non-demographic variables.

Population studies is often referred to as ***population analysis***, ***social demography*** or ***substantive demography***. Substantive demographers are usually non-mathematicians and non-statisticians. They are mostly social scientists, especially, sociologists, economists and geographers who use concepts, measures and methods of formal demography and statistics to determine the effect of non-demographic variables on a demographic variable on the basis of available data. Thus, unlike demographic analysis, population studies focuses on relations with other fields. It is interested in, say, how changes in income or education can affect fertility or mortality.

Though demography shares many of its methods with and depends so much on statistics, the techniques of data collection, data evaluation and adjustment and data analysis in demography are peculiar to it.

EXERCISES

1.1 What is demography in its narrower and broader senses?.

1.2 What is demographic statistics in its plural and singular senses?

1.3 Distinguish between the following terms as used in demography:

a) Demographic event and vital event;
b) Demographic statistics and vital statistics;
c) Population static and population dynamics.

1.4 Explain the term “demographic process”.

1.5 Explain the following terms as used in demography.

a) Characteristic;
b) Household.

1.6 Discuss in detail the two branches of demography.

2 SOURCES AND USES OF DEMOGRAPHIC DATA

2.1 INTRODUCTION

2.1.1 USES OF DEMOGRAPHIC DATA

The size, territorial distribution, composition and changes in future of population form one of the most important sources of demographic data, needed for:

1) determining the parameters of the development of all spheres of human life;
2) allocating resources to various parts of the country;
3) allocating parliamentary seats;
4) estimating the future population (in terms of size and distribution) which are necessary for the planning of health, educational and manpower needs of the country, and also the social services of government or private organisations;

Demographic data should, therefore, be sufficiently reliable, comprehensive and diverse, reflecting the real picture of the demographic events that are taking place in a country as a whole and in separate geographical areas. Consequently, they should be collected not incidentally but systematically.

2.1.2 CHARACTERISTICS OF DEMOGRAPHIC DATA

Demography is one of the observational, as distinguished from the experimental sciences. Demographic data have the following characteristics:

1) They are "spread out" in time and space, and only some aspects of demographic events can be observed by any investigator at any one time.
2) They are dependent of observation and recording of events occurring in the external world rather than on experiments under more or less controlled conditions in the laboratory.
3) They may relate to totals (single personal data are not relevant), social, economic and health characteristics, and to geographical distribution; they may have references to one point in time or to changes in the course of time.
4) The primary statistical unit of observation in demography is the individual; the secondary unit is the family or the household.

2.1.3 SOURCES OF DEMOGRAPHIC DATA

Demographic secondary data may be obtained from Governmental and non-governmental agencies. They may also be obtained from publications of some international organisations, such as the United Nations and its allied agencies, such as the World Bank. The United Nations publish demographic data in documents including, the United Nations Demographic Yearbook, the United Nations Statistical Yearbook which includes only population data of general interest, World Population Prospects, Estimates and Projections. The World Bank publishes the World Bank Development Reports and Population Reference Bureau Data Sheet. The African Development Bank also publishes African Statistical Year Book.

The limitation of these sources of demographic data is that they do not provide demographic data at the micro level. Information is available only for the country as a whole. The data are, therefore, useful if we require information at the national level and/or require to compare information with that of other countries of the world.

The major sources of demographic data of a country are censuses and vital registration. Certain countries maintain elaborate register where vital events and other details like movement of people are continuously recorded. Other sources, such as, school enrollment registers, national identification records, employment records, unemployment records and electoral registers, are secondary in nature and may provide the needed demographic data. When these data sources are not adequate or when we need to do an in-depth study on specific problems of demographic nature, special demographic sample surveys are carried out.

Basically there are four main sources of demographic primary data:

1) Population Censuses,
2) Vital Statistics Registration Systems,
3) Demographic Sample Surveys,
4) Population Registers.

In developing countries, demographic data come mainly from censuses and surveys and, to a limited extent, from the vital registration system. These three data collection systems must be seen to be complementary to each other; no one system alone can adequately serve the needs of potential users.

In Africa, in particular, the non-conventional sources, such as, parish registers, school registers, labour registers, baptismal records, maternity clinic and child welfare service records, medical records, to a large extent, continue to serve as demographic data sources. However, it is not easy for these sources to serve as potential sources of useful demographic data for the study of the various components of demographic change because of poor coverage due to transport and communication difficulties, low literacy levels, inadequate funding and so on.

2.2 POPULATION CENSUS

The population census is the official survey conducted periodically by a government to count the people of its country and collect relevant information about them.

2.2.1 DEFINITION OF POPULATION CENSUS

The word "census" is derived from the Latin word "censere", which means to assess. In modern usage, the term "census" refers to a nation-wide counting of population. According to the United Nations, ***A population census is the total process of collecting, compiling, evaluating, analysing and publishing or otherwise disseminating demographic, economic and social data pertaining, at a specified time, to all persons in a country or delimited part of a country*** (United Nations, 2006).

2.2.2 PURPOSES OF POPULATION CENSUS

A population census is the most comprehensive source of data on basic demographic characteristics at the macro and micro level for a country. It is aptly called the inventory of the human population of a country at a specified period of time. It collects information on

1) demographic and social characteristics;
2) educational characteristics;
3) migration;
4) economic characteristics; and
5) other characteristics.

A population census which provides data on size, structure, composition and growth of population, are extremely rich database for:

1) Planning and for evaluating the various government policy intervention programmes at national, regional and district levels:
2) Providing governments the essential data required for:
   i) the provision of services for the people and
   ii) determining the requirement of services from the people;
3) Monitoring progress toward the Millennium Development Goals (MDGs);
4) The study of social and economic trends by government, industry, the academic world, and individual private researchers;
5) Allowing sound estimates of the country as a whole to be made from sample surveys;
6) Providing the foundation for deciding on the structure for political representation.

2.2.3 FEATURES OF CENSUS

A modern population census has several essential features, five of them being considered as essential.

Essential Features of Censuses

Individuality

The individual is the primary unit of enumeration and hence, each individual is enumerated separately and his/her characteristics recorded separately. For characteristics that are common to the whole households, or family, the unit of enumeration may be the household or the family itself.

Territoriality

The census covers a precisely defined territory (the entire country or well-delimited part of it) within which the population count is to be made.

Universality

The census includes every person present and/or residing within its boundary, irrespective of their ethnic and socio-economic characteristics. Non-nationals temporarily residing in the country at the time of the census will have to be enumerated although they may not be included while defining the country's population.

Simultaneity

The population is enumerated with respect to a well-defined reference period. That is, everybody is enumerated at a well-defined reference time, known as the census night.

Other Features of Censuses

Government Sponsorship

Censuses have government sponsorship, that is, the conduct of a census rests on the national government. This is because it is only the government that has the adequate administrative apparatus, substantial financial resources and supreme legal authority over the entire population.

Defined Periodicity

Censuses are taken at regular intervals of time, usually ten years. Strict adherence to periodicity tends to improve the quality of data and makes analysis of data easier.

Confidentiality

The census original record sheets and the raw data on the computer are treated confidential and are not given out even for legal purposes.

Comparability

Census results must be comparable within a country overtime and among countries. To ensure maximum comparability on a national basis, a set of core questions are chosen and additional questions are added to suit regional or country needs.

Compilation and Publication

Plans are made towards compilation and publication of census results. Almost all modern population censuses, indeed, have resulted in some statistical publications. However, occasionally

1) the results are suppressed because they are unsatisfactory for technical or political reasons;
2) not all the planned tables are published especially when data on some subjects turn out to be too deficient.

2.2.4 ORGANISATION OF POPULATION CENSUS

The organisation of a census is very involving and, therefore, requires careful and long-term planning. Some of the activities start as far back as three to five years before the census is taken.[4] We shall identify three phases of census organisation: the pre-enumeration (preparatory) phase, the enumeration phase and the post-enumeration phase.

Pre-enumeration Phase

The pre-enumeration phase involves the following preparatory activities: the establishment of legal basis, putting in place administrative arrangements, undertaking cartographic work, development of census questionnaire and forms and conducting pilot census. We discuss each in detail.

Establishing Legal Basis

Every census requires a legal backing. In some countries, laws are enacted prior to every census but in others there is a permanent legislation to that effect.

1) The law establishes the institutional framework. It specifies the primary administrative authority responsible for the conduct of the census. In most African countries, it is the National Statistical Offices and, in a few other countries, such as Nigeria, it is the Census Offices which conduct censuses.
2) The law determines the scope and timing of the census. The timing depends on the climatic conditions of the country and social and economic activities of the people. When the census date is found to be satisfactory in the first census, subsequent censuses are taken at the same time of the year, unless there is a strong cause to change it. In Ghana, censuses are undertaken in March even though the 2010 census was taen in September because of some challenges.
3) The law gives the administrative power to place legal obligation on the public to cooperate with the census officials, especially the enumerators and to give them truthful answers. It also gives the administrative responsibility to the census officials and places legal obligation on the enumerators to record the responses faithfully and to ensure the confidentiality of the data. This aspect of the law cannot be taken for granted because the quality of census data depends on the public cooperation, the efficiency of enumerators and the arrangement done for the fieldwork.

Putting in Place Administrative Arrangements

The activities involved in the conduct of a census is multifaceted and, therefore, its organisation is not left to only one institution. A national census planning committee with institutional and sectorial representation is, therefore, established to plan and direct the conduct of the census. The committee reviews and finalises the budget, mobilises funds, prepares the advocacy plan and draws up the time schedule for the various operations. This committee also sets up other committees and specifies their responsibilities.

Undertaking Cartographic Work

Census cartographic work is one of the major preparatory activities that require longer time and large proportion of the total census budget. The cartographic work:

1) helps determine the number of enumerators, supervisors and other personnel;
2) helps determine the amount of logistics needed for the enumeration.

The main work here is the demarcation of internal boundaries and the detailed division and subdivision of areas into enumeration units, called enumeration areas. An enumeration area (EA) is the smallest areal unit that will be covered by a single enumerator. To ensure equality of workload among enumerators, the EAs are demarcated on the basis of:

1) population distribution,
2) topography of the terrain, and
3) communication and transportation facilities available in the area.

In each of the EAs, households are identified and numbered, and household members listed, indicating head of household. The respective locations of the households are appropriately recorded on maps. The list of EAs and the EA maps:

1) guide enumerators, during the enumeration phase, to dwellings and other places where people are likely to be during the enumeration period to ensure that every individual and housing unit in the area are enumerated without omission and duplication;
2) provide the basis for producing thematic maps for spatial analysis of the census;
3) provide statistical frame for future sample surveys.

In addition to EAs, supervisory boundaries to contain the detailed division and subdivision of a group of EAs are also demarcated. These are referred to as supervisory areas. A supervisory area (SA) is a group of EAs put together that will be under one supervisor. The demarcation of SAs takes into consideration:

1) topography of the terrain, and
2) communication and transportation facilities available in the area.

Similarly, supervisory maps are also drawn and these guide supervisors to the location of enumerators under their supervision and to ensure that every area under his or her supervision is canvassed and so properly done by the enumerator.

Development of Census Questionnaire and Forms

The census questionnaire is the principal document of the census. It is accompanied by instruction manuals for various categories of field workers and the quality control forms. These instruments are prepared and pretested sufficiently early to enable them to be used in the pilot census stage (see below). The preparation of the questionnaire is guided by the:

1) method of enumeration;
2) cultural and socio-economic set-up of the population;
3) educational level of the population;
4) exposure of the population to the type of information sought from them.

The questionnaire, when completed has always to be pretested to allow defects to be rectified before it is finalised for the pilot census. The pretest assesses:

1) the number of questions;
2) the wording and sequence of questions;
3) the relative importance of questions;
4) the interpretation of questions into the local languages;
5) definition of concepts;
6) the shape, size and spacing between items of the questionnaire which now appears to have gained importance because of the use of the scanning technology for data processing.

Sufficient training is given to the field workers in the use of the questionnaire, the meaning and definition of concepts used and the possible variations for each situation.

The final topics included in the census is determined after consultations with data users and after the pretest. In some cases two types of questionnaires are produced: a short questionnaire (a questionnaire with a few core questions) and a long questionnaire (a questionnaire with detailed questions or special modules). The short questionnaire are administered to the entire population while the long one covers only a sample of the population.

Conducting Pilot Census

At a point in time in the pre-enumeration phase, a pilot census, also called ***trial census***, ***census test***, ***experimental census*** or ***micro census***, is conducted. It is a recent development and has now become an integral part of any modern census. A pilot census is a small-scale study organised to assess:

1) the suitability of the census instruments (questionnaires, manuals and control forms);
2) the quality of the EA and SA maps produced during the demarcation exercise;
3) the publicity level achieved;
4) the adequacy of the arrangements made for the field work, including the recruitment and training procedures for enumerators and supervisors;
5) the implementation of the method and system of enumeration, including the estimation of age from a list of historical events;
6) the data processing strategies, from data capture to tabulations, data analysis and report writing.

After the pilot census has been completed, a subsample of EAs covered in the pilot census is taken for a re-interview, called ***pilot post enumeration survey*** (PPES). The PPES is undertaken to:

1) fully test all of the post enumeration survey operations, forms and manuals;
2) test the matching operation between the pilot census and the PPES questionnaire in order to finalise the matching procedures;
3) evaluate the pilot census;
4) estimate response error for several census questions;
5) provide an early indication of any coverage problems resulting from the census enumeration procedures.

Census Enumeration Phase

The enumeration phase is the most crucial of the entire census-taking but lasts for only a few days. It involves the collection of information concerning each individual in households and recording it on the census questionnaire. It is required that when enumeration is completed, all persons of the country shall be counted once and only once without any exception.

Methods of Census Enumeration

There are conventionally two main methods on census enumeration, namely, the canvasser method and the householder method. Sometimes some countries attempt to combine these two methods.

In the ***canvasser (enumerator) method***, a census official (a trained canvasser/enumerator) visits each of the households in the enumeration area assigned to him/her and collects information about all the persons in the household from one or more of the responsible members of the same household through interviews using the standard questionnaire. The advantages and disadvantages of this method is similar to those of the face-to-face interview (for detail, read Nsowah-Nuamh, 2005). In particular, the canvasser method is useful in largely illiterate populations such as we have in Africa.

In the ***householder method***, the census questionnaire with the corresponding instructions are distributed well in advance of the census date to all the households. The head or any responsible member of the household completes and returns the questionnaire to the census secretariat. The advantages and disadvantages of this method is similar to those of the self-administered questionnaire (read Nsowah-Nuamh, 2005). The householder method is useful in largely literate populations

Evaluation of Data

In census enumeration, there is a reference period called ***census moment***. This is usually the midnight which is usually referred to as the ***census night***. In view of this, census is considered as the photographic impression of the population of a country at a given instant of time as the count of individuals are made in reference to this night.

The completed questionnaires and other census documents are collected and reviewed by supervisors and control centre supervisors and transported to the Head office for data processing.

Post-enumeration Phase

The post-enumeration phase, which constitute the final phase of the census, includes the following:

Post-enumeration Survey

A post-enumeration survey (PES), undertaken shortly after enumeration and after all census documents have been returned to the census office, can be defined as the complete re-enumeration of a representative sample of the census population. Each individual who is enumerated in the post enumeration survey is matched with information from the main enumeration in order to estimate:

1) the error of coverage of the census at the national level and for major subnational domain or population sub-group; and
2) the error of content for specific census items.

When a PES is designed to measure content error only, it is usually known as ***re-interview***.

Data Capture and Tabulation

The technology of census data capture has shifted from manual to scanning to improve accuracy and speed and to reduce investment in terms of equipment, personnel and space. The Optical Mark Reader (OMR) scanner is the technology used by most countries. When the scanned data have been edited and validated, statistical tables are generated.

Census Publication and Dissemination

The final activities of the census-taking is the publication and dissemination of census reports. The publication programme is usually prepared during the pre-enumeration phase in consultation with data users. The numerous publications include:

1) highlights of census results for decision makers and general readers;
2) statistical tables;
3) special reports.

The type and number of special reports differ from countries to countries and also from one census to the other. For instance, the publcations from the 2000 and 2010 censuses of Ghnan were quite different as can be seen in Subsection 2.2.8. Census results may also be disseminated through

1) workshops and seminars;
2) press releases;
3) radio and TV discussions.

Post-enumeration Cartographic Work

The actual formation of the national sample frame is undertaken after census work has been completed. At this stage, the EAs created during the pre-enumeration phase are checked against the population defining criteria of creating an EA, and where necessary, further segmentation or amalgamation of EAs is made.

2.2.5 SYSTEMS OF POPULATION CENSUS-TAKING

A census may be taken on either a *de facto* or a *de jure* basis.

The De Facto Method

Under the *de facto* method, persons are counted at the place where they are found at the census night, irrespective of whether or not they belong to the place. When a person has been counted once somewhere, he/she is not counted again. This method was more common in the English-speaking countries in Africa but of late most countries in the world uses this method. Ghana, for instance, has used the *de facto* method in all her modern censuses.

Advantages of De Facto Method

The *de facto* method:

1) Provides a simple and clear-cut definition for enumerating persons;
2) Offers less chance of double-counting or omission of persons;
3) Is appropriate for a highly mobile population;

Disadvantages of the De Facto

The *de facto* method has the following disadvantages:

1) It is difficult to obtain information about persons in transit (persons travelling on census day or who work at night and consequently would not be found in any of the places where people usually live). This is because it does not make a distinction between the place at which a person is enumerated and the place where he/she maintains his/her residence which determines ones legal and political rights and obligations.
2) It provides incorrect picture of the usual population of a community. It tends to inflate or deflate a population due to temporary movements of population. But for administrative and other purposes, such as, taxation and delineation of constituencies, the usual population is essential.
3) The resident population is needed for many technical purposes like estimating vital rates. These rates may be distorted and would not agree with the definition of a rate since the population base for calculating them will include floating or non-resident population (for example, some persons normally return to their usual residence for the birth of a child). It is, therefore, necessary to identify individuals by their place of "usual residence" and include them as members of the population of the area.

The De Jure Method

Under the *de jure* method persons are counted according to their usual place of residence so that temporarily absent members/persons would be counted as if at home. Similarly, temporary visitors who have an usual residence elsewhere are excluded from the listing but are counted at their usual residence. Most of the French-speaking countries in Africa followed this system. However, of late all censuses conducted in Africa have tended to follow the *de facto* method.

Advantage of De Jure Method

The *de jure* has two main advantages.

1) It gives a picture of the permanent population of the communities enumerated;
2) It gives more accurate estimates of vital rates.

Disadvantages of De Jure Method

The *de jure* method has the following disadvantages:

1) Some persons may be omitted from the count when they are absent from their usual residence (under-coverage).
2) Some persons could be counted twice, once at their temporary residence and again at their usual residence (over-coverage).
3) Foreigners and temporary visitors are excluded and this violates the universality condition of the census.
4) Secondhand information obtained about persons who are temporarily absent may be incomplete or incorrect.

Combined Method

As we have seen, both the *de facto* and *de jure* methods have their advantages and disadvantages so in modern censuses, the two systems are used to obtain maximum advantage. In the combined method, the following terms are used in place of *de facto* and *de jure*.

1) *Place where found at census night*
   This is the geographic place where each individual spent the census night whether or not this was his usual place of residence.

2) *Place of usual residence*
   This is the geographical place where the enumerated person usually resides. This may be the same as, or different from, the place where he spent the census night or his legal residence.

2.2.6 PROBLEMS OF POPULATION COUNTS

As stated earlier, population censuses are conducted to obtain the total population of an area but there are a whole lot of problems associated with achieving this. These include the following:

1) Persons may appear to have more than one usual residence. Examples of such category of people include:
   i) persons who maintain two or more residences;
   ii) students living at a school away from their parental home;
   iii) members of the armed forces living at a military installation but still maintaining private living quarters away from their installation;
   iv) persons who sleep away from their homes during the working week but return home at the end of each week.

2) Persons who stay at the place where they are enumerated for some time but do not consider themselves to be residents of this place because they intend to return to their previous residence at some future time.
3) Persons who have left the country temporarily but are expected to return after some time.

In these instances, clearly stated time limits of presence in, or absence from, a particular place must be set, in accordance with the prevailing circumstances in the country, to determine whether or not the person is usually resident at that place.

2.2.7 EXPERIENCES OF POPULATION CENSUSES IN AFRICA

African societies had known and practised population counting even before their encounter with their colonial masters. The motives behind the headcounts were for military and taxation purposes.

In modern times, census-taking has been a characteristic feature of demographic data collection in non-French-speaking Africa (English, Portuguese and Swahili), at least since the fifties. Censuses were held in 1948 in the then British Africa countries, namely, Ghana, Uganda and Kenya. However, the first serious attempt to conduct a modern population census in Africa is the first population census of Ghana in 1960, three years after the country had attained political independence. In that year, a few other countries also conducted population censuses of some sort. Other English-speaking countries followed later (see Table 2.1).

Country	1960 Round (1955–64)	1970 Round (1965–74)	1980 Round (1975–84)	1990 Round (1985–95)	2000 Round (1995–2004)	2010 Round (2005–14)
English						
Botswana	1964	1971	1981	1991	2001	2011
Gambia	1963	1973	1983	1993	2000	2013
Ghana	1960	1970	1984	–	2000	
Kenya	1962	1969	1979	1989	1999	2009
Lesotho	1956	1966	1976	1986	1996	2006
Liberia	1962	1974	–	–	–	2008
Malawi	–	1966	1977	1987	1998	2008
Mauritius	–	1972	1983	1990	2000	2011
Namibia	1960	1970	1981	1991	2001	2011
Nigeria	1963	1973	–	1991	–	2006
Sierra Leone	1963	1974	–	1985	2004	
South Africa	1960	1970	1980/85	1991/95	2001	2011
South Sudan*	1955-56	1973	1983	1993	–	2008
St Helena				1987	1998	2008
Swaziland	1956	1966	1976	1986	1997	2007
Tanzania	1957	1967	1978	1988	2002	2012
Uganda	1959	1969	1980	1991	2002	2014
Zambia	–	1969	1980	1990	2000	2010
Zimbabwe	1962	–	1982	1992	2002	2012

TABLE 2.1 Inventory of Censuses in English-speaking African Countries
* The Census of erstwhile Sudan covered South Sudan.

The French-speaking countries had, to a large extent, relied on administrative records and sample surveys for demographic data. It was only in the 1970s (during the 1980 round of censuses) that most of the Francophone countries conducted their first modern censuses (Table 2.2). Table 2.3 also gives the years in which population censuses were conducted in the Arab- and Portuguese-speaking countries.

One main deficiency in the conduct of population censuses in Africa is that many of them have failed to maintain the 10-year periodicity. Many African countries postpone or do not conduct their census in some rounds of censuses because of:

1) Funding shortages or complete lack of it;
2) Lack of technical expertise;
3) Lack of political will or commitment to the development of statistics;
4) Desire to guard the status quo, that is, fear of the outcome of the population count (with respect to spatial distributions and the balance of power).

Country	1970 Round (1965–74)	1980 Round (1975–84)	1990 Round (1985–95)	2000 Round (1995–2004)	2010 Round (2005–14)
Benin	–	1979	1992	2002	2013
Burkina Faso	–	1975	1985	1996	2006
Burundi	–	1979	1990	–	2008
Cameroun	–	1976	1987	–	2005
Central African Rep.	–	1975	1988	2003	–
Chad	–	–	1993	–	2009
Comoros	–	1980	1991	2003	–
Congo	1974	1984	–	1996	2007
Cote D'Ivoire	–	1975	1988	1998	2014
DR Congo	–	—-	–	—-	–
Gabon	–	1980	1993	2003	2013
Guinea	–	1983	–	1996	2014
Madagascar	–	1975	1993	–	–
Mali	–	1976	1987	1998	2009

Country	1970 Round (1965–74)	1980 Round (1975–84)	1990 Round (1985–95)	2000 Round (1995–2004)	2010 Round (2005–14)
Mauritania	–	1976–77	1988	2000	2013
Niger	–	1977	1988	2001	2012
Rwanda	–	1978	1991	2002	2012
Senegal	–	1976	1988	2002	2013
Seychelles			1987	1997	2010
Togo	–	1981	–	–	2010

TABLE 2.2 Inventory of Censuses in French-speaking African Countries

Country	1960 Round (1955–64)	1970 Round (1965–74)	1980 Round (1975–84)	1990 Round (1985–95)	2000 Round (1995–2004)	2010 Round (2005–14)
Arab						
Algeria				1987	1998	2008
Comoros*			1991	–	2003	
Djibouti**						2009
Egypt				1986	1996	2006
Libya					1995	2006
Morocco				1994	2004	2014
Somalia				1987	–	–
Sudan***	1955-56	1973	1983	1993	–	2008
Portuguese						
Angola	1960	1970	–	–		2014
Cape Verde	1960	1970	1980	1990	2000	2010
Guinea Bissau	1960	1970	1979	1991	2001	2009
Mozambique	1960	1970	1980	–	1997	2007
Sao Tomé and Principé				1991	2001	2012

TABLE 2.3 Inventory of Censuses in Arab- and Portuguese-speaking African Countries
∗ **Comoros** has three official languages including Comorian and French.
∗ ∗ French is also official languages.
∗ ∗ ∗ The Census of erstwhile Sudan covered South Sudan.

2.2.8 HISTORY OF POPULATION CENSUS IN GHANA

The first population census in Ghana was undertaken in 1891 by the British Administration. Since then, censuses had been conducted at ten yearly intervals except in 1941 when the Second World War (1939–1945) interrupted the series. The census was conducted in 1948, the last to be conducted by the British Administration. It is worth noting that all these censuses were conducted in the same years censuses were conducted in the United Kingdom.

Post-independence Ghana chose to conduct its population censuses in the beginning of each calendar decade, that is, the years ending in "zero". However, like other African countries, Ghana failed to maintain regularity. Beginning 1960, Ghana has had five instead of six population censuses by the year 2010. The census in 1960 was the first serious attempt to conduct a modern population census in Africa. Ten years later in 1970, there was another census. However, the next census was in 1984 after fourteen years time had elapsed. No population census was carried out in the 1990 decade because of political instability.

The principle of conducting a census in the beginning of each decade was restored in 2000 when a census was taken in that year after sixteen years time lag. This was also the first time a ***housing census*** was conducted along side a population census in Ghana. That is, the 2000 Population and Housing Census (PHC) was the enumeration of all persons and all structures (permanent and temporary), including the occupied and unoccupied dwelling units and the occupants thereof in the country. The issues covered on housing included the type of dwelling, the number of occupied and unoccupied dwelling units, the main materials used in constructing houses, methods of waste disposal, utilities and household facilities.

The information obtained from the census on the materials used for constructing various housing units and the availability of facilities such as water and the practices of sanitation, serve as important indicators of the occupants' quality of life, which can aid the government in designing housing improvement programmes.

The information from housing censuses also serves as a basis for planners and policy makers to

1) formulate realistic and relevant housing policies and design appropriate programmes to meet Ghana's housing needs;
2) plan housing and human settlement programmes and policies;
3) determine the adequacy of housing stock and assess the need for additional housing;
4) obtain information on living conditions of those residing in temporary or substandard housing at the national, regional and district levels;
5) evaluate and monitor housing conditions and needs of the population within the context of the Millennium Development Goal Seven (MDG 7).

The publications which came out of the 2000 Population and Housing Census of Ghana were:

1) Preliminary Report;
2) Summary Report of Final Results;
3) Special Report on 20 Largest Localities;
4) Special Report on Urban Localities;
5) Administrative Report;
6) Summary Report on Stock of Houses and Housing Conditions;
7) Gazetteer (list of all localities with available facilities) (3 volumes);
8) Demographic and Economic Characteristics (11 volumes – one for each of the 10 regions and one for the total country);
9) Special Reports on Localities (10 volumes – one for each region);
10) Analysis of District Data and Implications for planning (10 volumes – one for each region);
11) Population Data Analysis Vol. 1 and 2.

The latest PHC in Ghana was conducted in 2010 and this restored the decennial process of census taking, following the 2000 census. Just as the 2000 PHC, the 2010 PHC provided an official count of all living quarters (occupied and vacant) within the nation. In addition to repeating all the modules and concepts of the 2000 PHC, the 2010 PHC included five new modules, namely, Maternal mortality, Disability, Emigration, Information Communication Technology (ICT) and Agriculture.

The publications which came out of the 2010 Population and Housing Census of Ghana are:

1) Summary of Final Results;
2) National Analytical Report;
3) Regional Analytical Reports (*for all 10 Regions*);
4) Demographic, Social, Economic and Housing Report (*mainly tables*);
5) Census Atlas;
6) Women and Men in Ghana;
7) Children, Adolescents and Youth in Ghana;
8) The Elderly in Ghana;
9) Non-Monetary Poverty in Ghana;
10) Millennium Development Goals in Ghana;
11) Population Projections;
12) Post Enumeration Survey (PES) Report;
13) Housing;
14) Urbanisation;
15) Mortality;
16) Fertility;.
17) Migration;
18) Disability;
19) Economic Activities;
20) Education and Literacy.

2.2.9 ADVANTAGES AND DISADVANTAGES OF CENSUSES

Advantages of Censuses

1) Censuses are the best source of information for the total number of people present in an area at a particular time. Data from censuses are very comprehensive in coverage.
2) Census data combined with the Living Standards Measurement Survey (LSMS) are used to produce what has come to be termed poverty map of a country.
3) Censuses provide
    a) data on size, structure, composition and growth of population which constitutes an extremely rich data base for planning and policy-making;

b) data for population projections and calculation of vital rates;
c) adequate socio-economic data at the local level which can be used for grass-root planning and decision-making.
d) a sample frame for subsequent field inquiries.

Disadvantages of Censuses

1) Censuses are gigantic and expensive operations and so most African countries face technical, financial and human resource problems. As a result,
   a) data from censuses may not be timely (the complete report of the 1984 Census of Ghana, for example, came out only in 1991);
   b) censuses have been carried out infrequently in most developing countries (for example, Ghana was unable to keep to the pattern of decennial census: 1960, 1970, 1984, 2000).

2) A census provides data only as at the time of the census and fails to give any information about the population in the intercensal period.
3) There is no perfect or "correct" method for obtaining the total population. The two standard schemes for enumerating a population (*de facto* and *de jure*) may
   a) yield a different size of population for the same subdivision of a country (for example, "people temporarily away from home" would be counted in one place by the *de facto* method and in another place by the *de jure*);
   b) give slightly different total population figures for a country. The selection of either one of these standards (or, more commonly, some mixture of the two) has an effect which is present in every figure of the census.

2.3 VITAL REGISTRATION SYSTEM

2.3.1 USES OF VITAL REGISTRATION

Vital registration is the most important source of obtaining vital statistical data on birth and death.

1) When a country has a good system of registering births and deaths, it can estimate its population size at any given time, assuming that there is no effect of migration.
2) Vital statistics are also used as legal evidence.
3) In addition, the aggregates of vital events serve as basis for
   a) planning public health administration,
   b) implementing social welfare measures, and
   c) for distributing public facilities.

The birth registration is the first legal acknowledgement of a child's existence.

1) It is fundamental to the realisation of a number of rights, including the right to a name and the right to nationality;
2) It is used to ascertain the correct age of the individual to enroll in school, to engage in employment, to enter into marriage, to enjoy special protection under the law, etc.
3) Records on births are used in the generation of the numerator required in the calculation of some demographic rates.

Equally important is the registration of deaths.

1) Death registration is important in demographic analysis. Records on deaths are used in the:
   i) Generation of the numerators required in the calculation of some demographic indicators, such as, mortality rates (crude death rate, infant mortality rate, age-specific death rates);
   ii) Construction of life tables.
   iii) Estimation of life expectancy.

2) Governments and other stakeholders use the incidence and causes of death to:
   i) initiate measures and monitor interventions;
   ii) prioritise health needs of the population.

2.3.2 NATURE OF VITAL REGISTRATION SYSTEMS

Vital registration system (synonymn is ***civil registration system***) consists of continuous and permanent recording of vital events pertaining to births, adoptions, deaths, marriages, divorces, legal separations, and annulments, etc. The recording of vital events is done shortly after they occur and within a specific period of time (for example, one year) in order to facilitate the calculation of vital rates.

Registration of births provides information on place of birth, sex, age and religion of the parents, legitimacy, number of previous births and their sexes, father's occupation and birth place of parents. Similarly, death registration gives information on place of death, sex, age, marital status, number of births, birth place, occupation and cause of death. Similar information is obtained with respect to marriages, divorces, etc. This detail information may differ from country to country.

By legal requirement, the data are usually recorded at the time of the occurrence of the event. In fact many countries require compulsory registration of births and deaths under the law, as for example, the 1965 Birth and Death Registration Act of Ghana. However, only a few African countries, mainly small island countries, like Mauritius, Sao Tome, Cape Verde, and Seychelles, have attained nation-wide registration coverage. A few other North African countries such as Algeria, Egypt, Tunisia and Libya have fairly satisfactory registration systems. In the rest of Africa vital registration is for selected areas, mainly towns or selected populations.

In actual fact, in most developing countries, especially in sub-Saharan Africa, vital registration is at a rudimentary stage of development and the legislation of compulsory registration of vital events is not adhered to. This situation can be traced to:

1) the passive nature of registration systems;
2) prohibitive administrative costs of a vital registration system and insufficient funding by governments to cover such activities;
3) high illiteracy rate;
4) low quality of registration staff;
5) a large proportion of births and deaths which do not take place in hospitals or which are not attended by a doctor;
6) a sizeable rural population scattered throughout the country;
7) the general perception by individuals that they derive little or no benefit from the registration of births and deaths.

Thus, statistics collected through the vital registration system remain deficient in content and coverage and for which reason its data are hardly quoted.

2.3.3 METHODS OF COLLECTING VITAL REGISTRATION DATA

There are two main methods of collecting vital registration data: the active and passive approaches.

Active System of Registration

For the active system of registration, the staff member visits households to collect information concerning births and deaths. In Africa, where illiteracy is very high, vital registration systems depend on active staff effort. Sometimes institutions, like hospitals, churches, or clinics record events and submit to a central agency. This is a semi-active approach.

Passive System of Registration

In the passive system of registration, a member of the household or a person closely associated with the household, registers the event himself/herself at a registration centre. This approach can work well among a literate population which understands and appreciates the importance of registration of vital events.

2.3.4 VITAL REGISTRATION SYSTEM IN GHANA

Vital registration in Ghana (then called the Gold Coast) started with the passing of the Cemeteries Ordinance in 1888, which was limited to the registration of deaths, mostly of expatriate workers of the colonial government. It was amended in 1891, and in 1912, the Births, Deaths and Burials Ordinance was passed which was also amended in 1926. In 1965, the Registration of Births and Deaths Act, Act 301, was passed to establish the Births and Deaths Registry within the Ministry of Local Government with the core business to "provide accurate and reliable information on all births and deaths occurring within Ghana for socio-economic development of the country through their registration and certification".

Birth Registration Form

In Ghana, the Birth Registration Form ("Form A") asks information on particulars of child, mother and father. Information about the child includes full name, sex, date of birth, type of birth, place of delivery and detailed address of place of delivery, and name of district and region.

Information about mother and father includes full name, nationality, hometown, ethnicity and occupation. In addition, information is elicited on mother's age, mother tongue and place and address of usual residence.

The Death Registration Form

The Death Registration Form ("Form B") elicits information about the deceased on full name, sex, date of death, age, nationality, full name of mother, place of death, detailed address of place of death, and usual place of residence of deceased.

2.3.5 ADVANTAGES AND DISADVANTAGES OF VITAL REGISTRATION SYSTEM

Advantages of Vital Registration System

1) It is a smaller task than a national census, since the clerical work is spread throughout the year and is more easily reduced to routine.
2) Vital registration data provide the flow data which are used to update the demographic profile of a country as it moves through time.

Disadvantages of Vital Registration System

1) Vital statistics registration requires "permanent" structures and personnel, which increase cost.
2) A large part of information is obtained from people who were not immediately concerned with the event. For instance, a death is reported by someone other than the deceased, and there may be little incentive to give an accurate report.
3) Recording may be done by other officers (for example, physicians, nurses, etc.) only as subsidiary to their main schedules.
4) It is poor in coverage in most developing countries because
   i) registration of births and deaths is non-existent in most areas because there are no registration offices or reporting centres;
   ii) even in areas with offices and centres some events are not reported because there is no rigid implementation of the legislation which makes registration of vital events and reporting of epidemics compulsory;
   iii) people have a general tendency to withhold information regarding their diseases particularly in respect of infectious or contagious diseases;

iv) people do not bother to report to registration offices (where they exist) vital events, such as, the ages of mothers at the time of marriage, at the birth of first child and of subsequent children. As a result of poor coverage vital statistics in developing countries suffer from errors of underestimation.

5) In vital registration system, the same person may be recorded several times during the year, or not at all, depending on the events which he has experienced. For example, one person may marry more than once a year while another may not experience any marriage at all within the same year.
6) Vital statistics registration may either be overloaded with many unnecessary facts or some vital information may be lost because it is collected also for other purposes (for example, legal).

2.3.6 VITAL REGISTRATION SYSTEM AND CENSUS COMPARED

Similarity Between Vital Registration System and Census

Vital registration and the census as methods of getting demographic data are not so distinct because:

1) Both are types of record-taking;
2) Both are concerned with the individual and treat the one as the primary unit;
3) Both have government sponsorship and also the universality and territoriality features;
4) The results of both census and vital registration are published in aggregates and no individual particulars are ever published;
5) Vital statistics can be obtained from a census and the composition of a population can be determined from a registration system;
6) It is possible to obtain both kinds of information by means of special survey rather than total inventories on a national scale (see Subsection 2.4.1).

Distinctions between Census and Vital Registration System

1) *Method of Obtaining Data*
   Census implies a direct canvass but vital registration does not. The vital events are reported to the registration institution by an informant. Unless this is done, many vital events may go unregistered.

2) *Difference in Results*
   As indicated earlier, there are two schemes for enumerating population at census: *de facto* population and *de jure* population; these two methods may give different results while vital registration may produce only one result.

3) *Unit of Analysis*
   Census data are records of persons but vital registration statistics are records of events. The census is concerned with what the individual is but the vital registration is interested in what happens to the individual. The vital event associated with the individual is, therefore, the unit of recording.

4) *Simultaneity*
   Simultaneity is not an applicable feature of vital registration system. Vital events are spread over time and are recorded as and when they occur[5]. Thus, vital registration goes on continuously, but a census has a definite date, usually at regular intervals of five or ten years. Continuity is rather an essential feature of the vital registration system.

5) *Confidentiality*
   Vital events cannot be kept completely confidential. In many instances, they serve as legal evidence or proof of the occurrence of the events.

2.4 DEMOGRAPHIC SAMPLE SURVEY

2.4.1 USES OF DEMOGRAPHIC SAMPLE SURVEY

Given the fact that in developing countries people do not willingly register vital events, the collection of vital event data requires active procedures such as household surveys in which trained interviewers visit dwelling units and follow a specific protocol to obtain demographic data.

The demographic sample survey is used to:

1) Collect vital statistics;
2) Collect supplementary demographic and other data where it is not feasible to collect the same from the population census;
3) Test the accuracy of the traditional sources of demographic data (e.g. census pretesting of questionnaire and census post-enumeration of validity check in a sample of census block);
4) Conduct a sample census (e.g. collecting data for only 10 percent of the population, collecting age and sex data for the whole population but socio-economic data for only a sample of the population, processing only a part of the information collected to save time and money and provide a wider range of tabulation.)

The quality of studies from demographic sample surveys depends heavily on:

1) the size of the sample;
2) the design of the sample;
3) the way the survey is carried out.

Unlike population censuses, the history of sample surveys as sources of demographic statistics in Africa is relatively recent; first employed in the Francophone countries and later in the Anglophone countries.

Three types of demographic sample surveys have been used in Africa: single round surveys, multi-round surveys, and dual record systems.

2.4.2 SINGLE-ROUND SURVEY

The term single round surveys (SRS) is used to refer to a one-time data collection effort. In this case, the selected respondents are interviewed once only. Single-round surveys are better known as ***single round retrospective surveys*** to emphasize that they ask questions about past events. The two approaches employed in a SRS for collecting birth and death data is the reference period procedure and the pregnancy/birth history procedure.

The Reference Period Approach

The reference period approach asks a knowledgeable adult member of a household to report information concerning events that occurred in the household within a fixed (specified) reference period, usually the 12 months preceding the survey. An example of a survey, referred to as *retrospective survey* that employs the reference period approach is the International Program of Laboratories for Population Statistics (POPLAB).

The main limitation of this procedure is the under-reporting of events, especially, infant and childhood death reporting due to errors of dating (reference period error and age misreporting) and, errors of omission (respondent's forgetfulness or concealment of events).

The Pregnancy/Birth History Approach

The pregnancy/birth history approach attempts to record the time sequence and spacing of all pregnancies/births and their survivorship along with the date of death (or age at death) for births which did not survive. This information is usually collected from the women whose fertility experience is being documented.

The World Fertility Survey (WFS), a major fertility and mortality measurement effort involving demographic surveys in about 40 developing countries follow the SRS format and rely on the pregnancy/birth history approach for data collection. Of late the Demographic and Health Survey (DHS) has been conducted in many countries. It uses both the reference period approach for some questions and the pregnancy/birth history approach for some other questions. This survey is conducted every five years. Ghana has already conducted six DHS in 1988, 1993, 1998, 2003, 2008 and 2013. The next Ghana DHS will be conducted in 2018.

The pregnancy/birth history approach is generally considered to collect more complete period-specific data than does the reference period approach because:

1) it attempts to obtain data pertaining to all reproductive events directly from the women concerned;
2) it employs probing questions to assist respondents in recalling events.

Pregnancy history data, however, do not necessarily yield accurate period-specific estimates of fertility levels or time trends in fertility because it can suffer from the omission of events, and the misreporting of dates of occurrences.

For the recent developments in the field of indirect estimation, SRS approach has been and will continue to be the primary source of fertility and mortality data in developing countries in the near future.

Advantages of Single Round Survey

1) It has a short time span and is less expensive.
2) It is flexible and easy to administer.

Disadvantages of Single-round Survey

1) Birth and deaths are seriously under-reported.
2) Non-sampling errors are high.

2.4.3 MULTI-ROUND SURVEY

The multi-round survey (MRS), also called ***follow-up survey*** is an attempt to overcome some of the sources of errors inherent in the single-round approach as regards the estimation of births, deaths and migration. Basically, the idea is to carry out a first round survey during which the enumerator prepares a list of persons to be followed by the survey (selected as a probability sample) and record the composition of the household and characteristics of household members. The enumerator visits the same sample after an interval to record the changes that have occurred in the household composition. In addition, the enumerator asks specific questions about events which have occurred since the previous round. Further visits may follow at appropriate intervals, so that the sample of households then becomes a sort of vital-rate "panel".

MRS has been used in some African countries, for example, in Nigeria (Rural Demographic Sample Survey, 1965–66) and in Ghana (Ghana Living Standards Survey, 1987/88, 1988/89, 1992/93, 1998/99, 2005/2006 and 2012/2013).

Advantages of Multi-round Survey

1) Since MRS approach collects information for a bundle, recall period errors of dating is reduced.
2) It allows information to be obtained on vital events, namely, deaths and births and also migration.
3) The follow-up procedures after the first round represent a very light burden for the enumerator and this makes it an ideal vehicle for other quite different types of enquiry.
4) Error of omission is reduced since the second and later rounds habitually reveal the existence of persons omitted at the first round.
5) It gives the age of a person as stated by him before he dies.

Disadvantages of Multi-round Survey

1) They are more costly than the single-round surveys. They require such scarce resources in Africa as:
   i) substantial inputs of money,
   ii) skilled manpower, and
   iii) high level of administrative organisation.
2) It is complex to design and administer (dwellings have to be clearly identified so that they can be located again, and instructions covering them are necessarily more complex, requiring longer training of enumerators).
3) The periodic visits to the same household may result in respondent or interviewer fatigue.
4) The quality of field work may vary from round to round, with the danger of interviewers reporting "no change" without serious interviewing.
5) There is no built-in mechanism for the measurement of coverage error.
6) In a highly mobile group such as populations of many urbans area in sub-Sahara Africa, there are difficulties in detecting events associated with persons who leave the sample area between rounds.

2.4.4 DUAL RECORD SYSTEM

The dual record system (DRS) represents a further refinement in the effort to collect accurate and complete reference period data. It obtains data on events in well-defined areas by two independent data collection systems. These are registration system (civil registration or wholly new ones) and periodic household survey (or data gathered by any independent agency). It is often called "PGE" following its first large scale application in the Population Growth Estimation project in Pakistan. In Africa, the method has been experimented in countries including Liberia 1969–73, Kenya 1972–77 and Morocco 1970–75. The DRS is based on the principle that if two independent data collection operations are set up to cover the same areas, most of the events they collect will be the same; in relatively few cases will one operation not detect an event missed by the other.

The registration exercise involves the nomination of a local inhabitant as a registrar in each sample area. He is normally assisted by sub-registrars, each of whom is responsible for reporting all vital events in a small part of the area. Reports, including nil returns, must be made monthly by all registrars and sub-registrars. A multi-round survey is conducted in the same area at fixed intervals, generally every 3 or 6 months, using enumerators who operate independently of the registrars.

Reported events must be identified in sufficient detail to allow "matching" between systems; the events reported by one system are matched against those reported by the other. Matching will help to avoid double counting. In case of discrepancy during the matching operation, a further visit to the household concerned may be made.

Chandrasekaran-Deming Method

Several kinds of checks may be used to verify the accuracy of published reports of the DRS. These include the study of trends, use of balancing equations and census data (see Section 3.1), comparing with other records and sample surveys.

Chandrasekaran and Deming have suggested a method for evaluating the coverage of vital registration and for estimating the undercount. In this method, all events are classified into four categories:

1) events reported in both systems, denoted by a;
2) events reported in system 1 (civil registration), denoted by b;
3) events reported in system 2 (sample survey), denoted by c;
4) events missed by both systems, denoted by d.

These categories may be represented in a 2×2 table shown in Table 2.4. Thus, $(a+b)$ is the total number of events registered and $(a+b)$ is the total number of events collected at the survey or by any independent agency. It is expected that a few events could have been missed by both the civil registration and the sample survey, and d is that unknown number. Once we estimate d, the amount of undercount could be estimated. The estimate of d is:

$$\widehat{d} = \frac{bc}{a}$$

This formula is valid only when the chance of detecting an error is independent of not detecting that event for both systems.

Survey	Civil Registration		Total
	Yes	No	
Yes	a	c	$a+c$
No	b	d	$b+d$
Total	$a+c$	$b+d$	$N=a+b+c+d$

TABLE 2.4 Classification of Events in Dual Record System

Advantages of Dual Record System

1) The DRS has a built-in mechanism for detecting and correcting coverage errors.
2) When the system continues to operate over a considerable period of time the accumulated experience from multiple repetitions of the same operations improves the system and results.

Disadvantages of Dual Record System

1) The DRS approach can result in an incorrect (bias) estimate of the total number of events if the two systems are not independent, if events are geographically or temporarily out-of-scope, and if there are matching errors.
2) Like the MRS, DRS is complex to design and administer, and suffers from the problems of cost, skilled labour, respondent and interviewer fatigue, and other non-technical problems.
3) Matching is a problem in DRS. It is difficult to know what constitutes a match and results are sensitive to the match rate.
4) It takes much longer to publish results in DRS than it is with the multi-round survey.

2.4.5 ADVANTAGES AND DISADVANTAGES OF SAMPLE SURVEY

Advantages of Demographic Sample Survey

1) The principles of sampling open up new and more flexible possibilities in the gathering of statistical information.
2) By reducing the number of individuals to be surveyed, sampling survey reduces costs, provides greater emphasis on the accuracy of the information about each individual and enables much information to be collected for more comprehensive analysis.
3) Data from the special surveys are timely as the reports are published soon after the survey.
4) Some surveys of late, such as the World Fertility Survey (WFS) and the Demographic and Health Survey (DHS) have shown that though surveys are one time events it is possible to determine trends over time.
5) Surveys are used to discover errors in the census, to provide information between censuses at more frequent intervals, and, to provide information on topics not included in the census.

Disadvantages of Demographic Sample Survey

1) The data from a sample survey are liable to some fluctuations owing to the incomplete coverage of the population. This adds another problem in reaching conclusions from sample data for, the "observed" figures may deviate from the "true" figures.

2) When improperly drawn, samples are "biased" and will yield erroneous results.
3) Sample surveys are of an ad hoc nature and, therefore, provide data only for relatively short periods before censuses.
4) The data of a sample survey may not be detailed enough to cover the groups of interest in other later studies. For example, the nutritional data in Ghana Demographic and Health Survey (GDHS) covers only children aged 0–36 months. If interest in another study is children aged 0–5 years, the data are not appropriate.

While surveys may usually be accurate on a national basis, they should be used with care on a sub-national basis.

2.4.6 COMPARING SURVEY WITH CENSUS AND VITAL REGISTRATION

Similarity Between Survey and Vital Registration System

A demographic survey may be nearly the same as census and vital registration system.

1) A sample survey may resemble a census, by asking respondents to report their characteristics.
2) These two methods of collecting demographic data are a record of persons while the vital registration is a record of events.

3) Sometimes a survey and a census ask respondents to record vital events, as for example, asking people to report the vital events of the past month or year in their own family or neighbourhood.
4) A survey may also set up a system for registering the events as they occur.

Distinction Between Survey and Vital Registration System

A survey differs from a census and a vital registration system. A survey has a more specific purpose. Most surveys are designed to provide some definite statistical information, as opposed to the wider scope and larger scale of a census or vital registration system.

2.5 POPULATION REGISTER

A population register is defined as a mechanism which provides for the continuous recording of information about the population in such a way that data on particular events that occur to each individual as well as selected characteristics describing him/her are maintained on a current basis. The information that is recorded include births, deaths, marriages, movements of people and other related events that have relevance to population size and change.

2.5.1 PURPOSES OF POPULATION REGISTER

Population registers are useful in providing a permanent up-to-date information for

1) administrative purposes, such as legal identification of persons, election rolls, and call for military services;
2) providing a comprehensive account of all changes in a population;
3) population estimates;
4) internal and international migration;
5) sampling for special studies;
6) genetic studies.

2.5.2 WHEN IS POPULATION REGISTER FEASIBLE?

Not all populations can maintain population registers so easily. Population registers are feasible for

1) sufficiently educated population;
2) motivated population;
3) small population.

This system was first started in Sweden in the 17th Century and developed successfully by the Church. Only few countries, including Italy, Israel, the Netherlands, Belgium, Germany, Bulgaria and the Scandinavian countries, maintain this system of continuous registration of their population. These countries, however, still conduct periodic censuses.

2.5.3 APPROACHES OF POPULATION REGISTER

There are two approaches to this method:

1) The first approach operates on community basis. Each community keeps a permanent record of the characteristics of its population and is informed by registration of changes as they occur.
2) The second approach operates on a national basis. By means of a continuous population register, information is obtained at any time comparable to that derived from a census. This approach is rarely applied in practice.

2.5.4 LIMITATIONS OF POPULATION REGISTER IN DEVELOPING COUNTRIES

Population register is not appropriate for developing countries.

1) They are expensive to maintain and may become defective.
2) They are usually used only in countries with a high literacy rate and low migration.
3) Migration data are less credible because only information on legal immigrants and emigrants are collected at the point of entry or exit.

2.6 ERRORS IN POPULATION DATA

2.6.1 TYPES OF ERRORS AND BIASES IN POPULATION DATA

Any mistakes in judgment or any carelessness during the planning and preparation for a census affect the tabulated results. The extent of the planning feasible for accuracy and completeness is governed by economic and social factors and with due regard to budget limitations.

Errors in census and vital statistics are of two types: those of coverage and those of content. In the case of vital statistics, there is evidence that some births, deaths, marriages, and divorces escape registration. In taking a census of population, some individuals and even some households may be missed. Over-counts in population are possible by the *de jure* method as indicated earlier. The term *errors of content* refers to instances where the characteristics of a person counted in a census enumeration, sample survey or in the registration of vital event are incorrectly reported or tabulated.

2.6.2 SOURCES OF ERRORS IN POPULATION DATA

Aside from errors due to carelessness or mistakes in judgment in planning, errors of coverage or content or both in census statistics, sample statistics and vital statistics can arise at any step from that requiring initiation of the original record to the publication of final tabulations. The five principal sources of errors are the respondent, the recorder, coding process, the editing process, and the data entry process.

Respondent

In reporting on herself, the informant:

1) may actually be ignorant of facts pertaining to herself, such as her true age;
2) may purposely misrepresent facts for reasons of her own, such as the tendency of some widowed or divorced persons to report themselves as a single, and of some foreign-born to claim native birth;
3) may have a tendency toward upgrading, as in the statement of occupation or education;
4) may not remember certain events as in the case when data are sought regarding illness or medical care within some prior period;
5) is especially likely to make an error when reporting for someone else.

Recorder

The quality of a census enumeration or a sample survey, including the recording, depends in part upon the care taken in the selection and training of the enumerators. The enumerator may also classify a person incorrectly even though the correct information has been given by the respondent. There is the possibility that physicians may sometimes state the cause of death incorrectly or incompletely on the death certificate. The local registrars are another possible source of recording error in the vital records, since they may not be carefully selected or supervised.

Coding Process

A particular source of error lies in the classification of verbal items, such as occupation, industry, or cause of illness or death, that require coding. In the classification of such verbal items, there may even be differences of opinion among experts in some instances. Much depends upon the detail of the coding system, that is, the number and arrangement of the classifications and also upon the training given to the coders.

Editing of Data

Clerical checking is an important stage in population data collection exercise. The analysis clerks scrutinise the returned questionnaire as they arrive to eliminate manifest absurdities. Some of the things to check would include:

1) uniformity in the interpretation of the questions by the enumerators and respondents;
2) completeness, that is, whether there is an answer to every question and that skips are obeyed strictly; when questionnaires are incomplete there may be the need to follow up;
3) adherence of instructions, that is, whether every instruction is obeyed;
4) omission of questions;
5) wrong answers;
6) proper ticking of codes, if the questions are precoded.

The editing clerks may fail to do proper editing. For example, if a respondent or an enumerator forgets to fill the code for sex, but fills the code for "occupation" as "housewife", it can be deduced that the respondent is a female. Of course, things do not always turn out this easy. So if necessary, validation is obtained by re-contacting subjects whose answers appear inconsistent or unusual.

2.6.3 DETECTION OF ERRORS IN POPULATION DATA

Basically, there are two ways of detecting errors in population data: comparison of tabulated data or comparison of individual records. These comparison approaches are useful before the data are published.

Comparisons of Tabulated Data

There are three approaches of detecting errors in tabulated population data.

1) Internal comparisons may reveal inconsistencies among related data in tabulated results from a census and a sample survey. For instance, the ratios of males to females for ages at some stage of life may appear out of line.
2) Comparisons can be made with a preceding census and with vital statistics. For example, from the balancing equation (Chapter 3), it may be observed that the census tabulation may indicate more or fewer persons than expected.
3) Comparisons can be made with independent collateral data. The census data may be compared with the results of a post-enumeration survey for the same area. This is discussed in Section 2.6.4.

Comparisons of Individual Records

Comparisons of census data can be made with such independent collateral records as individual birth and death certificates, immigration files, income tax returns, social security files on a small sample basis. In such cases, special care must be taken to maintain the confidential nature of the records involved.

2.6.4 POST-ENUMERATION SURVEY

Post-enumeration Surveys (PES), often referred to as re-enumeration survey or quality checks, are usually taken after a census to provide considerable information on the nature and causes of errors of reporting census data. Carefully selected interviewers are given special training and are directed to seek information from the best informed persons in the household. The reasons behind PES are to measure

1) the extent to which households are missed from the census;
2) the extent of error in
   i) omitting individuals from the household,
   ii) including individuals in the household,
   iii) the characteristics of those enumerated.

Among all the errors in demography, errors in age data have probably been examined more intensively than the others. The three factors that account for this intensive study are the following:

1) Many of these errors are readily apparent;
2) Measurement techniques can be more easily developed for age data;
3) Actuaries have had a special practical need to identify errors and to refine the reported data for the construction of life tables.

Errors in age data will, therefore, be discussed in much detail in this text (see Chapter 5).

EXERCISES

2.1 What is demography in its narrower and broader senses?

2.2 What is demographic statistics in its plural and singular senses?

2.3 Distinguish between the following terms as used in demography:

a) Demographic event and vital event;
b) Demographic statistics and vital statistics;
c) Population static and population dynamics.

2.4 Explain the term "demographic process".

2.5 What is demographic statistics in its plural and singular senses?

2.6 Explain the following terms as used in demography.

a) Characteristic;
b) Household.

2.7 Discuss in detail the two branches of demography.

3 BASIC ANALYTICAL TOOLS IN DEMOGRAPHY

3.1 INTRODUCTION

Depending on the nature of data available, purpose and elegance desired in interpretation, all statistical techniques of pictorial and tabular representation of data (refer to Nsowah-Nuamah, 2005) are employed by demographers in the study of population. These techniques include the bar chart, pie chart, histogram, frequency polygon, bivariate table and other more sophisticated techniques.

Generally, the vital statistics are available in the form of frequencies of the vital events. However, to compare characteristics and to study changes in demographic phenomena, frequencies (absolute numbers) can convey only limited meaning. For example, in 1989 in Mauritius, 6,949 persons died, 20,875 children were born and 11,197 persons got married. For such information to be meaningful to a statistician or a demographer, we need to use relative numbers. Relative numbers are obtained by relating two absolute numbers. In demographic analysis, the two main relative numbers often used are rates and ratios.

There is, however, a misunderstanding and misuse of these terms. It is necessary, therefore, to start by clarifying them before introducing any of the well-known measures. This chapter is devoted to this clarification.

3.2 RATIOS

To better understand ratios, we shall first discuss th terms "fraction", "decimal", "proportion" and "percentage"[6].

3.2.1 FRACTION

When a number A is divided by another number B we obtain a fraction. That is,

$$\text{fraction} = \frac{A}{B}$$

where A and B are measurable quantities (such as weight, age, height, volume, space, income) or frequencies (counts).

Example 3.1

Of an income of 10,000 Naira, a father spends 1,000 Naira to pay his child's school fees. What fraction of his income is spent on school fees.

Solution

$$\text{Let} \quad A = 1,000$$
$$B = 10,000$$

Therefore, the fraction of income spent on school fees is:

$$\text{fraction} = \frac{A}{B} = \frac{1000}{10,000} = \frac{1}{10}$$

Example 3.2

The population of Ghana in the 2010 Population and Housing Census was 24,658,823 out of which 12,024,845 were males. Calculate the fraction of the male population.

Solution

Let

$A = 12,024,845$ be the population of males;

$B = 24,658,823$ be the total population.

Then

$$\text{Fraction of male} = \frac{a}{b} = \frac{12,024,845}{24,658,823}$$

Decimals

A fraction is usually expressed in a decimal form. For example, we may express the results in Examples 3.1 and 3.2 as 0.01 and 0.488, respectively.

3.2.2 RATIO

A ratio is a relative measure that expresses the size of one quantity to the size of another quantity of the same unit.

When the quantities are frequencies of two non-overlapping groups possessing some common characteristic, a ratio is the relative measure of the sizes of the two groups. Suppose there are *a* elements in group *A* and *b* elements in group *B*, then the quantity

$$\text{Ratio} = \frac{a}{b}$$

is called the ratio of the number of elements in group *A* to the number of elements in group *B*.

A ratio, thus, may be expressed in three forms:

In words:	a to b
odds notation:	$a : b$
fractional notation:	$\frac{a}{b}$

where *a* and *b* refer to two quantities measured in the same unit or the frequencies of two items with a common characteristic.

Example 3.3

The male population of Ghana in the 2010 Population and Housing Census was 12,024,845 and that of the female was 12,633,978. Calculate the ratio of males to females.

Solution

Let
$a = 12,024,845$ be the population of males;
$b = 12,633,978$ be the population of females.
Then

$$\text{Ratio of male to female} = \frac{a}{b} = \frac{12,024,845}{12,633,978} = 0.952$$

That is, the ratio of men to women in Ghana in 2010 was 0.952 to 1 or the male population in Ghana in 2010 was 0.952 of the female population. In other words, for every 95 males in Ghana in 2010, there were 100 females.

Note

1) The expression ***"the ratio of*** a ***to*** b is ***40 to 80"*** means that the characteristic "*a*" corresponds to the number 40 and "*b*" to the 80. That is, whichever characteristic comes first, its number must come first. This order must be respected.

2) In demography, the fractional notation is the most common form used to express ratios. Thus,

$$\text{Ratio} = \frac{a}{b}$$

That is, ratios are usually first expressed in fractions as may be seen in Examples 3.2. Here, $A = a$ and $B = b$.

3) The ratio is usually expressed in its simplest form as a reduced fraction. Thus, the ratio expressed in (1) is:

$$\text{Ratio} = \frac{a}{b} = \frac{40}{80} = \frac{1}{2} = 0.5$$

Advantages of a Ratio

1) A ratio is very easy to calculate.
2) It is an index used to compare the relative strength of each group in populations.

3.2.3 PROPORTION

A proportion is a relative number that expresses the size of one sub-group to the total of all sub-groups:

$$\text{Proportion} = \frac{X_i}{\sum_{i=i}^{n_i} X_i} = \frac{n_i}{N}$$

where

$$X_i = \text{the size of the sub-group } i$$

$$\sum_{i=1}^{n_i} X_i = N = \text{the total of the sizes of all the } n_i \text{ sub-groups}$$

Example 3.4

For the data in Example 3.3, calculate the proportion of males in the entire population of Ghana.

Solution

$$\begin{aligned} \text{Let} \quad a &= 12,024,845 \quad \text{be the population of males;} \\ b &= 12,633,978 \quad \text{be the population of females.} \end{aligned}$$

Therefore, the entire population, *N*, is

$$\begin{aligned} N &= a + b \\ &= 12,024,845 + 12,633,978 = 24,658,823 \end{aligned}$$

The proportion of men in the entire population is:

$$\begin{aligned} \text{Proportion of men in the population} &= \frac{a}{a+b} \\ &= \frac{12,024,845}{24,658,823} = 0.488 \end{aligned}$$

That is, the proportion of males in the entire population of Ghana is 0.488.

Note

1) A proportion is a special type of ratio where the denominator is the total number in groups *A* and *B*:

$$\text{Proportion} = \frac{a}{a+b}$$

2) A proportion is usually first expressed in a fraction as may be seen in Examples 3.4. Here, $A = a$ and $B = a + b$.

3) A proportion is usually finally expressed in decimals.

Characteristics of a Proportion

1) A proportion is a relative frequency. This is because the numerator and the denominator represent frequencies of certain events.
2) The numbers *a* and *b* are never negative, and thus a proportion is a number between 0 and 1.
3) In a large population, a proportion may determine the probability of a certain event; in a sample (experiment), a proportion can be used as an estimate of probability of an event.

3.2.4 PERCENTAGE

A percentage is a relative number that expresses the size of one sub-group to the total of all sub-groups, which is equated to 100. Thus:

$$\text{Percentage} = \frac{X_i}{\sum_{i=i}^{n} X_i} \times 100 = \frac{n_i}{N} \times 100$$

where

$$X_i \quad = \quad \text{the size (in absolute numbers) of the sub-group } i;$$

$$\sum_{i=1}^{n_i} X_i = N \quad = \quad \text{the total of the sizes of all the } n_i \text{ sub-groups.}$$

Example 3.5

For the data in Example 3.4, calculate the percentage of males in the entire population.

Solution

From Example 3.3,

$$a = n_i = 12,024,845; \quad \text{and} \quad N = 24,658,823$$

The percentage of males in the entire population is:

$$\text{Percentage of males in the population} \quad = \quad \frac{n_i}{N} \times 100$$

$$= \quad \frac{12,024,845}{24,658,823} \times 100 = 48.8\%$$

Note

1) The term "percentage" is from the Latin "per centum", that is, per hundred units.

2) A percentage is a special type of proportion, one in which the proportion is multiplied by 100:

$$\text{Percentage} = \text{Proportion} \times 100$$

3) The choice of using proportions or percentages depends on the preference of the user. However, for interpretation of measures, percentages are used in which case the total is equal to 100. For mathematical manipulations, proportions expressed in decimals are used. In this case the total is maintained at 1.

3.3 RATES

Generally, ratios and proportions are useful measures for describing phenomena of a static population at a point in time (or over a specific, short period of time) that occurred under certain conditions. Particularly, in demography, the conditions are commonly determined by demographic factors such as age, sex, race, occupation, education, or ethnicity, and are often averaged over a definite period of time (usually over a year).

The process of living organisms, such as human beings, are dynamic, proceeding from birth to death. It is, therefore, important that we describe the rapidity of change. For example, we need to measure how fast a population is dying off and replenishing itself by new births and immigration. The appropriate measures that are used for such phenomena are probabilities and rates.

3.3.1 DEFINITION OF RATES IN BROADER SENSE

Generally, the term "rate" can be defined as a measure of change in one quantity (Y) per unit of change of another quantity (X) on which (Y) depends. Both Y and X may be expressed in different units as for example, "monthly income of Gh¢4,000 (Y) per ten years of working experience (X)".

In vital statistics, a rate expresses the number of events f that occurs in a population N in a given period of time which is usually a year. Rates, in a broader sense, are of two distinct types.

1) Those that are truly rates in the mathematical sense of measuring how rapidly some attribute of a population changes with time. Thus, for example, the crude death rate measures how fast or slow the size of a population is decreasing due to deaths and the birth rate measures how fast or slow it is increasing due to new births. Other examples of the true rates include the fertility rate, age (or sex) specific death rate and cause-specific death rate.
2) Those that are more correctly termed risks, as they are the probabilities of occurrence of certain events. Thus, the case fatality rate measures the risk that a person, having contracted a particular disease, will die from it.

In practice, both types of measures are not differentiated. They all imply the probability of the occurrence of some event:

$$\frac{A}{N} \times k$$

where

N	=	the number of persons "exposed to the risk of" the event during some "specified period of time";
A	=	the frequency with which an event has occurred during some "specified period of time";
k	=	the radix (a larger multiple of 10 as it may apply).

As indicated above, both the numerator A and the denominator N are expressed in different units and that rates are usually expressed in terms of numerator-units per denominator-units. For example, the crude death rate, obtained by dividing number of deaths by mid-year population, is expressed as deaths per mid-year population.

For a better understanding of the formula, certain keywords have to be explained.

Specified Period of Time

The standard "specified period of time" is usually taken as one year so that we deal with annual rates. The one-year period may be for:

1) the calendar year at the beginning of the period to which it refers. This is known as the ***probabilities***, to be discussed in the sequel;
2) the 12-month period from July 1 to June 30, or at the middle of the period (or the average). These are rates in a narrower sense and known as ***central rates***, also referred to as ***observed rates***, to be discussed in the sequel.

Exposed to the Risk of

The term "exposed to the risk of" specified in the denominator (N) is frequently used in relation to the estimation of rates. Here "exposed" means currently exposed. For example, the "population exposed to the risk of death" is the population currently alive who are exposed to the risk of dying; a child who is not born or a person who is dead has no exposure at all (he has no risk of dying). Persons who have not moved to a particular country are not exposed to the risk of dying in that country.

It is not possible to have a group exposed to the risk of only one kind of event for a long period of time, like a year. The group will continue to be changing its membership due to various demographic events and a suitable adjustment will have to be made to obtain an appropriate exposed group. For example, a person who dies in the middle of the year is a member of the exposed group only for half a year, and so is to be counted as half a person. In other words, that person contributed half a 'person-year'. That means, all those persons who leave or enter the group will have contributed only fractions of person-years. It is only those who live through the entire year will have contributed one person-year each. The person-year lived by each individual after it has been individually ascertained should be added. This total would be a good approximation of the size of the exposed group. It is equivalent to the average of the size of the group observed continuously at each pont of time from the beginning to the end of the year.

The number of person-years lived by a group of persons will be approximated by:

$$P_o + (P_1 - P_o)a$$

where

P_o	=	the initial population size of the group;
P_1	=	the population size of the group at the end of the year;
a	=	the average fraction of years lived by those who entered or left the population during the course of the year.

Radix

Rates and ratios are expressed in fractions and may be very small. Such values can be confusing and meaningless for ordinary use which may give a false sense of accuracy. Rates and ratios are, therefore. conveniently express as per 100 persons or 1,000 persons instead of per person. The multiplier 100, 1,000, etc., used to round off decimals is called the radix. The purpose of the radix *k* is

1) to avoid results involving the very small numbers that may arise in the calculation of rates and ratios;
2) to facilitate understanding of the rate and ratios;
3) to provide a common base for comparing several rates and ratios.

Unless otherwise specified, *k* is taken to be 1,000; for mathematical manipulation, the *k* is kept as one.

3.3.2 DEFINITION OF RATES IN NARROWER SENSE

In a narrower sense, the rate of a vital event is defined as the ratio of the total number of occurrences of the demographic event in a given interval of time to the total number of persons "exposed to the risk of" occurrence of that event at the ***middle of that period***. Mathematically,

$$R(A) = \frac{A}{N} \quad \times \quad k$$

where

N	=	$\frac{1}{2}(a+b)$ = number of persons "exposed to the risk of" the event at the middle of the period;
a	=	the number of persons "exposed to the risk of" the event at the beginning of the period;
b	=	the number of persons "exposed to the risk of" the event at the end of the period, (i.e. $b = a - A$).

Example 3.5

If 10 people die in one year out of a population of 1,000 at the start of the year, calculate the crude death rate[7].

Solution

The population at the beginning of the year $a = 1000$.
The number of deaths during the year (total deaths at the end of the year) $A = 10$.
The population at the end of the year:

$$\begin{aligned} b &= a - A \\ &= 1000 - 10 = 990 \end{aligned}$$

The population in the middle of the year is, therefore,

$$N = \frac{1}{2}(1000 + 990) = 995$$

Hence the rate of death is:

$$\begin{aligned} R(A) &= \frac{A}{N} \times 1,000 \\ &= \frac{10}{995} \times 1,000 = 10.05 \end{aligned}$$

Interpretation

That is, the rate of dying for this population during that year was 10.05 per 1,000 population (if the deaths were evenly distributed).

3.3.3 PROBABILITIES

In demography, a probability expresses the likelihood that an event will occur to a person or a group of persons in a particular population at risk during a particular period. Thus, the probability of a vital event is defined as the number of occurrences of the demographic event in a given interval of time divided by the total persons "exposed to the risk of" occurrence at the ***beginning of that period***. Mathematically,

$$P(A) = \frac{A}{N} \quad \times \quad k$$

where N = the number of persons "exposed to the risk of" the event at the beginning of the period;

A = the frequency with which an event has occurred during

It is important to note that a probability of a vital event is generally a proportion and, therefore, has all the characteristics of proportions. However, it differs from a proportion in the sense that probabilities must relate to a specified unit of time.

Commonly computed probabilities include mortality rates (see section 6.5). For example, infant mortality (probability of dying between birth and age one) is the probability that a child will die between birth and exact age one.

Probabilities might answer the question, such as, "what is the likelihood that a newborn child will die before it reaches its first birthday?".

Example 3.6

Refer to Example 3.5. Calculate the probability of death.

Solution

The population at the beginning of the year, $N = 1,000$.
The number of deaths during the year (total deaths at the end of the year) $A = 10$.

Therefore, the probability of death is

$$P(A) = \frac{10}{1,000} = 0.01000$$

Interpretation

The probability of dying for this population during that year was 0.01. This means that 1 percent of the population was likely to die in that year.

3.3.4 COMPARISON BETWEEN PROBABILITIES AND RATES OF EVENTS

Probabilities are similar to central rates. For short periods, there is not much difference between them for an open population. The differences are the following:

1) In the case of probabilities, the denominator ("population at risk of experiencing the event during the period") is composed of all those persons in the given population at the beginning of the period of observation but in the case of rates, the "population at risk" is the average number exposed (middle of the period).

2) Since probabilities take as their base, populations or persons at the beginning of the year who are six months younger than in the central rate, they do not take into consideration the effect of seasonal or monthly variations on the comparability of rates.
3) The (central) rates assume that the event is evenly distributed but probabilities do not.

3.3.5 TYPES OF RATES

In demography, besides the ***observed rates*** (central rates and probabilities), we may identify another type of rates referred to as ***standardised rates***. The distinction is only approximate. The observed rates are typically the simpler rates and computed directly from actual data in a single brief calculation. The standardised rates (synonym is ***adjusted rates***), on the other hand, are more complex with respect to method of calculation and also to interpretation. The standardised rates are hypothetical representations of level of the demographic measures of interest for a given population sub-group, in a summary form based on a set of specific rates and various procedures of combining them.

In the following two sections (Sections 3.4 and 3.5), we shall discuss in detail the general concepts of central and standardised rates.

3.4 CENTRAL RATES

Central (synonym is average) rates fall into one of two categories: crude (synonyms are total or overall) rates and specific rates.

3.4.1 CRUDE RATES

A crude rate (R) is usually used to describe the frequency of a demographic event across the total population, without regard to age or sex. It is defined as the ratio of the total number of a demographic event of a given type which occurred in a defined population in one calendar year, to the total population defined at a certain point in time (usually the mid-year population). Symbolically, it is given as:

$$R = \frac{E}{P} \times k$$

where E is the number of events during a year and P and k are as defined above.

The mid-year population is used to approximate the average size of the population over the year[8] (see Chapter 4). It invariably will be an estimate, as a population is not counted at every mid-year. To avoid estimation of population, crude rates may be calculated by referring to the census date as the mid-year in a census year.

Advantages of Crude Rates

The crude rate is:

1) a simple index that is highly suitable for the study of annual rates;
2) the rate that is often used in national and international publications to compare roughly the levels of vital events in various populations.

Disadvantages of Crude Rates

The crude rate is so called because of the following reasons.

1) It does not satisfy the conditions of an ideal rate; it is only an approximation. While the data for the numerator are usually obtained from registration of vital events as they occur throughout the year, the denominator represents the population for a fixed date.

2) It ignores the variations in the population structure and, therefore, includes in the denominator some individuals who are not susceptible to the event. By this, it presumes that all individuals in the population are equally exposed to the "risk" but this is not always true. A population is heterogeneous in its exposure to risk. For example,
   i) the very young and the very old are exposed to greater risk of death than those of the middle age group;
   ii) the chances of childbearing among the first reproductive age (15–19) and the last few reproductive ages (40–44 and 45–49) are very low as compared to the other reproductive ages;
   iii) among the unmarried women, the chances of getting married decreases as age increases.

3) It is misleading to make a comparison of crude rates between populations.

3.4.2 SPECIFIC RATES

Crude rates are not useful for comparison because of the variation in composition by age in different populations. As indicated above, the frequency of human death in a population, for example, varies considerably, other things being equal, with the population's age composition. For the measure of risk to have greater validity, it is necessary to calculate rates for homogeneous sub-groups. Such rates are referred to as "specific rates."

Specific rates for a characteristic X (say death or birth) are crude rates defined for special sub-groups j in a population (say, for age, sex, race, particular disease, or other classification variable). Thus, a specific rate is defined as:

$$r_j = \frac{E_j}{P_j} \times k$$

where

r_j	=	the specific rate of the sub-group j;
E_j	=	number of events occurring in the sub-group j possessing the characteristic X_j;
P_j	=	mid-year population size of the sub-group j possessing the characteristic X_j.

The term "specific" is used because the exposed population is so chosen that it is homogeneous with respect to the specified characteristic as far as exposure to risk to the given event is concerned. Examples of specific rates that will be discussed later include age-specific death rate and age-specific birth rate, where the sub-group j is the age group and the characteristic X_j are the death and birth, respectively.

Except in the case of age, which always defines a rate as specific, there is no clear dividing line between a crude rate and a specific rate; a rate is understood to be "crude" unless specifically indicated otherwise (e.g. sex-specific, urban-rural specific, race-specific, etc.) or unless it is specific with respect to age. Rates may be specific for more than one characteristic, as for example, age-sex specific death rate.

Limitations of Specific Rates

Unreliability

If some segments (strata) of the population consist of small numbers of people, the associated specific rates may be too imprecise and unreliable for use in detailed comparisons.

Nonexistence

For some populations or for some groups of special interest, specific rates may not exist. The only available data may be number of events (for example, deaths) and not their subdivision by segment.

3.4.3 RELATIONSHIP BETWEEN CRUDE AND SPECIFIC RATES

A crude rate may be expressed in terms of specific rates as:

$$R = \frac{\sum P_j r_j}{\sum P_j}$$

where the symbols are as defined above.

Thus, the crude rate is defined as a weighted mean of specific rates, where the populations in the sub-group j represent the weight.

3.4.4 ADVANTAGES AND DISADVANTAGES OF CENTRAL RATES

Advantages of Central Rates

1) Central rates are easy to calculate and their meaning can easily be communicated.
2) They require minimum data for calculation.

Disadvantages of Central Rates

The disadvantages of both the crude and specific rates discussed above are the disadvantages of the central rates.

3.5 STANDARDISED RATES

3.5.1 WHAT IS STANDARDISATION?

Generally, standardisation is a technique which reduces variables to the same scale and thus allows for differences in the composition of groups to be compared in respect of a particular measure. In demography, standardisation is the process of adjusting the rates to eliminate from them the effect of differences in population composition with respect to age, sex, marital status, race, or other categories of the population.

3.5.2 WHY STANDARDISATION?

One of the most frequently occurring problems in vital statistics is the comparisons of the rate for some event or characteristic. The comparison may be:

1) across different populations (that is, comparison of rates of two or more communities at a point in time);
2) for the same population over time (that is, comparison of rates of a community over a period of time).

If the populations were similar with respect to factors associated with the event under study (factors such as age, race, sex, or marital status), there would be no problem in comparing crude rates. If the populations are not similarly constituted, however, the direct comparison of the crude rates may be misleading. Crude rates, for example, will not be comparable if in one population there is a higher proportion of individuals in the older age groups, since most vital events (for instance, mortality and fertility) depend strongly on age. Thus,

1) crude death rate of a community may be relatively high merely because the community has a large proportion of persons in the older ages, where death rates are high; or may be relatively low because the community has a large proportion of children and young adults where death rates are low;
2) the crude death rate of a country may actually rise even though death rates at each age remain constant, if population is getting older.

Since differences in composition of the population will influence the total rates, it is preferable to use specific rates. Specific rates reconcile the population differences with respect to those associated with the event under study. Such investigation involves a mass of data and, although necessary for some purposes, produces more detail than the mind can conveniently grasp. Furthermore, comparisons using such a collection of rates quickly become awkward when they involve several countries. Unless there is a steady difference at all ages, a collection of rates does not let us rank a pair of populations.

For ready comparison, what is wanted is a single summary index which will use some acceptable standard of weights in averaging the rates for the various segments of the population. The index thus found is called the "standardised" (synonyms are "adjusted" or "corrected") rate. The standardised rate, therefore, controls the influence of the composition of the various populations with respect to certain characteristics such as age, sex and marital status thus making it possible to compare rates in populations of different distributions.

3.5.3 METHODS OF STANDARDISATION

There are two common methods of standardisation, namely, direct and indirect standardisation[9]. In both cases, the object is to calculate the number of deaths or births to be expected in one population on the basis of some information from another population.

Direct Standardisation

Direct standardisation is a simple and straightforward method of adjusting rates (death and birth) for age and/or sex structure. The basic idea is to choose a standard population distributed by, say, age with a fixed structure of the characteristic under study. The specific rates for the study populations are then adjusted to allow for discrepancies in the distribution between the standard and the study populations. The result is the number of cases expected in the study population, if the standard population had the same population distribution as the study population. In so doing, we are assuming that the study population has the same structure as the standard. The total expected number of cases divided by the total number of people in the standard population gives the directly standardised rate for the study population.

Indirect Standardisation

Non-availability of the specific rates of the study population presents difficulty in the use of the direct standardisation. In such cases, the technique of indirect standardisation is used. That is, to apply the indirect method of standardisation, it is not necessary to define a standard population but rather a standard set of specific rates. These standard specific rates are applied to the corresponding groups of the study population to determine the number of cases expected in each group. The expected cases in each group are summed to show how many cases would have occurred in the study population if it had experienced the rates of the standard population. Implicit in the method is the assumption that the study population experiences the specific condition as the standard rates. The ratio of the total observed cases to the total expected cases multiplied by the crude rate in the standard population gives the indirectly standardised rate.

3.5.4 CHOICE OF METHOD OF STANDARDISATION

The choice of method of standardisation is usually governed by the availability of data and by their (relative) accuracy. For example, in comparing mortality rates from different countries the indirect method will be preferable if it is difficult to obtain national data on age-specific mortality rates. In general indirect standardisation are more commonly employed for mortalities.

Table 3.1 summarises the characteristics of the two methods of standardisation.

Requirements	Direct Standardisation	Indirect Standardisation
1. Data Required *Study Population*	Age-sex specific rates	Age-sex composition + total number of events (deaths or births)
Standard Population	Age-sex composition	Age-sex specific rates (+ overall rate)
2. Method	Study rates applied to standard population	Standard rates applied to study population
3. Result	Age-sex adjusted rate	Age-sex adjusted rate
4. Extension	To adjust for other factors besides age and sex, such as different ethnic composition of the study groups	To adjust for other factors besides age and sex, such as different ethnic composition of the study groups

TABLE 3.1 Comparison of Direct and Indirect Methods of Standardisation

3.5.5 INTERPRETATION OF STANDARDISED RATES

A standardised rate, no matter which method of adjustment is used, has no direct meaning in itself. Its magnitude means little in and of itself. It is meaningful only in comparison with other similarly computed rates. Since standardised rates are useful only for comparisons, the commonest application of the procedure is to compute such rates for the areas or population groups whose characteristic is to be compared and to calculate the relative differences of the resulting rates. The meaningful measure then is a ratio, index, or percent difference between rates similarly adjusted.

Comparison of two standardised rates has some meaning only in connection with the standard population. The choice of a standard population should, therefore, be made with care.

3.5.6 SELECTION OF STANDARD POPULATION

There is no population that can be considered as "standard" per se, because no two populations can be expected to have identical population composition in the real situation. Opinions differ as to what population should be used as the standard. There are a number of possible choices. The standard selected should have a fixed structure of the characteristic under study which may be the distribution (of the characteristics of interest) of one of the areas or dates (e.g., earliest, middle, or latest in a series) being compared or an "external" real or theoretical distribution of some sort.

When the rates of two populations are to be compared, one of them is taken as the standard population. However, if the two populations exhibit wide variation in their structure, a third population which is similar to them is sometimes used. If such a population cannot be found, we use that average of the two populations as the standard population.

The problem of defining a standard population is similar to that of fixing a suitable base year for an index number in economic statistics. There is clearly no conceptual justification for choosing one standard over another. Theoretically, any population would do, as all comparisons are made relative to the standard. There are advantages, however, in choosing one which is representative of the general situation as it actually exists. Hence, the general rule is to select as a standard any distribution (of the characteristic) that is similar to the distribution of the various populations under study.

All said and done, the choice of the standard population is eventually arbitrary and will completely be at the discretion of the researcher, recognising that the choice of a standard can markedly alter comparisons between populations.

Note

1) Standardisation techniques are more common in mortality analysis than in any other branch of demographic analysis.
2) The most important and most common variable for which the adjustment or standardisation of rates is carried out is the age.
3) The standardised rate, a single summary measure, is more frequently compared with other summary measures than are the entire schedules of specific rates. However, when the specific rates in the groups being compared do not bear consistent relations across the segment, then any kind of overall standardisation is questionable;

The methods of direct and indirect standardisation will be applied, in later chapters, to mortality, fertility and nuptiality (see Chapters 6, 8 and 10, respectively).

3.6 TERMINOLOGY USAGE IN DEMOGRAPHY

The distinction between the use of the ratio, rate, and probability of demographic or vital events is not always clear-cut. For various historical reasons and also because demographers come from many academic disciplines, the measures used by demographers are not always as we have so far described.

Rate

Restriction of the term "rate" to fractions, where the numerator is part of the denominator and relate to a specified unit of time, is not always observed in actual practice. By convention, some ordinary percentage figures are called rates as in "literacy rate" which is defined as:

$$\frac{\text{number of literates}}{\text{number of literate} + \text{number of illiterates}} \times 100$$

This is simply the percentage of the population sub-group who are classified as literate in some specified language. The acquisition of literacy may have occurred many years back. Here, the term *rate* is loosely used to refer to the percentage of a population sub-group in the total population where the definition of the sub-group reflects a prior event.

Again, "total fertility rate" discussed in Chapter 8 are measures which are not rates in the sense defined earlier but are summary numbers that serve very adequately the purpose of comparison. Furthermore, in demography some probabilities are referred to as rates. These include mortality rates (e.g. maternal mortality rate and infant mortality rate). Though the term "rate" is commonly used, the measure in fact represents a probability of dying, and should appropriately be called the "maternal probability of dying" and the "infant probability of dying", respectively.

Ratio

Again, restriction of the term "ratio" to the fraction of two frequencies of occurrence is also not adhered to in practice. An example is the concentration ratio (see Chapter 5). Again, expectation of life discussed in Chapter 7 are measures which are not ratios in the sense defined earlier but are summary numbers that help us to very adequately make comparison.

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Sometimes,, the term "index" is used in situations where we are dealing with absolute differences, as in say, indices of concentration and dissimilarity. These may best be described as "measures".

4 POPULATION SIZE AND GROWTH

The basis of demographic analysis is the size of the population of the entire nation, of its sub-national areas or of particular sub-sections which possess certain characteristic. It is usually the first demographic fact that a government tries to obtain.

4.1 THE BALANCING EQUATION

In order to determine the population at any time 't' after census or between two censuses a number of methods have been devised. We discuss here a suitable method which makes use of births, deaths and migration statistics.

4.1.1 DECOMPOSITION OF POPULATION CHANGE

The population at time 't' may be determined by the "balancing equation" or the ***demographic booking equation*** which is defined as:

$$P_t - P_o = (B - D) + (I - O)$$

where

P_t = the population at any time t (later date);
P_o = the population at the beginning of the period (earlier date);
B = births during the period;
D = deaths during the period;
I = immigrants during the period;
O = emigrants during the period.

The *balancing equation* may be written as:

$$\triangle P = NI + M$$

where

$\triangle P = (P_t - P_o)$ is the change in population between two dates (the current and initial dates)[10];
$NI = B - D$ is the natural change (increase or decrease) of population;
$M = I - O$ is the net migration (mechanical change of population).

This method decomposes the population change into its components or conversely, the synthesis of the components to estimate the total population change. It is sometimes called the ***component equation*** or the ***inflow-outflow relationship***.

When the population is closed[11], the balancing equation is given as:

$$P_t - P_o = B - D$$

4.1.2 USES OF THE BALANCING EQUATION

Estimation of Net Error in System of Demographic Statistics

The balancing equation may be used to estimate the net error in this system of demographic statistics when we have accurate information on the various components.

Thus,

$$e = (P_t - P_o) - (B - D) + (I - O)$$

or

$$e = \triangle P - (NI + M)$$

where e is the residual error or error of closure.

That is, the population change between two periods should be equal to the natural increase, NI, and net migration, M. Any difference is due to error, e.

In a closed population, the ***error of closure*** is given as

$$e = (P_t - P_o) - (B - D)$$

Estimation of Post-censal Current Population

The balancing equation may be used to estimate a post-censal current population. Thus,

$$P_t = P_o + (B - D) + (I - O)$$

This equation may be referred to as the basic estimating equation.

Example 4.1

A town had a population of 450,000 persons on 1st January, 1986. In that year, 6,000 children were born and 2,000 persons died. In addition, 400 persons came to the town to farm and 250 school leavers left for the nearest city to work in the newly opened factory in that same year. Determine the population of the town at the end of the year.

Solution

Population on 1st January 1986	P_t	=	$450,000$
Number of births	B	=	$6,000$
Number of deaths	D	=	$2,000$
Number of immigrants	I	=	400
Number of emigrants	O	=	250

Population at the end of the year (31st December, 1986):

$$\begin{aligned} P_t &= P_0 + (B - D) + (I - O) \\ &= 450,000 + (6,000 - 2,000) + (400 - 250) = 454,150 \end{aligned}$$

Estimation of Intercensal Net Migration for Small Subdivisions of Country

The balancing equation may be used to estimate the intercensal net migration for small subdivisions of a country when data on births and deaths are available and there is no statistics on internal migration. Thus,

$$M = (P_t - P_o) - (B - D)$$

where M is the net migration to or from the area.

4.2 TOTAL POPULATION

A census is usually the best source to determine the size of a population (total population) of an area at a particular time. As discussed earlier, there are two ideal types of total population counts – *de facto* and *de jure* counts (see Chapter 2).

The rates of vital events are ideally expressed in relation to the total population. For the census year, census counts of population are commonly employed. As has been pointed out earlier, censuses are taken at a long period of time, say 10 year intervals. The population P_t, of an area at any moment of time t, is not constant but keeps on changing from time to time due to births, deaths and migration. There are two ways of obtaining the total population at any given time t:

1) To combine data on births, deaths and migration for the period between the census date and the estimated count from the last census (see Section 4.1). Such data are difficult to come by. In most developing countries, the estimates are questionable because information about births, deaths and migration is not reliable.
2) To estimate the population at time t from its initial size. Thus

$$P_t = P_o + \triangle P$$

It is equally difficult to obtain the population after a lapse of $\triangle t$

Which population figure should be used then when census data are not available? The practice is to use the average annual population which is a sufficient approximation for the computation of annual rates.

The average annual population is defined in many ways depending on the type of data available.

4.3 MEAN POPULATION IN A YEAR

The mean annual population, when monthly data on births, deaths and migration within a year are not available, may be defined in one of the following ways.

4.3.1 SIMPLE MEAN POPULATION

When we have population data for only the beginning of the year (i.e. 1st January) and the end of that year (i.e. 31st December) or the beginning of the following year (i.e. 1st January) we find the *simple mean* population as:

$$\overline{P} = \frac{P_0 + P_1}{2}$$

where

$\overline{P}$	=	mean population;
P_0	=	population for January 1 of the year;
P_1	=	population for January 1 of the following year.

This formula shows exactly what the mid-year population would be if all vital events were distributed evenly throughout the year.

Example 4.2

The population of a country on 1st January, 1999 was 25 million and was 27 million on 1st January, 2000. Find the annual mean population.

Solution

$$\begin{aligned} P_0 &= 25\,\text{million} \\ P_1 &= 27\,\text{million} \\ \overline{P} &= \frac{P_0 + P_1}{2} \\ &= \frac{25 + 27}{2} = 26\,\text{million} \end{aligned}$$

Alternatively the mean population may also be interpolated as:

$$P = P_0 + \frac{1}{2}\,(P_1 - P_0)$$

Example 4.3

For the data of Example 4.2, calculate the annual mean population for the country using the interpolation approach.

Solution

$$\begin{aligned} P_0 &= 25\,\text{million} \\ P_1 &= 27\,\text{million} \\ \overline{P} &= 25 + \frac{1}{2}(27 - 25) \\ &= 26\,\text{million} \end{aligned}$$

4.3.2 CHRONOLOGICAL MEAN POPULATION

When monthly population figures for the entire period of the year are available, then we use the *chronological mean* population defined as:

$$\overline{P} = \frac{\frac{1}{2}P_1 + P_2 + \cdots + P_{12-1} + \frac{1}{2}P_{12}}{12-1}$$

or

$$\overline{P} = \frac{\frac{P_1 + P_{12}}{2} + \sum_{i=2}^{12-1} P_i}{12-1}$$

where $1, 2, \cdots, 12$ correspond to January, February, through to December.

In general, the chronological mean population for any period in a year may be obtained as:

$$\overline{P} = \frac{\frac{1}{2}P_1 + P_2 + \cdots + P_{m-1} + \frac{1}{2}P_m}{m-1}$$

Or

$$\overline{P} = \frac{\frac{P_1 + P_m}{2} + \sum_{i=2}^{m-1} P_i}{m-1}$$

where m is the number of the last month of the period with population figures in which we are interested. If the period of interest is January to December then $m = 12$.

Example 4.4

The population of an area (in millions) in 1985 is given in Table 4.2.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
4	6	5	3	4	6	6	8	7	5	4	6

TABLE 4.2 Monthly Population of an Area in 1985 (in millions)

Find the annual mean population.

Solution

$$\overline{P} = \frac{\frac{1}{2}P_1 + P_2 + \cdots + P_{m-1} + \frac{1}{2}P_m}{m-1}$$

$$= \frac{\frac{1}{2}(4) + 6 + 5 + 3 + 4 + 6 + 6 + 8 + 7 + 5 + 4 + \frac{1}{2}(6)}{12-1} = 5.36$$

Alternatively,

$$\overline{P} = \frac{\dfrac{P_1 + P_m}{2} + \sum_{i=2}^{m-1} P_i}{m-1}$$

$$= \frac{\dfrac{4+6}{2} + 6 + 5 + 3 + 4 + 6 + 6 + 8 + 7 + 5 + 4}{12 - 1} = 5.36$$

4.3.3 WEIGHTED MEAN POPULATION

When some of the monthly figures are available and the intervals are not equal then the mean population is computed as:

$$\overline{P} = \frac{\dfrac{P_1 + P_2}{2} t_1 + \dfrac{P_2 + P_3}{2} t_2 + \ldots + \dfrac{P_{12-1} + P_{12}}{2} t_{12-1}}{t_1 + t_2 + \ldots + t_{12-1}}$$

This formula takes the form of a weighted mean:

$$\overline{P} = \frac{\sum_{i-1}^{m-1} \overline{P}_i t_i}{\sum_{i=1}^{m-1} t_i}$$

where

$$\overline{P}_i = \frac{P_i + P_{i+1}}{2} = \text{the mean population of two successive periods;}$$

$$t_i = \text{interval between two successive periods (in months).}$$

Example 4.5

The population of an area (in thousand) in 2005 was 12 on 1st January; 13 on 1st May; 15 on 1st November; 18 on 31st December. Find the mean population of the area for 2005.

Solution

Table 4.3 illustrates the calculation of the mean population of the area. Thus

$$\overline{P}_1 = \frac{12+13}{2} = 12.5$$

$$\overline{P}_2 = \frac{13+15}{2} = 14.0$$

$$\overline{P}_3 = \frac{15+18}{2} = 16.5$$

Date	Population (thousand)	Period of time (months)	$\overline{P}_i = \frac{P_i + P_{i+1}}{2}$	$\overline{P}_i t_i$
1	2	3	4	$5 = 3 \times 4$
01.01.85	12	4	12.5	50.0
01.05.85	13	6	14.0	84.0
01.11.85	15	2	16.5	33.0
31.12.85	18	–	–	–
Total		12		167.0

TABLE 4.3 Preliminary Calculation of Mean Population of Example 4.5

Hence

$$\overline{P} = \frac{\sum_{i-1}^{m-1} \overline{P}_i t_i}{\sum_{i=1}^{m-1} t_i} = \frac{167}{12} = 13.9$$

Simplifying the formula we obtain the following:

$$\overline{P} = \frac{(P_1 + P_2)\,t_1 + (P_2 + P_3)\,t_2 + \ldots + (P_{12-1} + P_{12})\,t_{12-1}}{2\,(t_1 + t_2 + \ldots + t_{12-1})}$$

or

$$\overline{P} = \frac{\sum_{i=1}^{m-1} (P_i + P_{i+1})\, t_i}{2 \sum_{i=1}^{m-1} t_1}$$

Example 4.6

For the data of Example 4.5, calculate the mean population.

Solution

Using the formula just stated, we have:

$$\overline{P} = \frac{(P_1 + P_2)\, t_1 + (P_2 + P_3)\, t_2 + ... + (P_{12-1} + P_{12})\, t_{12-1}}{2\,(t_1 + t_2 + ... + t_{12-1})}$$

$$\overline{P} = \frac{(12+13)4 + (13+15)6 + (15+18)2}{2(4+6+2)} = 13.9$$

4.4 MEAN POPULATION OF YEARS

4.4.1 MEAN POPULATION FOR TWO YEARS

When data are available for two years (earlier and later years) of a long interval and changes are assumed to be linear, then we use the simple mean population ($\overline{P}$) defined as:

$$\overline{P} = \frac{P_1 + P_n}{2}$$

where P_1 and P_2 are population for earlier and later years.

Example 4.7

The population of Ghana was 18.9 million in 2000 and 24.7 million in 2010. Calculate the mean population for the 10-year period.

Solution

$$\overline{P} = \frac{18.9 + 24.7}{2} = 21.8$$

When the interval between the initial year and the current year is long and the changes are assumed to follow a geometric progression[12], then

$$\overline{P} = \frac{P_n - P_1}{lnP_n - lnP_1}$$

where $ln =$ natural logarithm[13].

Example 4.8

Calculate the mean population of Ghana for the period 1984–2000 using the data in Example 4.7, assuming that the population changes follow a geometric progression.

Solution

$$\begin{aligned} P_1 &= 18.9; \quad P_n = 24.7 \\ \overline{P} &= \frac{P_n - P_1}{\ln P_n - \ln P_1} \\ &= \frac{24.7 - 18.9}{\ln 24.7 - \ln 18.9} = 21.7 \end{aligned}$$

4.4.2 MEAN POPULATION FOR EQUAL INTERVALS

When the population figures of years are available and the intervals between them are equal then we use the chronological mean, defined as:

$$\overline{P} = \frac{\frac{1}{2}P_1 + P_2 + \cdots + \frac{1}{2}P_n}{n-1} = \frac{\frac{P_1 + P_n}{2} + \sum_{i=2}^{n-1} P_i}{n-1}$$

where $n =$ number of years in which population figures are available.

Example 4.9

The population of an area on 1st January 1970 was 15,300 persons; on 1st January 1975 was 16,800 persons; on 1st January 1980 was 18,200 and on 1st January 1985 was 22,000 persons. Calculate the mean population for the period under study.

Solution

$$\begin{aligned} \overline{P}_1 &= 15,300; \quad P_2 = 16,800; \quad P_3 = 18,200; \quad \overline{P}_4 = 22,000 \\ n &= 4 \ \ \text{(that is, 1970, 1975, 1980, 1985)} \end{aligned}$$

The intervals between years are the same. Hence,

$$\overline{P} = \frac{\frac{1}{2}(15,300) + 16,800 + 18,200 + \frac{1}{2}(22,000)}{4-1} = 17,883$$

4.4.3 MEAN POPULATION FOR UNEQUAL INTERVALS

When population figures for years are available and the intervals between the years are not the same, then the *weighted mean* is used. Thus,

$$\overline{P} = \frac{\sum_{i=1}^{n-1} \overline{P}_i t_i}{\sum_{i=1}^{n-1} t_i}$$

where $\overline{P}_i$ = the average population of the two successive periods;
t_i = the interval between two successive periods (in years).

Example 4.10

The population of Ghana (in million) at censuses in 1960 was 6.7; 1970 was 8.6; 1984 was 12.3; 2000 was 18.9 and 2010 was 24.7. Calculate the mean population for the period 1960–2010.

Solution

$$\begin{aligned}
P_1 &= 6.7; \quad P_2 = 8.6, \quad P_3 = 12.3; \quad P_4 = 18.9 \\
t_1 &= 1970 - 1960 = 10 \text{ years} \\
t_2 &= 1984 - 1970 = 14 \text{ years} \\
t_3 &= 2000 - 1984 = 16 \text{ years} \\
t_4 &= 2010 - 2000 = 10 \text{ years} \\
\overline{P}_1 &= \frac{6.7 + 8.6}{2} = 7.65 \\
\overline{P}_2 &= \frac{8.6 + 12.3}{2} = 10.45 \\
\overline{P}_3 &= \frac{12.3 + 18.9}{2} = 15.60 \\
\overline{P}_4 &= \frac{18.9 + 24.7}{2} = 21.8 \\
\overline{P} &= \frac{\sum_{i=1}^{n-1} \overline{P}_i t_i}{\sum_{i=1}^{n-1} t_i} \\
&= \frac{7.65(10) + 10.45(14) + 15.60(16) + 21.8(10)}{10 + 14 + 16 + 10} = 13.8
\end{aligned}$$

That is, the mean population of Ghana for the period 1960–2010 was 13.8 million.

4.5 THE MID-YEAR POPULATION

Vital rates have as their denominator the "population at risk". This population is usually only approximate. As indicated earlier, for the census year, census data are taken as the population at risk, especially, if it was carried out close to June 30 (July 1). For the non census year, the mean population over the period to which the rate relates is used. However, data may not be available for calculating the mean population. In such a case it is usually appropriate to use the population at the middle of the period (which is roughly the average population during the period), or the population at the beginning of the period. In demography, rates are usually expressed in relation to the former usually referred to as the mid-year population taken as the population at 1st July of the year in question. The reasons for using the mid-year population for calculating rates in demography are as follows:

1) The mid-year population would be exactly the same as the average population if the events that change the size of the population were evenly distributed throughout the year. Even though these events are not distributed evenly, but vary by season, sometimes in a very marked degree, the mid-year population gives a reasonable approximation of the average number of persons present during the year.
2) The middle of the year is the point at which roughly half of the year's changes in the population would have occurred.
3) The mid-year population is very convenient to compute.

4.6 POPULATION GROWTH

The rate of growth of a population is one of the most important single demographic facts about the population. The rate at which population is changing affects not only its size and numerical increase, but also its composition.[14]

Population growth refers to the change in a population of a given area over a specific period of time. In simple terms, it is the change of population size between two dates. The term "growth" is used irrespective of whether the change is negative or positive. A population increasing in size is said to possess a positive growth rate and the one declining, to possess a negative growth rate. It has been generally observed that all large populations have registered increases over long periods of time. Any decreases of population are usually transient.

There are three types of growth associated with population – natural growth, mechanical growth and overall growth. ***Natural growth*** refers to the difference between the births and deaths in an area's population and does not take into account migration. ***Mechanical growth*** refers to the difference between the immigration and emmigration of an area's population and does not take into account births and deaths. ***Overall growth*** refers to the changes in the population of an area taking into consideration both natural and mechanical growth.

The overall growth of a population may be computed in different ways and the rest of this chapter is devoted to them.

4.6.1 ABSOLUTE POPULATION GROWTH

The growth of a population at any given time t is largely determined by its own size at the moment. Suppose P_t is the population of a defined territory at time t, then the ***absolute population growth*** (change) at time t, denoted as $\triangle P_t$, is

$$\triangle P_t = P_t - P_0$$

where P_0 is the initial population. This measure enables us to know the absolute amount of population change that has taken place.

The simplest measure of absolute population change is the intercensal change, that is, the difference between two censuses:

$$\triangle P_t = C_1 - C_0$$

where

C_0 is the population of the earlier census;
C_1 is the population of the later census.

Example 4.11

The population figures of Ghana at two censuses in 2000 and 2010 were 18,912,079 and 24,658,823, respectively. Calculate the absolute population growth in the period 2000–2010.

Solution

$P_{2000} = 18,912,079;\ P_{2010} = 24,658,823$

$$\begin{aligned}\triangle P_t &= P_t - P_0 \\ &= P_{2010} - P_{2000} \\ &= 18,912,079 - 24,658,823 = 5,746,744\end{aligned}$$

That is, the population of Ghana increased by about 5.7 million between 2000 and 2010.

Limitation of the Absolute Population Growth

The estimates of the absolute population growth do not take into account the size of the population. We cannot, therefore, compare the growth of populations for different countries.

Example 4.12

The population figures of Ghana at two censuses in 1984 and 2000 were 12,296,081 and 18,912,079, respectively. The figures for censuses of the Republic of Mauritius (excluding Agalega and St. Brandon) in 1983 and 2000 were 999,945. and 1,178,848, respectively. Calculate the intercensal absolute population growth for each country and compare.

Solution

For Ghana,

$$P_{1984} = 12,296,081; \quad P_{2000} = 18,912,079$$

Hence

$$\begin{aligned}\triangle P_t &= P_{2000} - P_{1984} \\ &= 18,912,079 - 12,296,081 = 6,615,998\end{aligned}$$

For Mauritius,

$$P_{1984} = 999,945; \quad P_{2000} = 1,178,848$$

Hence

$$\begin{aligned}\triangle P_t &= P_{2000} - P_{1983} \\ &= 1,178,848 - 999,945 = 178,903\end{aligned}$$

The results are presented in Table 4.5.

Year	Ghana census population	Mauritius census population
1984	12,296,081	999,945
2000	18,912,079	1,178,848
Change	6,615,998	178,903

TABLE 4.5 Calculation Intercensal Absolute Growth for two Countries

The Ghana's population is far higher than that of Mauritius (more than ten times), and it is, therefore, logical to expect Ghana to have a larger absolute growth.

For comparison purposes, what is desired is a measure of growth relative to the size of the population. In the subsequent sections, we shall discuss various measures of relative growth of population.

4.6.2 ARITHMETIC POPULATION GROWTH RATE

The crudest measure of the growth rate of a population at the period t is the arithmetic growth rate (AGR) which is more appropriately referred to as ***rate of increase***. AGR may be determined as relative growth or percentage change.

Relative Growth

The relative growth of population (R_A) is given by

$$R_A = \frac{P_t}{P_0}$$

$R_A = 1$ denotes the same number of people at the two times;
$R_A > 1$ denotes an increasing population;
$R_A < 1$ denotes a decreasing population.

Example 4.13

For the data in Example 4.11, calculate the relative growth rate of the population of Ghana between the period 2000–2010.

Solution

From Example 4.11,

$$P_{2000} = 18,912,079; \;\; P_{2010} = 24.7$$

Hence,

$$R_A = \frac{24,658,823}{18,912,079} = 1.3039$$

That is, the population of Ghana in 2010 was about 1.3 times that in 2000.

Arithmetic Percentage Growth Rate

The arithmetic percentage growth rate (APGR) is the change in the size of population during period t as a percentage of population at the beginning of the period. It is defined as:

$$APGR = \frac{P_t - P_0}{P_0} \times 100$$

where the radix k is taken as 100.

The arithmetic growth rate between two census dates is called ***intercensal arithmetic growth rate*** and when the censuses are ten years apart it is called ***decadal*** arithmetic growth rate.

A simpler way to compute the percentage change of population may be expressed by:

$$APGR = \left(\frac{P_t}{P_0} - 1\right) \times 100$$

or

$$APGR = \left(\frac{P_t}{P_0}\right) \times 100 - 100$$

That is, to divide the later date population by the earlier one, multiply the result by 100, and subtract 100.

$APGR = 0$ denotes the same number of people at the two times;
$APGR > 0$ denotes an increasing population;
$APGR < 0$ denotes a decreasing population.

Example 4.14

For the data in Example 4.11, calculate the intercensal arithmetic growth rate of the population of Ghana between the period 2000–2010, using both methods.

Solution

From Example 4.11,

$$P_{2000} = 18,912,079; \;\; P_{2010} = 24,658,823$$

Method 1:

$$
\begin{aligned}
APGR &= \frac{P_t - P_0}{P_0} \times k \\
&= \frac{P_{2010} - P_{2000}}{P_{2000}} \times 100 \\
&= \frac{24,658,823 - 18,912,079}{18,912,079} \times 100 = 30.39
\end{aligned}
$$

Method 2:

$$
\begin{aligned}
APGR &= \left(\frac{P_t}{P_0} - 1\right) \times 100 \\
&= \left(\frac{P_{2010}}{P_{2000}} - 1\right) \times 100 \\
&= \left(\frac{24,658,823}{18,912,079} - 1\right) \times 100 = 30.39
\end{aligned}
$$

That is, the population of Ghana increased by 30.4 percent between 2000 and 2010. Or the population of Ghana recorded an increase of 30.4 percent in the intercensal period 2000–2010.

Annual Arithmetic Growth Rate

We may sometimes like to compare the growth of different populations or different periods. This can be done when the time interval between censuses is the same. When these intervals are not the same, the ratios must be put on a comparable basis. This is done by converting them into annual rates of growth (or the ***annualisation of growth rates***), somehow distributing the observed growth through the intercensal period.

Suppose t is measured in years, then the mean annual arithmetic percentage growth rate, denoted by r_A, is:

$$r_A = \frac{1}{t}\left(\frac{P_t - P_0}{P_0}\right) \times 100$$

where t is the interval between two periods;

P_0 and P_t are the populations at the beginning and end of period t.

If P_0 and P_t are the beginning and end of the same year, respectively, the formula for the annual rate of population growth is the same as that for the arithmetic rate of growth.

Example 4.15

For the data in Example 4.11, calculate the average annual arithmetic growth rate of the population of Ghana between the period 2000–2010.

Solution

From Example 4.11,

$$P_{2000} = 18,912,079;\ P_{2010} = 24,658,823$$

The length of time between 2000 and 2010 is 10 years ($t = 2010 - -2000$). Hence

$$\begin{aligned} r_A &= \frac{1}{t}\left(\frac{P_t - P_0}{P_0}\right) \times 100 \\ &= \frac{1}{10}\left(\frac{P_{2010} - P_{2000}}{P_{2000}}\right) \times 100 \\ &= \frac{1}{10}\left(\frac{24,658,823 - 18,912,079}{18,912,079}\right) \times 100 = 3.03 \end{aligned}$$

That is, the population of Ghana grew at an average arithmetic rate of 3.3 percent annually between 2000 and 2010.

4.6.3 GEOMETRIC POPULATION GROWTH

In a population, every addition has the potential to change the size of the population. Increased population contributes to further increase during their lifetimes. This implies that the growth of population mimics the principle of compound interest rate, from which we can write the geometric growth equation as:

$$P_t = P_0(1 + r_G)^t$$

where r_G is the ***average annual geometric (compound) growth rate.***

This formula calculates the future population given the current population and a growth rate, r_G. We may obtain r_G from the equation as:

$$\begin{aligned} r_G &= \left[exp\left(\frac{lnP_t - lnP_0}{t}\right) - 1\right] \times k \\ or \qquad r_G &= \left(\sqrt[t]{\frac{P_t}{P_0}} - 1\right) \times k \end{aligned}$$

where

the radix k is usually 100;
ln is the natural logarithm (see Section 4.4.1)[15].

Example 4.16

For the data in Example 4.11, calculate the geometric growth rate, using both methods.

Solution

$$P_{2000} = 18,912,079; \;\; P_{2010} = 24,658,823; \quad t = 2010 - 2000$$

Method 1:

$$\begin{aligned} r_G &= \left[exp\left(\frac{ln\ P_t - ln\ P_0}{t}\right) - 1\right] \times 100 \\ &= \left[exp\left(\frac{ln\ P_{2010} - ln\ P_{2000}}{10}\right) - 1\right] \times 100 \\ &= \left[exp\left(\frac{ln\ 24,658,823 - ln\ 18,912,079}{10}\right) - 1\right] \times 100 = 2.69 \end{aligned}$$

Method 2:

$$\begin{aligned} r_G &= \left(\sqrt[t]{\frac{P_t}{P_0}} - 1\right) \times 100 \\ &= \left(\sqrt[10]{\frac{24,658,823}{18,912,079}} - 1\right) \times 100 = 2.69 \end{aligned}$$

That is, the population of Ghana grew at an average geometric rate of 2.7 percent annually between 2000 and 2010.

4.6.4 EXPONENTIAL POPULATION GROWTH

Population data are usually available on an annual basis and the geometric growth rate is applicable since it assumes that t is a discrete variable. However, population growth is a continuous process, and we need to modify the geometric growth equation to take care of the instantaneous growth instead of annual growth. This gives the exponential growth equation:

$$P_t = P_0 e^{rt}$$

where r is the annual average exponential growth rate defined as:

$$r = \frac{ln\, P_t - ln\, P_0}{t}$$

where the radix k is usually 1.

The geometric growth rate r_G and the exponential growth rate r would usually give values very close to each other, at a certain level of approximation. The geometric growth rate is for practical use and the one usually used to measure population growth. The exponential growth rate, on the other hand, is more of mathematical interest.

Example 4.17

For the data in Example 4.11, calculate the exponential growth rate.

Solution

$P_{2000} = 18,912,079$; $P_{2010} = 24,658,823$; $t = 2010 - 2000$

$$\begin{aligned} r &= \frac{ln\, P_t - ln\, P_0}{t} \\ &= \frac{ln\, P_{2010} - ln\, P_{2000}}{10} \\ &= \frac{ln\, 24,658,823 - ln\, 18,912,079}{10} = 0.0265 \end{aligned}$$

That is, the population of Ghana grew at an annual exponential rate of 2.7 percent between 2000 and 2010.

4.6.5 DOUBLING TIME OF POPULATION

Demographers are often interested in the time that will be taken for a population to double itself for a given population growth rate r. This is given by:

$$t = \frac{ln\,2}{r}$$

Example 4.18

Ghana's population in 2010 was 24,658,823. Calculate the number of years required for Ghana's population to double.

Solution

From Example 4.17, $r =$. Hence

$$t = \frac{ln\,2}{r} = \frac{ln\,2}{0.0268} = 25.86 \text{ years}$$

That is, Ghana's population of 24,658,823 in 2010, growing at an exponential rate of 2.7 percent annually, will double in 26 years time; that is, in the year 2036.

Time of Population to Reduce to Half in Size

For a declining population, we can also estimate the number of years in which the population will reduce to half in size if the rate of decline is r. This is given as:

$$t = \frac{ln\,0.70}{r}$$

This has little significance in demography since it rarely happens in practice.

EXERCISES

4.1 The population of a certain country at the beginning of 2000 was 12 million. There were 960 births and 542 deaths in that year. If 120 people left the country but 305 came to the country to stay for two years, what is the estimated population at the end of 2000.

4.2 The monthly population figures of an area (in million) in 1990 are shown below:

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
50	52	52	55	57	65	67	68	77	75	65	88

Find the mean population of this area in 1990 using

1) the chronological approach;
2) the weighted approach.

4.3 The population of a community in 1992 was: 631,000 on 1st January; 715,000 on 1st April; 700,000 on 1st August; 819 on 1st January 1993. Find the average population of the area for 1992.

4.4 The mid-year population of Togo in 1981 was 2,691,000 and 3,531,000 in 1990. Calculate the mean population of Togo for the period.

4.5 The mid-year population of the Gambia was 635,000 in 1982; 745,000 in 1985; and 861,000 in 1990. Calculate the mean population of the Gambia for the period 1981–1990.

4.6 For the data in Exercise 4.4, calculate
1) relative growth;
2) arithmetic percentage growth;
3) annual arithmetic rate of population growth;
4) geometric growth rate;
5) exponential growth rate
of the population in the period 1982–1990.

4.7 For the data in Exercise 4.5, calculate
1) relative growth;
2) arithmetic percentage growth;
3) average annual arithmetic rate of population growth;
4) geometric growth rate;
5) exponential growth rate
of the population in the period 1982–1990.

4.8 Refer to Exercise 4.4. Estimate the number of years required for Togo's population in 1990 to double.

4.9 Refer to Exercise 4.5. Estimate the number of years required for the Gambia's population to double, using the growth rate between 1985 and 1990.

5 MEASURES OF POPULATION COMPOSITION

5.1 INTRODUCTION

5.1.1 POPULATION COMPOSITION

The population composition is the internal structure of a human population with respect to one or more demographic attributes or traits at a particular point of time. The traits which are of particular concern to demographers are sex, age, marital status, urban-rural residence, educational and economic characteristics, religion, and family types. In population literature, the term "population composition" is often used interchangeably with "population structure" or "population distribution".

In a broad sense, population distribution implies any division of the population entity into constituents arranged in a definite pattern. The term is used to include the distribution of other attributes (geographic, residential, marital, etc.). The term "population structure" is, however, usually used to relate only to age and sex composition of the population.

In this text, we shall consider only population structure (i.e. composition of population) by age and sex.

5.1.2 USES OF POPULATION COMPOSITION

The study of population composition constitutes a basic and the most relevant field of study for demographers. It is helpful in

1) describing and comparing the make-up of a population
   i) at a given date with some other populations at the same date;
   ii) at some previous dates with that of the same population;
2) determining the structural requirement of population at any time;
3) understanding and providing a more clearer vision of the dynamics of population change;
4) making population projections.

5.2 POPULATION COMPOSITION BY SEX

Sex is one of the important personal and natural characteristics of the population. It is the biological characteristic that divides the human race into males and females[16]. Unlike other demographic characteristics, sex

1) is determined at birth and it is dichotomous,
2) undergoes no change.

The number of males and females in a population expressed either in absolute numbers or as percentages of the total, is called sex distribution of the population.

5.2.1 USES OF POPULATION COMPOSITION BY SEX

Population data tabulated by males and females are important for:

1) the evaluation of the completeness and accuracy of census counts of population;
2) drawing several inferences regarding the dynamics of demographic phenomena;
3) its bearing directly on:
   i) racial and ethnic composition,
   ii) educational status, and
   iii) citizenship status,
   iv) fertility, mortality, migration, marital status, and economic characteristics;
4) the sole study of reproductivity;
5) the determination of family rights and responsibilities and the social status in a community, especially in Africa;
6) both public and private planning, such as military planning, planning of community institutions and services, particularly health services, and planning of sales programmes that require separate population data for males and females.

It is customary to consider only the female segment of the population when analysis is being done in the demographic field. This is because

1) during the course of one year, a woman can be responsible for at most two birth cycles, while a man may be responsible for several cycles;
2) the childbearing years for a woman range approximately from 15 to 50 years of age, while that of a man may range from 15 to 80 years of age;
3) registration of births more than often includes the age of the mother than that of the father, especially in the case of illegitimate births.

5.2.2 DEMOGRAPHIC MEASURES OF SEX COMPOSITION

Masculinity Proportion

The masculinity proportion (MP) is the simplest and most commonly used measure of sex composition in nontechnical discussions. It is the percentage of males in the population. It is defined as:

$$
\begin{aligned}
MP &= \frac{P_m}{P_t} \\
&= \frac{P_m}{P_m + P_f}
\end{aligned}
$$

where

P_m and P_f are population of males and females, respectively;
P_t is the total population.

$MP = 0.5$ denotes a balance of sexes;
$MP > 0.5$ denotes an excess of males;
$MP < 0.5$ denotes an excess of females.

Feminity Proportion

Similarly, we may define feminity proportion as

$$FP = \frac{P_f}{P}$$

or as a complement to the masculinity proportion:

$$FP = 1 - MP$$

Example 5.1

The 2010 Population and Housing Census of Ghana recorded 12,024,845 males and 12,633,978 females. Calculate the masculinity proportion.

Solution

$$
\begin{aligned}
&\text{Number of males} && (P_m) &&= 12,024,845 \\
&\text{Number of females} && (P_f) &&= 12,633,978 \\
&\text{Total population} && (P_t) &&= P_m + P_f = 24,658,823
\end{aligned}
$$

$$\begin{aligned} MP &= \frac{P_m}{P_t} \\ &= \frac{12,024,845}{24,658,823} = 0.488 \end{aligned}$$

That is, there is an excess of 0.012 $(0.5 - 0.488)$ or 1.2 percent of females in Ghana in the 2010 census.

Percent Excess (or Deficit) of Males

The excess (or deficit) of males as a percentage of the total population, is given by the following formula:

$$E = \frac{P_m - P_f}{P_t} \times 100$$

$E = 0$ denotes balance of the sexes;
$E > 0$ denotes an excess of males or deficit of females;
$E < 0$ denotes an excess of females or deficit of males.

Example 5.2

For the data in Example 5.1, calculate the excess or deficit of males as a percentage of the total population.

Solution

From Example 5.1,

$$P_m = 12,024,845 \quad P_f = 12,633,978 \quad P_t = 18,912,079$$

Hence

$$\begin{aligned} E &= \frac{P_m - P_f}{P_t} \times 100 \\ &= \frac{12,024,845 - 12,633,978}{24,658,823} \times 100 = -2.47 \end{aligned}$$

The result indicates that the excess of females (deficit of males) amounts to about 2.5 percent of the total population.

Sex Ratio

The sex ratio (SR), sometimes called ***masculinity ratio*** (MR), is the most widely used and principal measure of sex composition used in technical studies. It is used to compare the relative strength of the number of males and females in a population. It is usually defined as the number of males per 100 females in a population[17]:

$$SR = \frac{P_m}{P_f} \times 100$$

where P_m and P_f are as defined above.

$SR = 100$ denotes balance of the sexes;
$SR > 100$ denotes an excess of males;
$SR < 100$ denotes an excess of females.

Example 5.3

Using data in Example 5.1, calculate the sex ratio.

Solution

From Example 5.1,

$$\begin{aligned} SR &= \frac{P_m}{P_f} \times 100 \\ &= \frac{12,024,845}{12,633,978} \times 100 = 95.2\% \end{aligned}$$

That is, the male population is about 95.2 percent of the female population in Ghana according to the 2010 census.

Sex Ratio by Segments

Sex ratio can be calculated for various groups or segments of the population:

$$SR^s = \frac{P_m^s}{P_f} \times 100$$

where P_m^b and P_f^s are number of males and females, respectively in population segment s.

Sex Ratio at Birth

Of much importance and wide use, especially in mortality analysis is sex ratio at birth, defined as:

$$SR^b = \frac{P_m^b}{P_f^b} \times 100$$

where P_m^b and P_f^b are number of live births of males and females, respectively.

Note

The sex ratio of any population is affected by past fertility, mortality and migration. The evidence available indicates that many more males than females are conceived, but that the proportion of males among foetal deaths is also high.

Relationship among the Various Measures of Sex Composition

When basic data on the number of males and females are not available, it is possible when required, to convert the masculinity proportion into the masculinity ratio or the per cent excess (or deficit) of males, and the vice versa.

1) *Deriving Masculinity Proportion from Sex (Masculinity) Ratio*

The ***masculinity proportion*** may be defined from sex (masculinity) ratio as follows:

$$MP = \frac{SR}{100 + SR}$$

Example 5.4
For the data in Example 5.1, calculate the masculinity proportion from the masculinity ratio calculated in Example 5.2.

Solution
From Example 5.2, $SR = 95.2$. Therefore,

$$\begin{aligned} MP &= \frac{SR}{100 + SR} \\ &= \frac{95.2}{100 + 95.2} = 0.495 \end{aligned}$$

2) *Deriving Sex (Masculinity) Ratio from Masculinity Proportion*

The ***sex (masculinity) ratio*** may be derived from masculinity proportion as follows:

$$SR = \frac{MP}{1 - MP} \times 100$$

Example 5.5
Calculate the masculinity ratio from the masculinity proportion calculated in Example 5.1.

Solution
From Example 5.1, $MP = 0.488$. Hence,

$$\begin{aligned} SR &= \frac{MP}{1 - MP} \times 100 \\ &= \frac{0.488}{1 - 0.488} \times 100 = 95.2 \end{aligned}$$

which is similar to the result in Example 5.2.

3) The ***percent excess or deficit of males*** may be derived from masculinity proportion as:

$$E = [MP - (1 - MP)] \times 100$$

Example 5.6
Calculate the percent excess or deficit of males from the masculinity proportion calculated in Example 5.1.

Solution
From Example 5.1, $MP = 0.495$. Hence

$$\begin{aligned} E &= [MP - (1 - MP)] \times 100 \\ &= [0.488 - (1 - 0.488)] \times 100 \\ &= (0.488 - 0.505) \times 100 = -2.7 \end{aligned}$$

Comparing Results of Various Measures of Sex Composition

It is obvious that the various demographic measures of sex composition convey essentially the same information. Percent excess (deficit) is very useful in the sense that besides the information we get from masculinity proportion and sex ratio, it also tells us the quantum of the excess or deficit.

5.3 POPULATION COMPOSITION BY AGE

Age is an important study variable in any demographic, epidemiological[18] and clinical research and analysis. Every individual is, invariably, a member of a temporary class of a generation, progeny or an age class. Age is, however, a complex variable because collection and analysis of age data is not straightforward, especially in Africa.

5.3.1 DEFINITION OF CONCEPTS

Age

Age reflects the duration of life as a biological phenomenon and at the same time is the main ordinate of man's life. The age of an individual is defined as his duration of life from the moment of birth to the moment of death. A person's exact age at any given time is the time elapsed since the person was born.

Age is usually measured as a discrete variable given in completed years. A person's age in completed years is the greatest integral number $(0, 1, 2, \cdots)$ less than the persons exact age. An age less than one year is considered as age zero. In a census, age is commonly defined in terms of the age of the person at his last birthday in years for people of more than one year, in months for children of less than one year but more than one month, and in days for children of less than one month.

A class of individuals of the same age forms an ***age group***.

Generation

A generation is the progeny[19] of individuals born during the entire reproduction period throughout the lifespan. The length of a generation corresponds to the average reproductive age of a given population.

Cohort

A cohort is a group of persons experiencing the same demographic event during the same time interval, usually during the same year. The most common and most important type of cohort considered in demographic studies is a birth cohort. A birth cohort is the set of individuals born during a particular time interval. Thus the newborns of the year 2000 is readily referred to as "the 2000 birth cohort". Consequently, a population at a given time is the aggregate of the survivors of different cohorts of people born during different years. For the purpose of current life table (see Chapter 7), these surviving cohorts are pieced together to behave like a single cohort which is called a ***synthetic cohort***.

The term cohort can be used in respect of other vital events also. Thus, a marriage cohort is a group of persons who married during the same year or period. Thus, the marriage cohort of 1998 refers to all marriages contracted in the year 1998. A labour force cohort is a group of persons entering a labour force during a year. Another example is a migration cohort – a group of persons who moved into (or out of) an area during a particular period. In a census, the population distribution contains various types of cohort, such as the birth cohort and marriage cohort. When a birth cohort is composed specifically of individuals born during the same calendar year, it is called a ***generation cohort***. The size of this cohort in the life table is called the radix of the cohort and is denoted as l_0 (refer to Chapter 7).

Cohort Analysis

Cohort analysis (synonym is ***longitudinal analysis*** or ***generation analysis***) is the study of the sequence of events over time within a well defined cohort or group. Cohort analysis of demographic data is necessary to understand the process of changes over time. Data useful for cohort analysis may be obtained from any of the standard sources of demographic data. While the main source of data for cohort analysis is the survey, the other sources are not well developed, especially in Africa. Surveys can provide panel data (longitudinal), retrospective and repetitive observations.

Period

A period is an interval of time (usually between two dates) within which different or the same vital events take place. ***Period data*** relate to a type of demographic event which occurred over a great number of years (i.e. involving many cohorts) but which is observed during a specified brief period of time (usually one year). This is also referred to as ***cross-sectional data***.

Period Analysis

Period analysis (synonyms are ***cross-sectional*** or ***current analysis***) is the study of a category of events experienced by a group of cohorts during a given year, or in a period of years, covering different cohorts. For example, we may study deaths in the year 1990; these deaths take place among several different birth cohorts.

5.3.2 USES OF AGE COMPOSITION

The number of person against each age interval (1 year, 5 year, 10 year or any other equal or unequal classes) expressed in absolute numbers or as percentages is called the age composition of the population,

1) Age is the primary basis of demographic classification in vital statistics, census and survey work.
2) It is one of the most fundamental demographic characteristics interrelating all other characteristics of a person.
3) It is the most important variable in the study of mortality, fertility, nuptiality, migration, and certain areas of demographic analysis.
4) It determines the social role and place of a person in the society.
5) Age composition is important for the evaluation of the quality of the census counts of populations.
6) Age composition of a country has enormous socio-economic significance. It affects all spheres of life of its citizens: the level of consumptions, income, social services needed (schools, teachers, health services, food and housing), potential manpower, participation of people in productive work, taking part in reproduction process, potential voting population, etc.

5.3.3 TABULATION OF AGE

Detailed classification of age data collected in census or national sample surveys may consider intervals of a single year of age. Such classification consists of maximum position of 99 beginning from age 0 (0 is for persons of less than one year of age)[20]. The single-year distributions are necessary because

1) they allow a far more detailed scrutiny of the data for errors;
2) age data are highly demanded both for specific ages and special combinations of ages, making it also possible to re-group ages to minimize the effects of age misreporting;
3) age data are required for cross-classification with several characteristics which change sharply from age to age over parts of the age range (e.g. school enrollment, labour force status, marital status).

When each single years of age are cross-classified with other categories, the table will be too cumbersome for most purposes. If overly condensed, the data would cease to be useful. For analysis of concrete empirical population for most cross-classifications (for example, sex, ethnic groups, socio-economic status) a more compact classification in 5-year age groups is often employed. The 5-year age groups:

$$0-4,\ 5-9,\ 10-14,\ \ldots,\ 90-94,\ 95$$

have become almost a standard feature of demographic tables. Table 5.1 shows the population of Ghana by 5-year age groups for the five censuses.

Age group	1960	1970	1984	2000	2010
0–4	1,296,625	1,563,130	2,030,082	2,769,421	3,405,406
5–9	1,018,590	1,450,165	2,001,825	2,775,206	3,128,952
10–14	681,291	1,002,670	1,503,209	2,262,216	2,916,040
15–19	541,076	778,055	1,246,390	1,883,753	2,609,989
20–24	590,912	681,131	1,056,001	1,600,820	2,323,491
25–29	584,930	631,426	945,111	1,487,299	2,050,111
30–34	488,398	560,497	742,803	1,206,809	1,678,809
35–39	377,413	438,301	584,299	1,029,765	1,421,403
40–44	311,509	350,046	473,254	886,931	1,186,350
45–49	218,346	272,066	428,207	720,357	938,098
50–54	178,490	231,437	352,684	568,369	833,098
55–59	107,719	142,516	213,081	355,842	523,695
60–64	118,039	146,378	225,776	366,351	475,849
65–69	60,958	94,218	145,309	258,709	293,871
70–74	56,529	82,392	128,866	225,158	351,330
75–79	30,961	42,262	71,813	144,830	205,953
80–84	28,809	40,255	70,427	140,847	159,084
85+	36,220	52,368	76,944	229,396	157,294
Total	6,726,815	8,559,313	12,296,081	18,912,079	24,658,823

Table 5.1 Population of Ghana by Age, 1960–2010
Source: Various Population Censuses, GSS

For certain specific purposes, the age intervals could, however, be broader by condensing some of them or even be unequal. For instance, to study mortality, age intervals as $0,\ 1-4,\ 5-9,\ 10-19,\ 20-29\cdots$ would be more meaningful as risk and death is different among these age groups but is more, or less uniform within them. Similarly, age intervals $0-19,\ 20-44,\ 45+$ would be adequate to study migration as it is age selective.

For fertility analysis, the age groups for females have been generally chosen as $0-14$, $15-49$, $50+$. This broad division implies that these women under 15 and above 49 will not be generally able to give birth. For analysis of economic activities, the age groups are usually chosen as $0-14$, $15-64, 65+$. This implies that those under 15 and 65 and above will not in general be engaged in economically productive activities and so will be normally dependent on the economically active population, namely, those in the age group $15-64$.

5.3.4 ERRORS IN AGE DATA

Errors in age data occur in both census statistics and vital statistics. In each case there may be errors of coverage or of content, or a combination of both.

Error of Coverage

Errors of coverage in age data are of two types. These are *gross underenumeration*, when individuals of a given age may have been missed by the census; and *gross overenumeration*, when individuals of a given age may have been erroneously included in it. The balance of these two types of coverage errors represent *net underenumeration* (since underenumeration usually exceeds overestimation). Underenumeration in many censuses occur mostly in the younger ages.

Error of Content

Error of content are caused by misstatement of age of respondents and error by census officials.

Misstatement of Age
The ages of some individuals included in the census may have been

1) erroneously reported by the respondent owing to
   i) ignorance of correct age,
   ii) a general age preference,
   iii) a tendency to exaggerate length of life at advanced ages,
   iv) a possibly subconscious aversion to certain numbers,
   v) an economic, social, political or purely personal motive;

2) caused by the census enumerator by
   i) erroneous estimation of age;
   ii) incorrect computation of age from the given information;
   iii) not stating the age;

3) erroneously allocated by the census office in the course of correcting age misstatement.

A measure of misstatement or misreporting of age is referred to, in general, as ***response variability*** of age. The offsetting effect of reporting "into" and reporting "out of" a given age group is known as ***net misreporting*** or ***response bias***. The combination of net underenumeration and net misreporting for a given age is known as ***net census undercount*** or ***net census overcount***, if the number in the age is overstated, or generally termed ***net census error***.

Digit Preference

Age data have been found to be subject to ***tendentious error***. There is a strong tendency among persons while reporting their age, to exhibit digit preference. Digit preference is the unexplained liking for numbers ending in certain digits. The result of such preference is that there will be unusually large number of persons at ages ending with the preferred digit. This is known as ***age heaping***. The causes and patterns of age or digit preference vary from one culture to another, but preference for ages ending in "0" and "5" is quite widespread. Age heaping is most pronounced among populations or population subgroups having a low educational status.

Age heaping is considered to be a measure of data quality and consistency. It is the principal type of error in single-year-of-age data, although single ages are also affected by other types of age misreporting, net underenumeration, and nonreporting or misassignment of age. Fortunately, age data are usually presented in the form of 5 or 10 year age groups and age heaping disappears to a large extent but other biases will not be canceled out.

Another form of tendentious error is ***tendentious bias***. A tendentious bias is the tendency for people to report an age closer to their actual age because of certain importance or emotional reasons attached to it. For example, the tendency of a Ghanaian near 17 to report the age as 18 because 18 is significant for voting or marriage rights in Ghana. We may observe from Table 5.2 that females in Ghana are very eager to move to age 18. The ratio of males to females reduces sharply from age 17 to 18. On the other hand, males are eager to be 21 years old.

Age-group	1960	1970	1984	2000	2010
15	116.5	113.6	112.3	108.5	103.2
16	111.8	112.1	106.4	101.1	99.5
17	107.5	111.4	108.6	105.6	101.5
18	95.5	94.6	96.5	102.8	100.9
19	89.9	95.1	98.4	102.0	98.9
20	79.8	78.8	85.4	89.8	92.0
21	85.0	88.3	91.0	97.2	94.0
22	80.9	78.5	80.2	88.4	86.5
23	88.3	89.4	86.6	91.4	87.4
24	85.5	76.5	81.7	90.9	88.9

Table 5.2 Ratio of Males per 100 Females (Sex Ratio) in Single Years from age 15 to 24, 1960–2010
Source: Author's calculation from various Population and Housing Censuses, GSS

Ages Unknown or not Stated

Age tabulation from census and vital registration sources frequently contain a category of persons of unstated age. This occurs when an age is unknown, illegible, partially stated, inconsistent with other particulars on the enumeration schedule, or ambiguous, such as "over eighteen".

The issue of unknown or unstated age is troublesome to those who use population data. It also adds to the expense of publishing census reports since they require the same amount of space in print as a known age class. To eliminate this expense, a method is devised by the census office to assign an age to each case in the enumeration schedule where the problem occurs. In doing this, two basic principles should be followed. That we should

1) be guided by any information on the census schedule that may be indicative of age;
2) ensure that the estimated age was not inconsistent with any reliable information on the schedule.

5.3.5 DETECTION OF ERRORS IN AGE DATA

Post enumeration surveys or a sample reinterview study is a means of detecting errors in census data. This is particularly useful in providing information and causes of errors of reporting in single years. However, in practice, the size of sample of the reinterview survey ordinarily precludes any evaluation in terms of single ages. Various arithmetic techniques have been developed to handle errors on age reporting. These include age ratios, sex ratios and age accuracy index. The extent of age heaping is measured by indices of digit preference.

There are several measures including Baachi's Index and Ramachandran's Index to measure age heaping but the four measures that will be discussed in this text include age ratios, age-ratio-type index, Whipple's index and Myers' index. These measures are applicable where age is reported in single years.

5.3.6 AGE RATIOS

Age ratios (AR) are calculated from the census data to check the quality of the census returns by age groups. It may be defined as "the ratio of the population in the given age group to one half of the sum of the population in the two adjacent age groups multiplied by 100". That is, the age ratio for a 5-year group, ${}_5P_x$, is defined as:

$$AR_x = \frac{{}_5P_x}{\frac{1}{2}\left({}_5P_{(x-5)} + {}_5P_{(x+5)}\right)} \times 100$$

where

${}_5P_x$	is the population in age group $x+4$,
${}_5P_{(x-5)}$	is the population in the preceding age group,
${}_5P_{(x+5)}$	is the population in the following age group.

The lower the AR, the more adequate the census data on age.

$AR=100$

This implies that there is no error in age reporting.

$AR < 100$
This implies either that members of the group were selectively underenumerated or that errors in age reporting resulted in mis-classifying persons who belonged to the age group.

$AR > 100$
This implies either that members of the group were selectively overenumerated or that errors in age reporting resulted in mis-classifying persons into the age group who did not belong there, or both.

Example 5.7

Refer to Table 5.1. Calculate for the year 2010 the age ratio of the population in

1) age group 10–14 years;
2) age group 20–24 years

Solution

1) To calculate age ratio for age group 5-9, we require population figures for age groups 0–4, 5–9 and 10–14 which we obtain from Table 5.1:

$$\begin{array}{llcccc} \text{Age-group} & 5-9 & = & {}_5P_{x-5} & = & 3,128,952 \\ \text{Age-group} & 10-14 & = & {}_5P_x & = & 2,916,040 \\ \text{Age-group} & 15-19 & = & {}_5P_{x+5} & = & 2,609,989 \end{array}$$

Hence the age ratio of the population in the age group 10-14 years is

$$AR_{5-9} = \frac{2,916,040}{\frac{1}{2}(3,128,952 + 2,609,989)} \times \quad 100 = 101.62$$

About 1.6 percent of the members in the age group 10–14 were selectively over-enumerated.

Alternatively stated: "errors in age reporting resulted in 1.6 percent of mis-classifying persons in the age group 5–9 who did not belonged to that age group in the 2010 census.

2) To calculate age ratio for age group 20–24, we require population figures for age groups 15–19, 20–24 and 25–29 which we obtain from Table 5.1:

$$\begin{array}{lcl} {}_5P_x & = & {}_5P_{(20-24)} = 2,323,491 \\ {}_5P_{x-5} & = & {}_5P_{(15-19)} = 2,609,989 \\ {}_5P_{x+5} & = & {}_5P_{(25-29)} = 2,050,111 \end{array}$$

Hence the age ratio of the population in the age group 35-39 years is

$$AR_{35-39} = \frac{{}_5P_x}{\frac{1}{2}\left({}_5P_{x-5} +_5 P_{x+5}\right)} \times 100$$

$$= \frac{2,323,491}{\frac{1}{2}(2,609,989 + 2,050,111)} \times 100 = 99.22$$

About 0.8 percent of the members in the age group 20–24 were selectively underenumerated.

We may atate it as: "errors in age reporting resulted in 0.8 percent of mis-classifying persons in the age group 20–24 who belonged to that age group in 2010."

Generally, age ratios should be studied for a series of age groups, usually for the entire span of age up to, at least, age 70. Column 3 of Table 5.3 presents age ratios for all the age groups.

Alternative Definition of Age Ratios

Alternatively, age ratios have been defined as the ratio of the population in the given age group to one-third of the sum of the populations in the age group itself and the two adjacent age groups, times 100. Thus,

$$AR = \frac{{}_5P_x}{\frac{1}{3}\left({}_5P_{(x-5)} +_5 P_x +_5 P_{(x+5)}\right)} \times 100$$

This definition is preferred because it centres on the age group under study and heaps considerably on it. An index of 100 indicates no concentration on this age. The higher the index the greater the concentration on the age examined.

Age-group	2010 Population	Age ratio	Age Ratio (Alternative definition)
1	2	3	4
0–4	3,405,406	–	–
5–9	3,128,952	99.00	99.33
10–14	2,916,040	101.62	101.08
15–19	2,609,989	99.63	99.75
20–24	2,323,491	99.72	99.81
25–29	2,050,111	102.45	101.62
30–34	1,678,809	96.72	97.79
35–39	1,421,403	99.22	99.48

Age-group	2010 Population	Age ratio	Age Ratio (Alternative definition)
40–44	1,186,350	100.56	100.37
45–49	938,098	92.91	95.16
50–54	833,098	113.98	108.91
55–59	523,695	80.02	85.73
60–64	475,849	116.41	110.37
65–69	293,871	71.05	78.64
70–74	351,330	140.58	123.83
75–79	205,953	80.70	86.25
80–84	159,084	87.59	91.37
85+	157,294	–	–
Total	24,658,823		

TABLE 5.3 Age Ratio of 2010 Population of Ghana
Source: Author's calculation from 2010 Population and Housing Census, GSS

Example 5.8

Refer to Table 5.1. Calculate for the year 2010 the age ratio of the population in

1) age group 5–9 years;
2) age group 20–24 years

using the alternative definition

Solution

1) From Table 5.6,

$$\begin{array}{llcccc} \text{Age-group} & 5-9 & = & {}_5P_{x-5} & = & 3,128,952 \\ \text{Age-group} & 10-14 & = & {}_5P_{x} & = & 2,916,040 \\ \text{Age-group} & 15-19 & = & {}_5P_{x+5} & = & 2,609,989 \end{array}$$

Hence

$$\begin{aligned} AR &= \frac{{}_5P_{(5-9)}}{\frac{1}{3}\left({}_5P_{(5-9)} +_5 P_{(10-14)} +_5 P_{(15-19)}\right)} \times 100 \\ &= \frac{2,916,040}{\frac{1}{3}\left(3,128,952 + 2,916,040 + 2,609,989\right)} \times 100 = 101.08 \end{aligned}$$

2) From Table 5.1,

$$\begin{aligned} {}_5P_x &= {}_5P_{(20-24)} = 2,323,491 \\ {}_5P_{x-5} &= {}_5P_{(15-19)} = 2,609,989 \\ {}_5P_{x+5} &= {}_5P_{(25-29)} = 2,050,111 \end{aligned}$$

Hence

$$\begin{aligned} AR &= \frac{{}_5P_{(20-24)}}{\frac{1}{3}\left({}_5P_{(15-19)} +_5 P_{(20-24)} +_5 P_{(25-29)}\right)} \times 100 \\ &= \frac{2,323,491}{\frac{1}{3}\left(2,609,989 + 2,323,491 + 2,050,111\right)} \times 100 = 99.81 \end{aligned}$$

Column 4 of Table 5.3 presents age ratios of the alternative definition for all the age groups.

5.3.7 AGE-RATIO-TYPE INDEX

The age-ratio-type index of digit preference (ARTI) is based on the assumption that the true figures are uniformly distributed, that is, there are equal numbers in age over some age range which includes and, preferably, is centred on the age being examined. However, when the age range is large (for example, 11 years), the true figures may be assumed to be linear, that is, the true figures form an arithmetic progression, or that they decrease by equal amounts from age to age over the range. The age ranges often used are the 3-year, 5-year, or 11-year.

The ARTI on age x may be calculated as:

$$ARTI_x^{(3\,year)} = \frac{P_x}{\frac{1}{3}\left(P_{(x-1)} + P_x + P_{(x+1)}\right)} \times 100$$

for the 3-year age range and

$$ARTI_x^{(5\,year)} = \frac{P_x}{\frac{1}{5}\left(P_{(x-2)} + P_{(x-1)} + P_x + P_{(x+1)} + P_{(x+2)}\right)} \times 100$$

for the 5-year age range.

The ARTI on age x for 11-year age range is similarly calculated.

Example 5.10

Table 5.4 gives the 2010 population of Ghana for single ages from 20 to 70 years.

Calculate the ARTI for

1) 3-year range
2) 5-year range
3) 11-year range

of the total population within the ages 23 to 62 inclusive.

Solution

1) Calculation of ARTI for 3-year range:

To calculate age-ratio-type index for age 24, we require population figures for ages 23, 24 and 25:

$$\begin{array}{lcl} \text{Age } 23: & P_{23} & = & 401,890 \\ \text{Age } 24: & P_{24} & = & 412,380 \\ \text{Age } 25: & P_{25} & = & 556,140 \end{array}$$

Hence the age-ratio-type index of the population in the age group 24 year is

$$ARTI_{(24)}^{(3\,year)} = \frac{412,380}{\frac{1}{3}(401,890 + 412,380 + 556,140)} \times 100 = 90.3$$

That is, 9.7 percent less than expected have been reported at age 24 (assuming a 3-year range uniform distribution).

Age	Population	Age	Population
20	650,177	36	264,250
21	398,945	37	227,508
22	460,099	38	294,871
23	401,890	39	173,747
24	412,380	40	467,054
25	556,140	41	166,202
26	373,308	42	243,400
27	427,358	43	157,462
28	414,518	44	152,232
29	278,787	45	333,095
30	579,941	46	165,217
31	274,835	47	144,972
32	336,958	48	184,120
33	249,483	48	110,694
34	237,592	50	335,273
35	461,027	51	98,454
16	496,123	52	148,808
17	474,666	53	134,533
18	599,926	54	116,030
19	433,937	55	164,660
20	650,177	56	119,741

Age	Population	Age	Population
21	398,945	57	81,770
22	460,099	58	102,315
23	401,890	59	55,209
24	412,380	60	221,708
25	556,140	61	55,469
26	373,308	62	80,380
27	427,358	63	63,296
28	414,518	64	54,996
29	278,787	65	124,53897
30	579,941	66	43,228
31	274,835	67	42,573
32	336,958	68	53,220
33	249,483	69	30,312
34	237,592	70	170,36
35	461,027		

TABLE 5.4 2010 Population of Ghana for Single Ages from 20 to 70 years

To calculate age-ratio-type index for age group 25, we require population figures for age groups 24, 25 and 26:

$$\begin{aligned} \text{Age } 24: \quad P_{24} &= 412,380 \\ \text{Age } 25: \quad P_{25} &= 556,140 \\ \text{Age } 26: \quad P_{26} &= 373,308 \end{aligned}$$

Hence the age ratio of the population for age 25 is

$$ARTI_{(25)}^{(3\,year)} = \frac{556,140}{\frac{1}{3}(412,380 + 556,140 + 373,308)} \times 100 = 124.3$$

That is, 24.3 percent more than expected have been reported at age 25 (assuming a 3-year range uniform distribution).

The age-ratio-type indices of the remaining ages for 3-year range are similarly obtained. The results are presented in Table 5.5.

2) Calculation of ARTI for 5-year range

To calculate age-ratio-type index for age 25, we require population figures for ages 23, 24, 25, 26 and 27:

$$\begin{aligned}
\text{Age } 23: \quad P_{23} &= 401,890 \\
\text{Age } 24: \quad P_{24} &= 412,380 \\
\text{Age } 25: \quad P_{25} &= 556,140 \\
\text{Age } 26: \quad P_{26} &= 373,308 \\
\text{Age } 27: \quad P_{27} &= 427,358
\end{aligned}$$

Hence the age-ratio-type index of the population in the age 25 for 5-year range is:

$$ARTI_{(25)}^{(5\,year)} = \frac{556,140}{\frac{1}{5}(401,890 + 412,380 + 556,140 + 373,308 + 427,358)} \times 100 = 128.1$$

That is, 28.1 percent more than expected have been reported at age 25 (assuming a 5-year range uniform distribution).

To calculate age-ratio-type index for age 26, we require population figures for age groups 24, 25, 26, 27 and 28::

$$\begin{array}{llcl} \text{Age } 24: & P_{24} & = & 412,380 \\ \text{Age } 25: & P_{25} & = & 556,140 \\ \text{Age } 26: & P_{26} & = & 373,308 \\ \text{Age } 27: & P_{27} & = & 427,358 \\ \text{Age } 28: & P_{28} & = & 319,570 \end{array}$$

Hence the age ratio of the population in the age 26 for the 5-year range is

$$ARTI_{(26)}^{(5\,year)} = \frac{373,308}{\frac{1}{5}\left(412,380+448,420+373,308+427,358+414,518\right)} \times 100 = 85.5$$

The age-ratio type indices of the remaining ages for the 5-year range are similarly obtained. The results are presented in Table 5.5.

The indices for 11-year old age are similarly calculated and given in Table 5.5.

5.3.8 WHIPPLE'S INDEX

The Whipple's Index (WI) has been developed to detect preference for or avoidance of a particular terminal digit or each terminal digit. It measures the intensity of concentration at multiples of 5, namely, age heaping at the digits 5 and 0. The index dwells on the ages between 23 and 62 years inclusive. The choice of this age range is standard, but largely arbitrary. The two main reasons for this choice are:

1) Heaping was most evident within this period of life for a study in 1910 in the United States;
2) Ages of early childhood and extreme old age are more strongly affected by other types of errors of reporting than by preference for specific terminal digits.

The WI is defined as the ratio of the sum of the populations at the ages ending in '0' or '5' to one-fifth of the total population in the range 23 to 62. That is,

$$WI = \frac{P_{25}+P_{30}+P_{35}+P_{40}+P_{45}+P_{50}+P_{55}+P_{60}}{\frac{1}{5}(P_{23}+P_{24}+P_{25}+P_{26}+\cdots+P_{59}+P_{60}+P_{61}+P_{62})} \times 100$$

P_a is the number of persons reporting their age as a.

In a more compact form,

$$WI = \frac{\sum_{a=25}^{60} P_a^{05}}{\frac{1}{5}\sum_{a=23}^{62} P_a} \times 100$$

where P_a^{05} is the number of persons reporting ages ending in '0' or '5';

Age	2000 Population	3-year	5-year	11-year
23	401,890	–	–	–
24	412,380	90.3	–	–
25	556,140	124.3	128.1	–
26	373,308	82.5	85.5	–
27	427,358	105.5	104.2	105.9
28	414,518	111.0	99.9	174.1
29	278,787	65.7	70.6	7152.3
30	579,941	153.5	153.8	177.6
31	274,835	69.2	79.9	798.8
32	336,958	117.4	100.4	975.8
33	249,483	90.8	80.0	777.3
34	237,592	75.2	76.7	7142.2
35	461,027	143.6	160.1	192.2
36	264,250	83.2	89.0	980.2
37	227,508	86.8	80.0	8110.2
38	294,871	127.1	103.3	167.2
39	173,747	55.7	65.3	6174.7
40	467,054	173.6	173.6	169.1
41	166,202	56.9	68.8	6106.0

Age	2000 Population	3-year	5-year	11-year
42	243,400	128.8	102.6	169.8
43	157,462	85.4	74.8	672.9
44	152,232	71.0	72.4	7149.0
45	333,095	153.6	174.8	186.9
46	165,217	77.1	84.3	876.9
47	144,972	88.0	77.3	7103.1
48	184,120	125.6	97.9	163.3
48	110,694	52.7	63.4	6190.5
50	335,273	184.8	191.1	162.9
51	98,454	50.7	59.5	699.9
52	148,808	116.9	89.3	992.7
53	134,533	101.1	101.5	987.0
54	116,030	83.8	84.8	8114.7
55	164,660	123.4	133.5	1101.4
56	119,741	98.1	102.4	170.2
57	81,770	80.7	78.1	7
58	102,315	128.3	88.1	–
59	55,209	43.7	53.4	–
60	221,708	200.1	215.2	–
61	55,469	46.5	—	–
62	80,380	–	–	–
Total	9,803,391			

TABLE 5.5 Age-ratio-type Indices of the 2010 Population of Ghana
Source: Author's calculation from 2000 Population and Housing Census, GSS

Interpretation of WI

The Whipple's Index lies in the range 100 and 500, that is

$$100 \leq WI \leq 500$$

A value of $WI = 100$

Suppose there is no age heaping at 0 and 5, that is, there is no universal tendency to report ages as 25, 30, 35, 40, 45, 50, 55, 60. Assuming that the ages from 23 to 62 years are equally reported among the population. But the numerator of Whipple's index gives ages ending in only 2 digits ('0' and '5'), out of the 10 (0, 1, 2,$\cdots$, 9) digits. This represents approximately $\frac{1}{5}$ of the total population in that range, equivalent to the denominator, resulting in WI being equal to 100.

Thus, $WI = 100$ is the theoretical minimum which implies no age heaping at 0 and 5. That is, there is no preference for 0 or 5.

A value of $WI = 500$

Suppose there is a complete age heaping, that is, the entire population is concentrated at ages 25, 30, 35,$\cdots$, 60 with none reporting their ages as 23, 24, 26$\cdots$, 29, 31,$\cdots$, 59. The numerator should be approximately the same as the total population in the range 23 to 62 years given in the denominator.

Thus, $WI = 500$ is the theoretical maximum which means that there is complete age heaping, that is, a preference for only 0 and 5 (everbody reporting ages ending in 0 and 5).

A value of $100 < WI < 500$

That is, when the value of WI is greater than 100 but less than 500. Table 5.6 shows what meaning may be assigned to any value of WI within that range.

Whipple's Index	Quality of the Data
Less than 105	Highly accurate
105–109.9	Fairly accurate
110–124.9	Fairly acceptable
125–174.9	Inaccurate
175–499.9	Highly inaccurate

TABLE 5.6 Scale for detecting the reliability of the data

Example 5.11

Refer to Table 5.5. Calculate the Whipple's Index of digit preference.

Solution

The calculation of the Whipple's Index requires population in the age range 23 to 62 years. Preliminary calculations are in Table 5.12.

Age	2010 Population
25	556,140
30	579,941
35	461,027
40	467,054
45	333,095
50	335,273
55	164,660
60	221,708
Total	3,118,898

TABLE 5.7 Preliminary calculation of Whipple's Index for Ghana, 2010
Source: Computed from 2010 Population and Housing Census, GSS

From Table2 5.5 and 5.7,

$$\sum_{a=23}^{62} = P_{23} + P_{24} + P_{25} + P_{26} + \cdots + P_{59} + P_{60} + P_{61} + P_{62} = 9,803,391$$
$$\sum_{a=25}^{60} = P_{25} + P_{30} + P_{35} + P_{40} + P_{45} + P_{50} + P_{55} + P_{60} = 3,118,898$$

Hence,

$$WI = \frac{3,118,898}{\frac{1}{5}(9,803,391)} \times 100 = 159.1$$

That is, reporting of age data in the 2010 census was inaccurate.

Special Case of Whipple's Index

A special case of the WI is the case when we assume a uniform distribution in a 10-year range defined as:

$$WI_0 = \frac{P_{30} + P_{40} + P_{50} + P_{60}}{\frac{1}{10}(P_{23} + P_{24} + P_{25} + P_{26} + \cdots + P_{59} + P_{60} + P_{61} + P_{62})} \times 100$$

for an age ending in ‘0’; or

$$WI_5 = \frac{P_{25} + P_{35} + P_{45} + P_{55}}{\frac{1}{10}(P_{23} + P_{24} + P_{25} + P_{26} + \cdots + P_{59} + P_{60} + P_{61} + P_{62})} \times 100$$

for an age ending in ‘5’.

Limitations of Whipple’s Index

Whipple’s Index has the following limitations.

1) Heaping occurs on digits other than 5 and 0, usually with no particular pattern but WI is applicable when preference for age is limited to 0 and 5. Hence, WI is not an efficient method.
2) WI considers only the arbitrary interval 23 to 62 years and not the entire life span of 0 to ω where ω is the last age attainable.
3) WI does not take into account the decreasing nature of the age distribution due to depletion by death. That is, population at younger ages would receive greater weight than at the older ages which would bias the index.

5.3.9 MYERS’ INDEX

Myers has suggested a modified index to overcome the limitations of Whipple’s Index. It is the most widely known index among the more complex measures. Similar to the Whipple’s Index, Myers’ Index has been developed to reflect preferences or dislikes of terminal digits. Unlike Whipple’s Index, Myers’ Index is for each of the ten digits from 0 to 9.

A modification of the WI is to ascertain what proportion of the total population reports at an age ending in each of the digits, x $(x = 0, 1, 2, 3, \cdots, 9)$. The bias developed by starting only at a particular digit is minimised by beginning the count at each of the 10 digits in turn and then average the results. The method derives a ***blended population*** for the digit at age x which is essentially a weighted sum of the number of persons reporting ages ending in each of the ten terminal digits, denoted by P_x. The underlying assumption is that if there are no systematic irregularities in the reporting of age, the blended sum at each terminal digit should be approximately equal to 10 percent of the total blended population.

The blended population at each of the ten digits (b_x^P) can be compared with the total of the blended population, called the ***blended sum***, to obtain the percentage distribution of the blended sum (BSD_x):

$$BSD_x = \frac{b_x^P}{\sum_{x=0}^{9} b_x^P} \times 100$$

BSD_x is the percentage of the total population reporting on the given terminal digit. Any b_x^P must therefore be 10 percent of $\sum_{x=0}^{9} b_x^P$ when there is no preference for any digit x.

Interpretation of BSD_x

The blended sum BSD_x lies in the range 0 and 100, that is,

$$0 \leq BSD_x \leq 100$$

The point of balance of heaping is 10.

A value of $BSD_x = 10$
When $BSD = 10$, then there is no age heaping.

A value of $BSD_x > 10$
When $BSD_x > 10$, it indicates a tendency towards preference for a particular digit.

A value of $BSD_x < 10$
When $BSD_x < 10$ it indicates a tendency towards avoidance for a particular digit.

Overall Measure of Preference for all Terminal Digits

An overall measure of preference for all terminal digits, the Myers' Index, is one-half the sum of the absolute deviations from 10. That is[21]:

$$MI = \frac{1}{2}\left|\sum_{x=0}^{9} BSD_x - 10\right|$$

Interpretation of Myer's Index

The MI lies in the range 0 and 90, that is,

$$0 \leq MI \leq 90$$

A value of $MI = 0$
When $MI = 0$, there is no age heaping, that is, an indication of excellent age reporting.

A value of $MI = 90$
When $MI = 90$, then all recorded ages end in the same digit.

Steps in Computing Myers' Index

The intermediate calculations of Myers' Index are usually presented in two tables. The first table contains the first set of calculations which are done in three steps (steps 1–3). The second table starts with the first column (column 1) and the last two columns (columns 11–12) of the first table which are obtained in steps (2) and (3). What follow in this table are based on steps (4)–(8).

Step 1

Put the terminal (or preferred) digits from 0 to 9 in column (1) of the table and indicate the populations with ages ending in each of the terminal digits in nine subsequent columns (columns 2–10).

Step 2

Sum the populations with the ages ending in each digit over the whole range, starting with the lower limit of the range, namely, 10. Put the results in column (11).

Step 3

Sum the populations with the ages ending in each digit over the whole range, starting with the next lower limit of the range, namely, 20. Put the results in column (12).

Note

The results in steps (2) and (3) are transferred to columns (2) and (3) respectively of another table. Column (1) of this second table is transferred from column (1) of the first table.

Step 4

Assign weights to the sums in steps (2) and (3). For the terminal digits 0 to 9, the corresponding weights for the sums in step (2) are 1 to 10 while those for the sums in step (3) are 9 to 0. Thus, for each of the preferrred digits we have the corresponding "blended population". The weights represent the number of times the combination of ages in step (2) or (3) is included when the starting age is varied from 10 to 19.

Step 5

Apply the weights in step (4) to the sums in steps (2) and (3), that is, multiply the weights by their corresponding sums to obtain the weighted sums. The total of the weights is same for each digit.

Step 6

Add the two weighted sums in step (5) to obtain the blended sum.

Step 7

Calculate percentage distribution of the blended sum.

Step 8

Subtract each percentage in step (7) from 10 to obtain the extent of concentration on or avoidance of a particular digit.

Example 5.12

Table 5.8 shows the 2010 total population of Ghana for single ages. Calculate the Myers' Index of digit preference.

Age	Total Population	Age	Total Population	Age	Total Population
0	731,201	34	237,592	68	53,220

Age	Total Population	Age	Total Population	Age	Total Population
1	622,871	35	461,027	69	30,312
2	680,641	36	264,250	70	170,36
3	684,823	37	227,508	71	44,162
4	685,870	38	294,871	72	57,196
5	653,006	39	173,747	73	38,695
6	656,286	40	467,054	74	40,911
7	629,007	41	166,202	75	93,439
8	625,319	42	243,400	76	34,621
9	565,334	43	157,462	77	24,228
10	731,610	44	152,232	78	34,680
11	506,549	45	333,095	79	18,985
12	607,796	46	165,217	80	94,740
13	538,356	47	144,972	81	14,390
14	531,729	48	184,120	82	21,499
15	605,337	48	110,694	83	14,341
16	496,123	50	335,273	84	14,114
17	474,666	51	98,454	85	237,902
18	599,926	52	148,808	86	13,613
19	433,937	53	134,533	87	10,849
20	650,177	54	116,030	88	11,106
21	398,945	55	164,660	89	9,600
22	460,099	56	119,741	90	34,228
23	401,890	57	81,770	91	24,098
24	412,380	58	102,315	92	6,388
25	556,140	59	55,209	93	13,258

Age	Total Population	Age	Total Population	Age	Total Population
26	373,308	60	221,708	94	3,109
27	427,358	61	55,469	95	610,309
28	414,518	62	80,380	96	24,973
29	278,787	63	63,296	97	12,951
30	579,941	64	54,996	98	34,910
31	274,835	65	124,538	99	16432
32	336,958	66	43,228		
33	249,483	67	42,573		

TABLE 5.8 Total Population of Ghana for Single Ages, 2010

Solution

Step 1

Column (1) of Table 5.9 shows all the terminal digits from 0 to 9. Columns (2)–(10) are the populations with ages ending in each of the terminal digit. For example the terminal digit 0, has ages 10, 20, 30, $\cdots$, 80, 90. Also the terminal digit 1, has ages 11, 21, 31,$\cdots$, 81, 91. Finally, the terminal digit 9, has ages 19, 29, 39, $\cdots$, 89, 99.

Terminal digit x	10–19 (10+x)	20–29 (20+x)	30–39 (30+x)	40–49 (40+x)	50–59 (50+x)	60–69 (60+x)	70–79 (70+x)	80–89 (80+x)	90–99 (90+x)	Total (10+x) +	Total (20+x)+
1	2	3	4	5	6	7	8	9	10	11	12
0	557,528	476,467	489,751	394,941	269,635	199,256	133,417	80,737	32,144	2,633,876	4,710,224
1	383,529	247,983	134,853	84,440	52,604	36,675	18,413	13,509	6,643	1,115,119	1,710,239
2	503,320	333,629	266,155	177,915	105,010	52,901	34,021	18,500	7,289	1,552,948	2,548,368
3	414,386	267,135	154,745	140,264	67,841	39,474	21,377	14,294	5,753	1,094,412	1,805,295
4	403,453	275,606	161,305	89,371	73,279	38,045	17,930	13,807	5,413	1,178,076	1,852,832
5	465,317	448,420	367,789	306,235	131,707	125,276	72,378	37,154	13,036	1,916,381	3,418,376
6	360,009	276,284	196,335	111,798	77,179	31,461	21,793	18,589	8,008	1,144,900	1,886,347
7	333,064	249,139	135,229	83,096	41,479	29,819	12,527	11,216	4,191	1,032,013	1,598,709
8	425,225	319,570	208,158	139,399	66,702	45,698	23,700	20,476	9,513	1,193,225	2,026,441
9	300,138	193,886	122,254	79,829	38,775	26,455	14,432	20,123	29,848	858,666	1,384,268

TABLE 5.9 Preliminary calculation of Myer's Index

Steps 2 and 3

The last two columns are the totals. Column (11) is the sum of the populations with ages ending in each terminal digit over the whole range, starting with age 10. Column (12) is the sum of the populations with ages ending in each terminal digit over the whole range, starting with age 20.

Step 4

Columns (11) and (12) of Table 5.9 are transferred to columns (2) and (3) of Table 5.10. Column (4) are the weights for column (2) (that is, the sum of the ages at each terminal digit x starting from age 10). Column (5) are the weights for column (3) (that is, the sum of the ages at each terminal digit x starting from age 20). As indicated earlier, the sum of the weights in column (4) and column (5) (even though not shown in the table) are the same for each digit.

Terminal digit, x	Sum $(10+x)$	Sum $(20+x)$	Weights for Sum $(10+x)$	Weights for Sum $(20+x)$	Weighted Sum $(10+x)$	Weighted Sum $(20+x)$
1	2	3	4	5	6=(2) × (4)	7=(3) × (5)
0	2,633,876	2,076,348	1	9	2,633,876	18,687,132
1	978,649	595,120	2	8	1,957,298	4,760,960
2	1,498,740	995,420	3	7	4,496,220	6,967,940
3	1,125,269	710,883	4	6	4,501,076	4,265,298
4	1,078,209	674,756	5	5	5,391,045	3,373,780
5	1,967,312	1,501,995	6	4	11,803,872	6,007,980
6	1,101,456	741,447	7	3	7,710,192	2,224,341
7	899,760	566,696	8	2	7,198,080	1,133,392
8	1,258,441	833,216	9	1	11,325,969	833,216
9	825,740	525,602	10	0	8,257,400	-
Total						

TABLE 5.10 Final calculation of Myer's Index

Steps 5–8

The remaining calculations are shown in the table. Thus,

$$MI = \frac{1}{2}\left|\sum_{x=0}^{9} BSD_x - 10\right| = \frac{1}{2}(30.6) = 15.3$$

Since $MI > 0$ it indicates that there is age heaping.

The next step is to locate the digit preference. From column (8), $BSD_0 = 18.8 > 10$, $BSD_5 = 15.7 > 10$ and so also are BSD_2 and BSD_8 which are slightly greater than 10. Therefore, age heaping is at ages 0, 2, 5 and 8.

Blended Sum	Percentage Distribution $BSD_x = \frac{b_x^P}{\sum b_x^P} \times 100$	Deviation from 10 $BSD_x - 10$	Absolute Deviation from 10 $\lvert BSD_x - 10\rvert$
8=(6)+(7)	9	$10=(9)-10$	$(11)=\lvert(10)\rvert$
21,321,008	18.8	8.8	8.8
6,718,258	5.9	-4.1	1.4
11,464,160	10.1	0.1	0.1
8,766,374	7.7	-2.3	w2.3
8,764,825	7.7	-2.3	2.3
17,811,852	15.7	5.7	5.7
9,934,533	8.8	-1.2	1.2
8,331,472	7.3	-2.7	3.7
12,159,185	10.7	0.7	0.7
8,257,400	7.3	-2.7	2.7
113,529,067	100.0	0	30.6

TABLE 5.10 (*Continued*) Final calculation of Myer's Index

Note

In Exercise 5.4, the reader is asked to calculate the Myers' Index for the 2000 population. Compare that result with the result for this example. If the Myers' Index for the 2010 is smaller, then there is improvement of reporting of age data over time and vice versa.

Limitations of Myers' Index

1) Myers' Index is more complex to calculate than the Whipple's Index.
2) Similar to the Whipple's Index, Myers' Index too does not have any sound theoretical basis.
3) Myers' Index assesses only digit preference but we know that digit preference is not the only tendentious bias in reported age.

In spite of the limitations of Myers' Index, it is the simplest among other measures and is still in common use particularly because other indices have not established their superiority. It provides an overall measure of age heaping as well as an index of preference for each digit.

5.3.10 DEMOGRAPHIC MEASURES OF AGE COMPOSITION

Sometimes we may want to compare age distributions for different areas, for population subgroups in a single area, and for the same area at different dates. Caution must, however, be taken in making comparisons over populations, as they depend not only on the size of the differences but also on the number of age categories used.

A detailed representation of a society's age structure can be based on a division of its population into three main age groups: the child population, the active population and the aged population. For international comparisons, the child population is taken to be persons under 15 years, the active population to be 15–64 years and the aged population as persons of 65 years and over. The child population and the aged population are usually referred to as the ***dependent population*** and the active population as the ***productive population***.

Various measures that are used for this purpose include computing percentages in various age groups, age dependency ratio, the average age, index of ageing, index of relative difference and index of dissimilarity.

In the following six sections, we shall discuss these measures in detail.

5.3.11 PERCENTAGE AGE DISTRIBUTION

We can compare two populations over space and time in terms of their age structure. This can be done by comparing the percentage of numbers in various age categories, defined arbitrarily or defined on the purpose it serves.

A convenient way of showing the age distribution that provides sufficient information is to present the percentage distribution of the child population, the active population and the aged population. The percentage distribution of the three population segments are defined as:

$$\text{Percent Child Population} = \frac{C_{0-14}}{P} \times 100$$

$$\text{Percent Active Population} = \frac{A_{15-64}}{P} \times 100$$

$$\text{Percent Aged Population} = \frac{O_{65+}}{P} \times 100$$

where

C_{0-14} are population under 15 years;
A_{15-64} are population aged 15–64 years;
O_{65+} are population aged 65 years and over;
P is total population.

Example 5.13

The population in the 2010 Population and Housing Census of Ghana by age is given in Table 5.11. Calculate percent

1) Child population,
2) Active population,
3) Aged population.

Solution

From Table 5.11, the population aged 0–14 years was 7,806,843 in 2010, those 15–64 years was 10,106,296 and those aged 65 years and over was 998,940. Thus

$$\begin{aligned} C_{0-14} &= 9,450,398 \\ A_{15-64} &= 14,040,893 \\ O_{65+} &= 1,167,532 \end{aligned}$$

Consequently,

$$\begin{aligned} \text{Percent Child Population} &= \frac{C_{0-14}}{P} \times 100 \\ &= \frac{9,450,398}{24,658,823} \times 100 = 38.3\% \\ \text{Percent Active Population} &= \frac{A_{15-64}}{P} \times 100 \\ &= \frac{14,040,893}{24,658,823} \times 100 = 56.9\% \\ \text{Percent Aged Population} &= \frac{O_{65+}}{P} \times 100 \\ &= \frac{1,167,532}{24,658,823} \times 100 = 4.7\% \end{aligned}$$

That is, the population of Ghana in 2010 constituted 38.3 percent child population, 56.9 percent active population and 4.7 percent aged population.

5.3.12 AGE DEPENDENCY RATIO

The dependency ratio is an index of the age distribution. It is usually employed to stress the economic implications of the age distribution.

Child Dependency Ratio

The child dependency ratio describes the number of children that are supported by the active members of the population. It is defined as

$$DR_c = \frac{C_{0-14}}{A_{15-64}} \times 100$$

where C_{0-14} and A_{15-64} are as defined above.

Old Age Dependency Ratio

The old age dependency ratio describes the number of the aged population that are supported by the economically active members of the population. It is defined as

$$DR_o = \frac{O_{65+}}{A_{15-64}} \times 100$$

where O_{65+} and A_{15-64} are as defined above.

This ratio shows the burden of those in advanced ages on those in the working age group.

Total Dependency Ratio

The age dependency ratio or simply the dependency ratio (DR) shows the relative predominance of persons in the dependent ages in relation to those in the productive ages. It is defined as the number of youths under 15 years of age and the old aged of 65 years and over to the adults aged 15–64 years. That is,

$$DR_t = \frac{C_{0-14} + O_{65+}}{A_{15-64}} \times 100$$

where C_{0-14}, O_{65+} and A_{15-64} are as defined above.

Alternatively, the total dependent ratio may be calculated by adding the child dependent ratio and the old age dependent ratio. Thus,

$$\begin{aligned} DR_t &= \text{Child Dependency Ratio + Old Age Dependency Ratio} \\ &= DR_c + DR_o \end{aligned}$$

Example 5.14

For the data in Table 5.11, calculate

1) the child dependency ratio;
2) the old age dependency ratio;
3) the total dependency ratio.

Solution

From Example 5.13,

$$\begin{aligned} C_{0-14} &= 9,450,398 \\ A_{15-64} &= 14,040,893 \\ O_{65+} &= 1,167,532 \end{aligned}$$

1) Child Dependency Ratio

$$\begin{aligned} DR_c &= \frac{C_{0-14}}{A_{15-64}} \times 100 \\ &= \frac{9,450,398}{14,040,893} \times 100 = 67.3\% \end{aligned}$$

That is, there were about 67 child dependents per 100 persons in the working ages (i.e., in the 15–64 age group). That is to say that for every 100 productive persons there were about 67 child dependents in 2010.

2) Old Age Dependency Ratio

$$\begin{aligned} DR_o &= \frac{O_{65+}}{A_{15-64}} \times 100 \\ &= \frac{1,167,532}{14,040,893} \times 100 = 8.3\% \end{aligned}$$

That is, there were about 8 old aged dependents per 100 persons in the working ages. That is to say that for every 100 productive persons there were about 8 old aged dependents in 2010.

3) Total Dependency Ratio

$$\begin{aligned} DR &= \frac{C_{0-14} + O_{65+}}{A_{15-64}} \times 100 \\ &= \frac{9,450,398 + 1,167,532}{14,040,893} \times 100 = 75.6\% \end{aligned}$$

Alternatively,

$$\begin{aligned} DR &= \text{Child Dependency Ratio + Old Age Dependency Ratio} \\ &= 67.3 + 8.3 = 75.6 \quad \text{percent} \end{aligned}$$

That is, there were about 76 dependents per 100 persons in the working ages. That is to say that for every 100 productive persons there were about 76 dependents in 2010.

Note

1) In a typical African country 80 ≤ DR ≤ 100;
2) In a typical developed country 50 ≤ DR ≤ 70.

In most African countries, where retirement from workforce typically comes early, it is more applicable to employ the population aged 60 and over for the adult dependents and population 15 to 59 for adults of working age.

5.3.13 MEASURES OF AGEING POPULATION

Unlike an individual person who always grows older over time, the population of the aggregate may grow older or younger. The ageing of population can be described as the increase in the relative number of older people in the population. It may be measured by:

1) Percent aged population, discussed in 5.3.11;
2) Old age dependency ratio, discussed in 5.3.12, which shows the burden of those in advanced ages on those in the working age group;
3) Age-specific Sex Ratio, discussed in Section 5.4.3, for the elderly population which shows how relatively more serious the problem of ageing is impacting on one of the sexes.

4) Index of Ageing;
5) Average age of population (the mean and the median).

When there is an increase in a measure of ageing over time, then it means that the population is ageing. On the other hand, when the measure has a decreasing trend, we say that the population is rejuvenating.

5.3.14 INDEX OF AGEING

The index of ageing is defined as

$$IA = \frac{O_{65+}}{C_{0-14}} \times 100$$

where O_{65+} and C_{0-14} are as defined above.

Example 5.15

Refer to Table 5.11. Calculate the index of ageing for the population of Ghana in 2010.

Solution

From Example 5.13,

$$O_{65+} = 1,167,532; \quad C_{0-14} = 9,450,398$$

Hence the index of ageing is

$$\begin{aligned} IA &= \frac{O_{65+}}{C_{0-14}} \times 100 \\ &= \frac{1,167,532}{9,450,398} \times 100 = 12.4\% \end{aligned}$$

Interpretation

The older population of Ghana in 2010 was about 12 percent that of the child population. This implies that for every 100 children there were about 12 older persons in 2010.

Limitation of Index of Ageing

The index of ageing does not bring out the current ageing problem, rather the potential problem. It is influenced by two main factors, namely, fertility and mortality.

1) The decline in fertility may cause a population to grow older and vice versa. This factor, however, affects only the denominator of the ageing index and does not disturb the numerator for a long period of time.
2) The decline of mortality at younger ages makes the age distribution younger while mortality decline at older ages makes the age distribution older. But the decline of mortality at all ages produces only a marginal effect on the ageing of population.

5.3.15 MEAN AGE OF POPULATION

The ***mean age*** of population, $\overline{X}$, may be defined as

$$\overline{X} = \frac{\sum_{a=1} P_a X_a}{\sum_{a=1} P_a}$$

where X_a is a single age or the mid-point of the age interval a, if age groups are used; P_a is the population for age group a.

Example 5.16

Refer to Table 5.1. Calculate the mean age for the 2010 population of Ghana.

Solution

Table 5.11 presents the preliminary calculations for obtaining the mean value.

Age group	Mid-point P_a	Population X_a	$P_a X_a$
1	2	3	$4 = (2) \times (3)$
0–4	2	3,405,406	6,810,812
5–9	7	3,128,952	21,902,664
10–14	12	2,916,040	34,992,480
15–19	17	2,609,989	44,369,813
20–24	22	2,323,491	51,116,802
25–29	27	2,050,111	55,352,997
30–34	32	1,678,809	53,721,888
35–39	37	1,421,403	52,591,911
40–44	42	1,186,350	49,826,700

Age group	Mid-point P_a	Population X_a	$P_a X_a$
45–49	47	938,098	44,090,606
50–54	52	833,098	43,321,096
55–59	57	523,695	29,850,615
60–64	62	475,849	29,502,638
65–69	67	293,871	19,689,357
70–74	72	351,330	25,295,760
75–79	77	205,953	15,858,381
80–84	82	159,084	13,044,888
85+	87	157,294	13,684,578
Total		24,658,823	605,023,986

TABLE 5.11 Preliminary calculation of Mean Age of 2010 Population of Ghana
Source: Author's calculation from 2010 Population and Housing Census, GSS

Thus,

$$\overline{X} = \frac{\sum_{a=1} P_a X_a}{\sum_{a=1} P_a}$$

$$= \frac{605,023,986}{24,658,823} = 24.5$$

That is, the mean age of the population of Ghana in 2010 was 24.5 years.

Limitation of Mean Age of Population

There are two main limitations of computing the mean for the age distribution.

1) Age distributions are usually skewed to the left but the mean is appropriate if the distribution is bell-shaped.
2) In many cases, the age composition will have only an open interval for the last group as we have in Table 5.1, which makes it inappropriate to calculate the mean. As we might have noted, we made appropriate adjustment to the last group.

To overcome these limitation, the median is often used as the appropriate measure of the average age.

5.3.16 MEDIAN AGE OF POPULATION

The ***median age*** of population, M_{age}, is the most useful single figure summarising a population's age structure. It is the value which divides the total population into two equal parts, half older and half younger. It may be defined as[22]:

$$M_{age} = L_{me} + \frac{\frac{N}{2} - F}{P_{me}} \times h$$

where

L_{me} = lower class boundary of the median class, i.e the class interval upon which the median lies;

$\frac{N}{2}$ = one-half the total population; $N = \sum P_a$ is total population;

P_{me} = the number of persons in the age group containing the median;

F = total population up to the class immediately before the median class;

h = width (size of age interval) of median class.

Example 5.17

Refer to Table 5.1. Calculate the median age for the 2010 population of Ghana.

Solution

Table 5.12 presents the preliminary calculations for obtaining the median value.

Age group (Class Interval)	Population P_a	Cumulative frequency cf
1	2	3
0–4	3,405,406	2,769,421
5–9	3,128,952	6,534,358
10–14	2,916,040	9,450,398
15–19	2,609,989	12,060,387
20–24	2,323,491	14,383,878
25–29	2,050,111	16,433,989
30–34	1,678,809	18,112,798

Age group (Class Interval)	Population P_a	Cumulative frequency cf
35–39	1,421,403	19,534,201
40–44	1,186,350	20,720,551
45–49	938,098 47	21,658,649
50–54	833,098 52	22,491,747
55–59	523,695 57	23,015,442
60–64	475,849 62	23,491,291
65–69	293,871 67	23,785,162
70–74	351,330 72	24,136,492
75–79	205,953 77	24,342,445
80–84	159,084 82	24,501,529
85+	157,294 87	24,658,823
Total	24,658,823	

TABLE 5.12 Preliminary calculation of Median Age of 2010 Population of Ghana

We determine the class boundary that contains the median.

1) From column 2,

$$N = \sum P_a = 24,658,823$$

Hence

$$N/2 = \sum P_a/2 = 12,329,411.5$$

and the median lies in the class interval of the distribution which has 12,329,411.5 population.

2) Beginning at the first class interval and adding up the P_a in order (column 3), we find that class interval 15–19, inclusive, contains the median. This is the median class.

To convert the lower class limit of 15 to lower class boundary, we need to subtract 0.5 from 15 to get 14.5. Thus,

$$L_{me} = 14.5; \;\; P_{me} = 2,609,989; \;\; F = 9,450,398; \;\; h = 5$$

Consequently,

$$\begin{aligned} M_e &= L_{me} + \frac{\frac{N}{2} - F}{P_{me}} \times h \\ &= 14.5 + \frac{12,329,411.5 - 9,450,398}{2,609,989} \times 5 = 20.02 \end{aligned}$$

That is, half of the population of Ghana in 2010 were less than 20 years and half were more than 20 years[23].

5.3.17 INDEX OF RELATIVE DIFFERENCE

The index of relative difference (IRD) can be used to compare the composition of several populations with respect to age and other demographic characteristics such as marital status, economic, educational, social and ethnic characteristics.

In relation to age, the IRD is defined as

$$IRD = \frac{1}{2} \times \frac{\sum_{a=1}^{n} \left| \left(\frac{r_{a2}}{r_{a1}} \times 100 \right) - 100 \right|}{n}$$

where

$r_{ia}\,(i=1,2)$ refers to the proportion in ith population in age group a to the total number of persons in all age groups (total population);

n is the number of age groups and it is used as the divisor to derive the mean of the percent differences at each age.

If the age distributions are the same, $IRD = 0$. The larger the IRD the bigger the difference between the two age distributions.

Steps in Computing Index of Relative Difference

Step 1

Calculate the proportion of the two populations in each age group to the total population:

$$r_{ia} = \frac{P_{ai}}{\sum_{a=1}^{n} P_{ai}}, \qquad i = 1,2$$

Step 2

Divide the proportions in each age group of the later year (population 2) by those of the previous year (population 1) to obtain the relatives:

$$r = \frac{r_{a2}}{r_{a1}}$$

Step 3

Multiply the values in Step 2 by 100 to obtain percentages.

Step 4

Subtract 100 from the percentage values in Step 3 and sum the absolute values (i.e. ignoring sign) to obtain total relative percentage.

Step 5

Divide the total relative percentage in Step 4 by the number of age groups, n, to obtain the mean of the relative percentage at each age, which is the index of relative difference.

Example 5.18

The population of Ghana for 2000 and 2010 censuses by age group is given in Table 5.1. Calculate the index of relative difference of the composition of the two populations.

Solution

Table 5.13, columns (1)–(6) show the preliminary calculations of index of relative difference.

Age-group	1984 Population	2000 Population	Proportion distribution of 1984 Population (r_{a1})	Proportion distribution of 2000 Population (r_{a2})	$\frac{r_{a2}}{r_{a1}}$
1	2	3	4	5	6=(5)/(4)
0–4	2,769,421	3,405,406	0.1464	0.1381	0.9431
5–9	2,775,206	3,128,952	0.1467	0.1269	0.8647
10–14	2,262,216	2,916,040	0.1196	0.1183	0.9886
15–19	1,883,753	2,609,989	0.0996	0.1058	1.0626
20–24	1,600,820	2,323,491	0.0846	0.0942	1.1132
25–29	1,487,299	2,050,111	0.0786	0.0831	1.0572
30–34	1,206,809	1,678,809	0.0638	0.0681	1.0669
35–39	1,029,765	1,421,403	0.0545	0.0576	1.0586
40–44	886,931	1,186,350	0.0469	0.0481	1.0259
45–49	720,357	938,098	0.0381	0.0380	0.9988
50–54	568,369	833,098	0.0301	0.0338	1.1242
55–59	355,842	523,695	0.0188	0.0212	1.1287
60–64	366,351	475,849	0.0194	0.0193	0.9962
65–69	258,709	293,871	0.0137	0.0119	0.8712
70–74	225,158	351,330	0.0119	0.0142	1.1967
75–79	144,830	205,953	0.0077	0.0084	1.0906
80–84	140,847	159,084	0.0074	0.0065	0.8663
85+	229,396	157,294	0.0121	0.0064	0.5259
Total	18,912,079	24,658,823	1.0000	1.0000	17.9793

TABLE 5.13 Preliminary calculations of Index of Relative Difference

Hence,

$$IRD = \frac{1}{2} \times \frac{\sum_{a=1}^{n} \left| \left(\frac{r_{a2}}{r_{a1}} \times 100 \right) - 100 \right|}{n}$$

$$= \frac{1}{2} \times \frac{(17.99 \times 100) - 100}{18} = 46.94\%$$

Interpretation

The average relative difference of the age structure of the population of Ghana between 2000 and 2010 was 46.9 percent. That is, the percentage age distribution of the 2010 population of Ghana differs from that of 2000 by about 47 percent in relative terms.

5.3.18 INDEX OF DISSIMILARITY

The index of dissimilarity (ID) is another summary measure of the difference between two age distributions and can also be used to compare the composition of several populations with respect to other demographic characteristics such as marital status, economic, educational, social and ethnic characteristics. It is a measure of absolute percentage difference of any two comparable population distributions. It indicates the percent of one population that needs to be redistributed to have the age distribution of the other. When a spatial comparison is made for two dates, the index may be interpreted as the percentage of the population of the latter date that would have to be relocated to attain the spatial distribution of the population of earlier date.

The ID is defined as:

$$ID = \frac{1}{2} \times \sum_{a=1}^{n} |r_{a2} - r_{a1}| \times 100$$

Example 5.19

Refer to Table 5.1. Calculate index of dissimilarity of the composition of the 2000 and 2010 populations and interpret the result.

Solution

Table 5.14 (Col (7)) shows the preliminary calculations of index of dissimilarity. Hence,

$$ID = \frac{1}{2} \times \sum |r_{a2} - r_{a1}| \times 100$$

$$= \frac{1}{2} \times 0.08070 \times 100 = 4.0\%$$

Interpretation

The absolute difference of the age structure of the population of Ghana between 2000 and 2010 was 4 percent. That is, the percentage age distribution of the 2010 population of Ghana differs from that of 2000 by about 4 percent in absolute terms.

The implication of this result is that 4 percent of the population of Ghana in 2010 had to be redistributed to attain the age distribution of the population in 2000. This implies that the age distribution of population of Ghana had changed very little between the two census (2000 and 2010). This is due to the fact that neither fertility, mortality nor migration streams have changed drastically during this period.

Age-group	2000 Population	2010 Population	Proportion distribution of 1984 Population (r_{a1})	Proportion distribution of 2000 Population (r_{a2})	$\lvert r_{a2} - r_{a1} \rvert$
1	2	3	4	5	$6 = \lvert(5) - (4)\rvert$
0–4	2,769,421	3,405,406	0.1464	0.1381	0.0083
5–9	2,775,206	3,128,952	0.1467	0.1269	0.0199
10–14	2,262,216	2,916,040	0.1196	0.1183	0.0014
15–19	1,883,753	2,609,989	0.0996	0.1058	0.0062
20–24	1,600,820	2,323,491	0.0846	0.0942	0.0096
25–29	1,487,299	2,050,111	0.0786	0.0831	0.0045
30–34	1,206,809	1,678,809	0.0638	0.0681	0.0043
35–39	1,029,765	1,421,403	0.0545	0.0576	0.0032
40–44	886,931	1,186,350	0.0469	0.0481	0.0012
45–49	720,357	938,098	0.0381	0.0380	0.0000
50–54	568,369	833,098	0.0301	0.0338	0.0037
55–59	355,842	523,695	0.0188	0.0212	0.0024
60–64	366,351	475,849	0.0194	0.0193	0.0001
65–69	258,709	293,871	0.0137	0.0119	0.0018
70–74	225,158	351,330	0.0119	0.0142	0.0023
75–79	144,830	205,953	0.0077	0.0084	0.0007
80–84	140,847	159,084	0.0074	0.0065	0.0010
85+	229,396	157,294	0.0121	0.0064	0.0058
Total	18,912,079	24,658,823	1.0000	1.0000	0.0764

TABLE 5.14 Preliminary calculations of Index of Relative Difference

5.4 POPULATION COMPOSITION BY AGE AND SEX

The age-sex composition of the population is one of the most important characteristics of the population. It is the net result of the past trends of fertility, mortality and migration operating in an area at any specified time. It has many social and economic implications. It is rather this factor that significantly influences mortality and fertility, and socio-economic development of the society.

The sex composition, for example, of Ghana's population in 2010 was 12 million males and 12.6 million females. While this information may be important, it provides no information on the detailed structure of the Ghana population for that year. If, however, the population was presented not only according to sex, but also by age category, then there would be a new dimension of interpretation for this population. With this, certain vital analysis on the population, such as, reproductivity issues can be done. Table 5.15 shows the distribution of the population of Ghana in the 2000 census by age and sex. It shows 19 strata belonging to this population.

5.4.1 TYPES OF AGE-SEX POPULATION STRUCTURE

The age-sex population structure may be depicted using absolute numbers or percentages. The structure on the population figures themselves (see Table 5.15) can show differences or changes in the overall size of the total population and in the numbers at each age group by sex.

Age-group	Total	Male	Female
Less than 1 year	525,258	262,041	263,217
1–4	2,244,163	1,117,729	1,126,434
5–9	2,775,206	1,390,652	1,384,554
10–14	2,262,216	1,151,131	1,111,085
15–19	1,883,753	961,162	922,591
20–24	1,600,820	763,051	837,769
25–29	1,487,299	695,494	791,805
30–34	1,206,809	566,439	640,370
35–39	1,029,765	490,864	538,901
40–44	886,931	443,284	443,647

Age-group	Total	Male	Female
45–49	720,357	377,315	343,042
50–54	568,369	279,950	288,419
55–59	355,842	182,843	172,999
60–64	366,351	177,347	189,004
65–69	258,709	129,090	129,619
70–74	225,158	106,513	118,645
75–79	144,830	74,268	70,562
80–84	140,847	66,941	73,906
years and over	229,396	121,268	108,128
Total	18,912,079	9,357,382	9,554,697

TABLE 5.15 Population Composition of Ghana in 2000 by Age and Sex
Source: 2000 Population and Housing Census, GSS

For comparison purpose, however, percentages are calculated at each age group for both male and female on the respective totals of each sex (see Table 5.16).

Age-group	Total	Male	Female
Less than 1 year	525,258	262,041	263,217
1–4	2,244,163	1,117,729	1,126,434
5–9	2,775,206	1,390,652	1,384,554
10–14	2,262,216	1,151,131	1,111,085
15–19	1,883,753	961,162	922,591
20–24	1,600,820	763,051	837,769
25–29	1,487,299	695,494	791,805
30–34	1,206,809	566,439	640,370
35–39	1,029,765	490,864	538,901
40–44	886,931	443,284	443,647

Age-group	Total	Male	Female
45–49	720,357	377,315	343,042
50–54	568,369	279,950	288,419
55–59	355,842	182,843	172,999
60–64	366,351	177,347	189,004
65–69	258,709	129,090	129,619
70–74	225,158	106,513	118,645
75–79	144,830	74,268	70,562
80–84	140,847	66,941	73,906
years and over	229,396	121,268	108,128
Total	18,912,079	9,357,382	9,554,697

TABLE 5.16 Population Composition of Ghana in 2000 by Age and Sex
Source: Author's Calculation from the 2000 Population and Housing Census, GSS

There are basically two types of age-sex population structure, namely, progressive and retrogressive population structure.

The ***progressive population structure***, (also referred to as the ***pyramidal structure*** or ***expansive structure***) characterises a high proportion of young population (0–14 years) and low proportion of old population (50 years and above). This indicates that every generation of newly born babies outnumbers the previous generation as a result of high fertility while the generation vanishes fast as a result of high mortality. This is typical of developing countries.

The ***retrogressive population structure*** (also referred to as the ***cylindrical structure stable structure***), characterises low proportion of young persons (0–14) and relatively higher proportion of old (aged 50 years and over). This is an indication of a stable number of births in every year with all the newly born babies having the chance of living, and the chance of moving from one age to another because of low mortality rate. Each age cohort is approximately the same size. Also, with low mortality, every woman who reaches child bearing age lives at least until she exceeds childbearing age. This is typical of developed countries.

5.4.2 POPULATION PYRAMID

The age-sex composition of population is usually represented graphically by what is called a ***population pyramid*** or an ***age pyramid***, or simply a ***pyramid***. A population pyramid, also referred to as ***age-sex graph***, therefore, is a special type of graph that shows the distribution of the population of an area in terms of age groups, called cohorts, and sex. It consists of a number of bars representing successive age groups from the lowest age at bottom to the highest age at top by sex. It depicts a back-to-back histogram for ages of males and females in a given group.

Uses of the Population Pyramid

A population pyramid is one of the most widely used graphical displays in demography.

1) It displays and compares national and regional populations;
2) It conveys, at a glance, the entire shape of the age-sex structure;
3) It shows gross irregularities due to:
   i) widespread omission of people of some age group by the census enumeration,
   ii) special past events (such as war, epidemic or age-selective migration), fluctuations of fertility, inaccurate reporting of age.

Types of Population Pyramid

Since pyramids are just the graphical representation of age-sex population structure, Pyramids may be constructed on the population numbers themselves or on the percentages. For the purpose of comparing different population sizes, it is better to construct population pyramids using percentages. The percentages can either be percentages of men and women separately or percentages of the total population; the diagram should make clear which is intended. Pyramids on percentages are based on grand total of the population show the proper proportion of the size of each age-sex group to the total. This type of pyramid shows the differences or changes in the proportional size of each age-sex group.

To compare different pyramids, the pyramids should have the same scale or at least the same ratio between the units on the vertical scale and the units on the horizontal scale. If this relation is altered, even the same population looks very different.

With appropriate selection of scale, percent and absolute population pyramids are identical.

Construction of Population Pyramids

For the purposes of comparison, population pyramids are constructed in a standard manner. The technique of construction is as follows.

1) The abscissa (horizontal axis) indicates the population.
2) The ordinate (vertical axis) indicates the age groups.
3) A central vertical axis perpendicular to the horizontal axis indicates zero. On the left of the central axis is a correspondence of the male population and on the right is the female population. The central vertical axis may be marked with the age groups either at the right or left of the pyramid or on both the right and left. Most often pyramids show 5-year age groups though it can also be made in age groups of 10 or in single years of age. For some reasons, pyramids are truncated at an age group where the data begin to run thin.
4) A horizontal bar (rectangle) which is proportional to the population of the group is drawn from the central axis to correspond to each age group of each sex. The bars are placed on one another in ascending order (from the lowest to the highest age), that is, the age groups are arranged in strata, youth at the bottom, and old age at the top. The heights of these bars are proportional to the number of years in the age groups.

Example 5.20

For the data in Table 5.15, construct the population pyramid of Ghana.

Solution

The pyramid is displayed in Fig. 5.1.

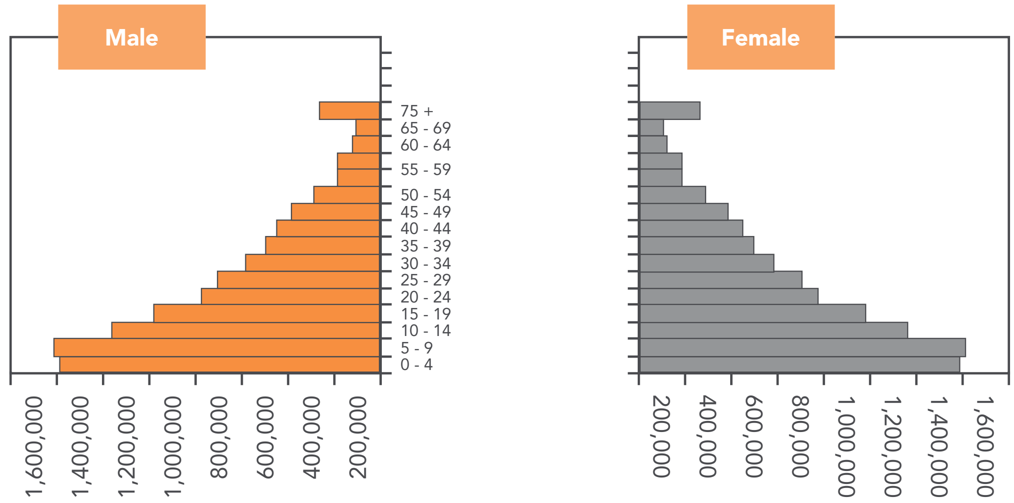

Fig. 5.1 Age-Sex Population Pyramid of Ghana, 2000

5.4.3 FORMS OF POPULATION PYRAMIDS

Three factors, which may vary within broad limits, determine the form of a population pyramid. These factors are fertility, mortality and migration.

The general shape of the population pyramid is a triangle from which its name is derived. It is broad at the base, tapering upward according to the past numbers of births and deaths that have formed it. This is simply because of the high fertility factor which increases the number at the base while the mortality factor progressively reduces the numbers of the generations as they get older, or, in other words, as they move towards the upper parts of the population pyramid. However, many other factors do intervene which explain the more or less irregular shape of these population pyramids.

The two forms of population structure discussed in Section 5.4.1, result in two forms of pyramids: progressive and retrogressive pyramids.

Progressive Population Pyramid

The classical ***progressive population pyramid***, is a triangular pyramid, with:

1) a broad base,
2) an acute base angle,
3) unbroken lateral sides, and
4) a rapid narrowing vertex.

The classical progressive population pyramid is illustrated in Fig. 5.2 (a).

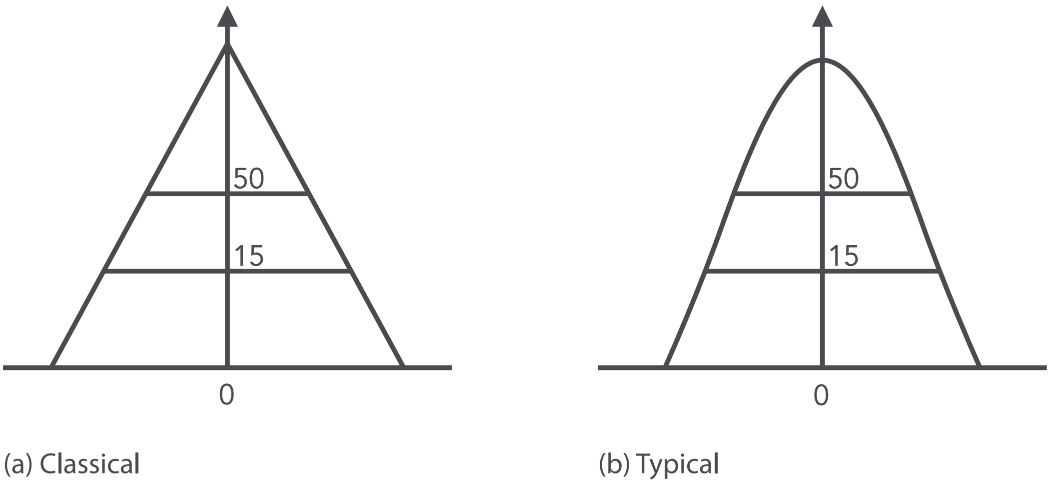

Fig. 5.2 Progressive Pyramid

However, as pointed out earlier, certain factors do intervene and we obtain a typical progressive pyramid as shown in Fig.5.2 (b).

The progressive population pyramid is typical of developing countries. The population pyramid for the population of Ghana in Figure 5.1 is an example of a typical progressive pyramid.

Retrogressive Population Pyramid

The classical ***retrogressive population pyramid*** is a cylindrical pyramid that has:

1) a narrow base with the middle section having the same dimensions,
2) right angle at the base,
3) smooth lateral sides, and
4) a slow narrowing vertex.

The classical retrogressive population pyramid is illustrated in Fig.5.3 (a). It assumes unchanging age population structure, that is, there is equal proportion of the young and the old ages. However, in practice we obtain a typical retrogressive pyramid that is narrower at the base than at the middle section (see Fig.5.3(b).

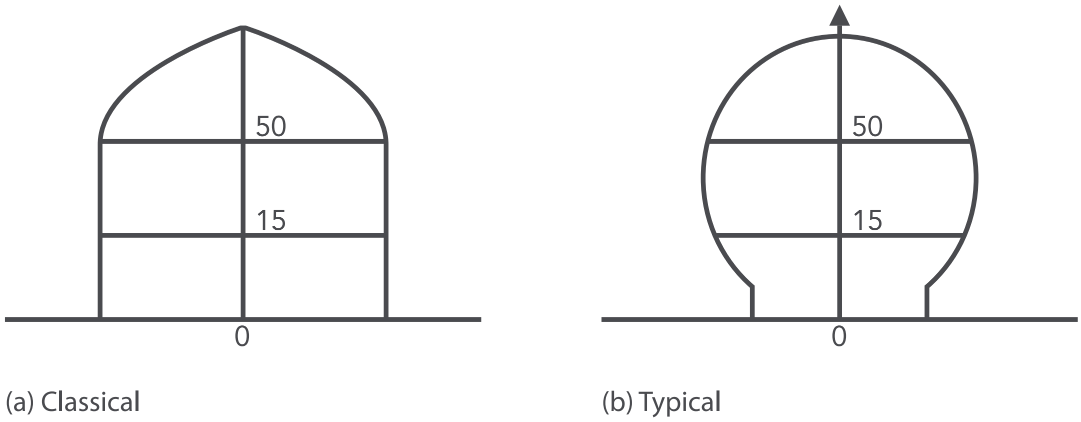

Fig. 5.3 Retrogressive Pyramid

This is typical of developed countries. Figure 5.4 shows the pyramid for the population of Switzerland, a developed country, in 2002. The classical shape appears in this case but has been perceptively modified, especially owing to the intervention of a multitude of factors in the past.

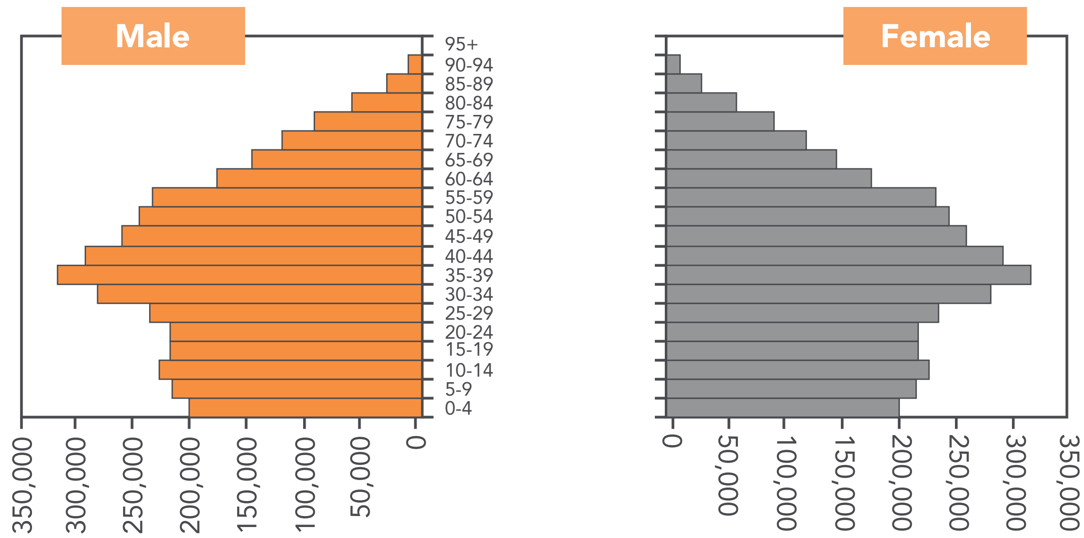

Fig. 5.4 Age-Sex Population Structure of Switzerland

5.4.4 MEASURES OF AGE-SEX COMPOSITION

Age-specific Sex Ratios

One measure for evaluating census age data is the age-specific sex ratios. This ratio is simply the number of males per 100 females at age x or in each age group a[24]. That is

$$SR_x = \frac{{}_5P_x^m}{{}_5P_x^f}$$

where ${}_5P_x^m$ is number of males in a 5-year interval of age group x;
${}_5P_x^f$ is number of females in a 5-year interval of age group x.

Usually, more males are born than females. But in life more males die than females. The continual higher mortality of males throughout most of life is due to the fact that

1) males are generally more susceptible to disease,
2) vocations of males are typically more dangerous.

So under normal situations[25] we would expect that the sex ratio would decline by age but very gradually. Any deviation from this trend or pattern suggests

1) relative under-enumeration or over-enumeration of ages of males relative to females, or
2) a tendency of the two sexes shifting in opposite directions; males usually exaggerate while females understate their ages.

The value less or more than 100 percent by 10 percent is a serious issue. This means that sex ratio is poorly recorded or reported.

Example 5.21

Refer to Table 5.15. Calculate the age-specific sex ratios for the 2000 population of Ghana.

Solution

The sex ratio by age is defined similar to the sex ratio given earlier (refer to Section 5.2.3). Thus:

$$SR_x = \frac{{}_5P_x^m}{{}_5P_x^f} \times 100$$

Thus, for the various age groups we have:

$$\begin{aligned}
SR_{<1} &= \frac{{}_5P_{<1}^m}{{}_5P_{<1}^f} \times 100 = \frac{262,041}{263,217} \times 100 = 99.6\% \\
SR_{1-4} &= \frac{{}_5P_{1-4}^m}{{}_5P_{1-4}^f} \times 100 = \frac{1,117,729}{1,126,434} \times 100 = 99.2\% \\
\vdots & \qquad \vdots \\
SR_{80-84} &= \frac{{}_5P_{80-84}^m}{{}_5P_{80-84}^f} \times 100 = \frac{66,941}{73,906} \times 100 = 90.6\% \\
SR_{85+} &= \frac{{}_5P_{85+}^m}{{}_5P_{85+}^f} \times 100 = \frac{66,941}{73,906} \times 100 = 112.2\%
\end{aligned}$$

Age-Sex Accuracy Index

The age-sex accuracy index (ASAI) is an overall measure of the accuracy of an age distribution.

Let SR_x be tha sex ratio at age or age group x, AR_x^m and AR_x^f the age ratios for males and females, respectively, at age x. Then ASAI is defined as:

$$ASAI = (3 \times \overline{SR^*}) + \overline{AR_x^m} + \overline{AR_x^f}$$

where

$$\begin{aligned}
\overline{SR^*} &= \frac{\sum_{x=1}^{n} |SR_x - SR_{x+1}|}{n} \\
\overline{AR^m} &= \frac{\sum_{x=1}^{n} |AR_x^m - 100|}{n} \\
\overline{AR^f} &= \frac{\sum_{x=1}^{n} \left|AR_x^f - 100\right|}{n}
\end{aligned}$$

This ASAI is not very exact and is usually regarded as an "order of magnitude" rather than as a precise measurement. The lower this index, the more adequate the census data on age. Table 5.16 shows the UN definition of ASAI.

ASAI	Quality of Reporting
Less than 20	Accurate
20–40	Inaccurate
More than 40	Highly inaccurate

TABLE 5.16 Scale for interpreting Age-Sex Accuracy Index

Steps in Computing Age-Sex Accuracy Index

Step 1

Calculate the age ratios for males AR_x^m and females AR_x^f in each age group, x (see Section 5.3.6).

Step 2

Calculate the sex ratios for each age, SR_x.

Step 3

Take the first difference of the sex ratios, that is, subtract each sex ratio from the sex ratio before it.

Step 4

Find the mean of the first difference of the sex ratios by summing the values in step 3 and dividing it by the number of age groups, n. This gives $\overline{SR^*}$.

Step 5

Take the deviation of the age ratios in step 1 from 100 and find the mean by dividing by n. This gives $\overline{AR^m}$ and $\overline{AR^f}$.

The values obtained in steps 3 and 4 should be put in the age-sex accuracy formula.

Example 5.22

Refer to Table 5.15. Calculate the age-sex accuracy index for the 2000 population of Ghana.

Solution

The preliminary calculations are given in Table 5.17.

For example, the age ratio for males for age group 5–9 is:

$$\begin{aligned} AR^m_{10-14} &= \frac{{}_5P_{(10-14)}}{\frac{1}{2}\left({}_5P_{(5-9)} +_5 P_{(15-19)}\right)} \times 100 \\ &= \frac{1,1,151,131}{\frac{1}{2}\left(1,390,652+961,162\right)} \times 100 = 97.89 \quad \text{per cent} \\ AR^m_{10-14} &= \frac{{}_5P_{(10-14)}}{\frac{1}{2}\left({}_5P_{(5-9)} +_5 P_{(15-19)}\right)} \times 100 \\ &= \frac{1,1,151,131}{\frac{1}{2}\left(1,390,652+961,162\right)} \times 100 = 97.89 \quad \text{per cent} \end{aligned}$$

The following values for AR^m_x are similarly calculated and the results given in col. 3 of the table.

Similarly, the age ratio for females, AR^f_x, are calculated and given in col. 6.

Now,

$$\begin{aligned} \overline{AR^m} &= \frac{\sum_{i=1}^{n} \left|AR^m_x - 100\right|}{n} = \frac{81.51}{13} = 6.27 \\ \overline{AR^f} &= \frac{\sum_{i=1}^{n} \left|AR^f_x - 100\right|}{n} = \frac{120.31}{13} = 9.25 \\ \overline{SR^*} &= \frac{\sum_{x=1}^{n} \left|SR_x - SR_{x+1}\right|}{n} = \frac{91.23}{18} = 6.52 \end{aligned}$$

Hence, the age-sex accuracy index is

$$\begin{aligned} ASAI &= (3 \times \overline{SR^*}) + \overline{AR^m} + \overline{AR^f} \\ &= (3 \times 6.52) + 6.27 + 9.25 = 35.08 \end{aligned}$$

By the UN definition, the age-sex accuracy index of 35.3 indicates that the reporting of ages by sex is inaccurate.

Age group	Male	Age Ratio for Males, AR_x^m	$\lvert AR_x^m - 100 \rvert$	Female	Age Ratio for Females, AR_x^f	$\lvert AR_x^m - 100 \rvert$	Sex Ratio, SR_x	$\lvert SR_x - SR_{x+}$
1	2	3	4	5	6	7	8	9
0–4	1,379,770	–	–	1,389,651	–	–	99.29	1.15
5–9	1,390,652	109.89	9.89	1,384,554	110.73	10.73	100.44	3.16
10–14	1,151,131	97.89	2.11	1,111,085	96.32	3.68	103.60	0.58
15–19	961,162	100.43	0.43	922,591	94.68	5.32	104.18	13.10
20–24	763,051	92.12	7.88	837,769	97.73	2.27	91.08	3.24
25–29	695,494	104.63	4.63	791,805	107.14	7.14	87.84	0.62
30–34	566,439	95.49	4.51	640,370	96.25	3.75	88.45	2.63
35–39	490,864	97.23	2.77	538,901	99.43	0.57	91.09	8.83
40–44	443,284	102.12	2.12	443,647	100.61	0.61	99.92	10.07
45–49	377,315	104.34	4.34	343,042	93.72	6.28	109.99	12.93
50–54	279,950	99.95	0.05	288,419	111.78	11.78	97.06	8.63
55–59	182,843	79.97	20.03	172,999	72.47	27.53	105.69	11.86
60–64	177,347	113.71	13.71	189,004	124.91	24.91	93.83	5.76
65–69	129,090	90.95	9.05	129,619	84.26	15.74	99.59	9.82
70+	106,513	–	–	118,645	–	–	89.77	–
Total	9,357,382	–	81.51	9,554,697	–	120.31	97.9	91.23

TABLE 5.17 Preliminary Calculations Age-Sex Accuracy Index
Source: Author's calculation from 2000 Population and Housing Census, GSS

5.5 MEASURES OF POPULATION DISTRIBUTION

The distribution of population within a country is important for social and economic planning. The basic information needed for the study of population distribution is the census enumeration of population by geographic subdivisions. The measures of population distribution over an area include population density, concentration ratio and concentration index.

5.5.1 POPULATION DENSITY

The density of population is the ratio of population to land and gives the population per square kilometre of the area. The area may be a country, a region, a district or a community.

The population density (PD) is defined as:

$$PD = \frac{\text{Population}}{\text{Area}}$$

Example 5.23

The population of Ghana at the 2000 census was 18,912,079 and the surface area is 238,533 square kilometres. Calculate the population density for Ghana in 2000.

Solution

$$\begin{aligned} \text{Total population} &= 18,912,079 \\ \text{Total area} &= 238,533 \\ \text{Population Density} &= \frac{\text{Population}}{\text{Area}} \\ &= \frac{18,912,079}{238,533} = 79.3 \end{aligned}$$

That is, in 2000 there were approximately 79.3 persons per one square kilometre in Ghana.

Example 5.24

Burundi with a surface area of 27,834 square kilometres had a mid-year population of 5,458,000 in 1990. Calculate the population density.

Solution

$$\text{Total population} = 5,458,000$$

$$\text{Total surface area} = 27,834$$

$$\text{Population Density} = \frac{5,458,000}{27,834} = 196.1$$

For any subdivision of a country the population density is defined as:

$$PD_i = \frac{P_i}{A_i}$$

where

P_i = the number of persons in the subdivision i of the area;
A_i = the area of the same subdivision i

Example 5.25

Table 5.18(a) shows the population and land area of the ten regions in Ghana in 2000. Calculate the population density for all the regions.

Region	Population	Area (sq. km.)
Western	1,924,577	23,921
Central	1,593,823	9,826
Greater Accra	2,905,726	3,245
Volta	2,106,696	20,570
Eastern	1,635,421	19,323
Ashanti	3,612,950	24,389
Brong Ahafo	1,815,408	39,557
Northern	1,820,806	70,384

Region	Population	Area (sq. km.)
Upper West	920,089	18,442
Upper East	576,583	8,876
Total	18,912,079	238,533

TABLE 5.18(a) Population and Land Area of Regions in Ghana, 2000
Source: Obtained from 2000 Population and Housing Census, GSS

Solution

$$PD_{\text{(Western)}} = \frac{1,924,577}{23,921} = 80.5$$

$$PD_{\text{(Central)}} = \frac{1,593,823}{9,826} = 162.2$$

$$PD_{\text{(Greater Accra)}} = \frac{2,905,726}{3,245} = 895.5$$

$$\vdots \qquad \vdots$$

$$PD_{\text{(Upper East)}} = \frac{576,583}{8,876} = 31.2$$

The population density for the other regions are similarly calculated. The results are presented in Table 5.18(b). The table indicates that there are variations in density among the various regions of Ghana. The density is lowest in the Northrn Region and highest in the Greater Accra Region.

Region	Population	Area (sq. km.)	Population Density
Western	1,924,577	23,921	80.2
Central	1,593,823	9,826	162.2
Greater Accra	2,905,726	3,245	895.4
Volta	1,635,421	20,570	79.5
Eastern	2,106,696	19,323	109.0
Ashanti	3,612,950	24,389	148.1
Brong Ahafo	1,815,408	39,557	45.9
Northern	1,820,806	70,384	25.9

Region	Population	Area (sq. km.)	Population Density
Upper East	920,089	8,842	104.1
Upper West	576,583	18,476	31.2
Total	18,912,079	238,533	79.3

TABLE 5.18(b) Calculation of Population Density of Example 14

Population density, defined above as population per square kilometre of land area, is often referred to as crude because it takes into consideration the gross area (land and water). Usually, density is computed as population per square kilometre of land area. However, countries with considerable parts of land area in deserts, mountains, tropical rain forest, icecaps, and so on may still have very low densities even though they may have a small land for agriculture. Hence, for some purposes, more meaningful densities are obtained for a country or region by relating the size of its population to the amount of agricultural (arable) land; that is,

$$PD = \frac{\text{Population}}{\text{Area of Agricultural Land}}$$

Example 5.26

For the data in Example 5.23, it is also known that the agricultural land of Ghana is 136,282 square kilometres. Calculate the population density of Ghana in relation to its agricultural land.

Solution

$$\begin{aligned} \text{Total Population} &= 18,912,072 \\ \text{Total agricultural land} &= 136,282 \\ \\ \text{Population density} &= \frac{18,912,072}{136,282} = 138.8 \end{aligned}$$

That is, there are about 139 persons per one square kilometre of agricultural land of Ghana in 2000.

5.5.2 CONCENTRATION RATIO

The concentration ratio (CR) measures the degree of unevenness of population distribution or the concentration of the population at a point in time. Suppose the areas or group of areas are arranged in order of increasing density.

Let

P_i Cumulative percentage distribution of the population in area i;
A_i Cumulative percentage distribution of area i or group of areas;
n Total number of geographical subdivisions of the population.

Then, the concentration ratio may be defined as:

$$CR = \left(\sum_{i=1}^{n} P_i A_{i+1} - \sum_{i=1}^{n} P_{i+1} A_i \right) \times k$$

where $k = 100$.

The concentration ratio may be interpreted as the proportion of the population that would have to be redistributed in order to attain a completely uniform distribution of persons over the area.

Example 5.27

For the data in Example 5.25, calculate the index of concentration and interpret the result.

Solution

Table 5.19 shows the preliminary calculations of the concentration ratio.

Region	Area (sq. km.) (in order of increasing size)	Population	Proportion of Area (a_i)	Proportion of Population (p_i)	Cumulative Proportion of Total Area (A_i)	Cumulative Proportion of Total Population (P_i)
1	2	3	4	5	6	7
G. Accra	3,245	2,905,726	0.01360	0.15364	0.01360	0.15364
U. East	8,842	920,089	0.03707	0.04865	0.05067	0.20229
Central	9,826	1,593,823	0.04119	0.08428	0.09187	0.28657
U. West	18,476	576,583	0.07746	0.03049	0.16932	0.31706
Eastern	19,323	2,106,696	0.08101	0.11139	0.25033	0.42845
Volta	20,570	1,635,421	0.08624	0.08647	0.33657	0.51493
Western	23,921	1,924,577	0.10028	0.10176	0.43685	0.61669

Region	Area (sq. km.) (in order of increasing size)	Population	Proportion of Area (a_i)	Proportion of Population (p_i)	Cumulative Proportion of Total Area (A_i)	Cumulative Proportion of Total Population (P_i)
Ashanti	24,389	3,612,950	0.10225	0.19104	0.53910	0.80773
B. Ahafo	39,557	1,815,408	0.16583	0.09599	0.70493	0.90372
Northern	70,384	1,820,806	0.29507	0.09628	1.00000	1.00000
Total	238,533	18,912,079	1.00000	1.00000		

TABLE 5.19 Preliminary Calculations of Concentration Ratio of Example 5.18(a)

Region	$(a_i)A_{i+1}$	$\overline{P_{i+1}}$	$\underline{P_iA_{i+1}}$	$\overline{P_{i+1}A_i}$
	8	9	10 = (7) × (8)	11 = (9) × (6)
Greater Accra	0.05067	0.20229	0.00779	0.00275
Upper East	0.09187	0.28657	0.01858	0.01452
Central	0.16932	0.31706	0.04852	0.02913
Upper West	0.25033	0.42845	0.07937	0.07255
Eastern	0.33657	0.51493	0.14420	0.12890
Volta	0.43685	0.61669	0.22495	0.20756
Western	0.53910	0.80773	0.33246	0.35286
Ashanti	0.70493	0.90372	0.56939	0.48719
B. Ahafo	1.00000	1.00000	0.90372	0.70493
Northern				
Total			2.32898	2.00038

TABLE 5.19 (Continued) Preliminary Calculations of Concentration Ratio of Example 5.18(a)

Therefore the concentration ratio is:

$$\begin{aligned} CR &= \left(\sum_{i=1}^{n} P_iA_{i+1} - \sum_{i=1}^{n} P_{i+1}A_i\right) \times k \\ &= (2.32898 - 2.00038) \times 100 = 32.9 \end{aligned}$$

Interpretation

About 33 percent of the population of Ghana in 2000 will have to be redistributed in order to attain a completely uniform distribution.

5.5.3 INDEX OF CONCENTRATION

The index of concentration (IC) is another technique for measuring the degree of population concentration. It is defined as

$$IC = \frac{1}{2} \times \sum_{i=1}^{n} |p_i - a_i| \times k$$

where p_i and a_i are uncumulated proportions of population and area respectively; and n are the number of geographical subdivisions of the population.

The IC is algebraically equivalent to the index of dissimilarity. Interpretation of this index is same as the concentration ratio.

Example 5.28

For the data in Example 5.25, calculate the index of concentration.

Solution

Table 5.20 shows the preliminary calculations of index of concentration.

Region	Population	Area (sq. km.)	Proportion of Total Population (p_i)	Proportion of Total Area (a_i)	$\|p_i - a_i\|$
1	2	3	4	5	6=(4)-(5)
Western	1,924,577	23,921	0.10176	0.10028	0.00148
Central	1,593,823	9,826	0.08428	0.04119	0.04308
Greater Accra	2,905,726	3,245	0.15364	0.01360	0.14004
Volta	1,635,421	20,570	0.08647	0.08624	0.00024
Eastern	2,106,696	19,323	0.11139	0.08101	0.03039
Ashanti	3,612,950	24,389	0.19104	0.10225	0.08879
Brong Ahafo	1,815,408	39,557	0.09599	0.16583	0.06984
Northern	1,820,806	70,384	0.09628	0.29507	0.19879
Upper East	920,089	8,842	0.04865	0.03707	0.01158
Upper West	576,583	18,476	0.03049	0.07746	0.04697
Total	18,912,079	238,533	1.00000	1.00000	0.63121

TABLE 5.20 Preliminary Calculations of Index of Concentration

Therefore the index of concentration is

$$\begin{aligned} IC &= \frac{1}{2} \times \sum_{i=1}^{n} |p_i - a_i| \times k \\ &= \frac{1}{2}(0.63121) \times 100 = 31.6 \end{aligned}$$

Interpretation

About 32 percent of the population of Ghana in 2000 would have to be redistributed in order to attain a completely uniform distribution of the population.

EXERCISES

5.1 The 1985 population census of Burkina Faso showed that there were 3,833,237 males and 4,131,468 females. Calculate the

1) masculinity proportion;
2) masculinity ratio;
3) excess or deficit of males as a percentage of the total population.

5.2 The population in the 1987 population census of Cameroon aged 0–14 years was 1,946,123, those aged 65 years and over was 387,366 and those aged 15–64 years was 8,112,920. For the given data, calculate

1) the child dependency ratio;
2) the old age dependency ratio;
3) the total dependency ratio;

and interpret your results.

5.3 The population of Congo in 1984 aged 15–19 years was 211,294; those aged 20–24 years was 172,226; and those aged 25–29 years was 136,523. Calculate the age ratio of the population in the age group 20–24.

5.4 The following is the 1984 population of Ghana by age. Construct a population pyramid. Comment on the structure of the pyramid.

Age-group	1984 Population
0–4	2,030,082
5–9	2,001,825
10–14	1,503,209
15–19	1,246,390
20–24	1,056,001

Age-group	1984 Population
25–29	945,111
30–34	742,803
35–39	584,299
40–44	473,254
45–49	428,207
50–54	352,684
55– 59	213,081
60– 64	225,776
65– 69	145,309
70–74	128,866
75– 79	71,813
80– 84	70,427
85+	76,944
Total	12,296,081

5.5 The table below shows the 2000 population of Ghana for single ages 20 to 62 years. Calculate for the male and the female populations:

1) age ratio;
2) the age-ratio-type indices for 3-year and 5-year ranges;
3) the Whipple's index;

Comment on your results.

Age	Male	Female	Total
20	225,446	251,021	476,467
21	122,227	125,756	247,983
22	156,551	177,078	333,629
23	127,599	139,536	267,135
24	131,228	144,378	275,606

Age	Male	Female	Total
25	206,931	241,489	448,420
26	128,952	147,332	276,284
27	120,942	128,197	249,139
28	145,334	174,236	319,570
29	93,335	100,551	193,886
30	217,548	272,203	489,751
31	68,281	66,572	134,853
32	125,865	140,290	266,155
33	76,549	78,196	154,745
34	78,196	83,109	161,305
35	172,670	195,119	367,789
36	93,591	102,744	196,335

Age	Male	Female	Total
37	66,821	68,408	135,229
38	98,109	110,049	208,158
39	59,673	62,581	122,254
40	184,795	210,146	394,941
41	44,808	39,632	84,440
42	90,285	87,630	177,915
43	75,845	64,419	140,264
44	47,551	41,820	89,371
45	154,179	152,056	306,235
46	59,789	52,009	111,798
47	45,775	37,321	83,096
48	73,233	66,166	139,399
48	44,339	35,490	79,829
50	120,093	149,542	269,635
51	29,312	23,292	52,604
52	54,823	50,187	105,010
53	37,602	30,239	67,841
54	38,120	35,159	73,279
55	62,975	68,732	131,707
56	40,366	36,813	77,179
57	23,199	18,280	41,479
58	34,227	32,475	66,702
59	22,076	16,699	38,775
60	87,040	112,216	199,256
61	20,821	15,854	36,675

Age	Male	Female	Total
62	27,037	25,864	52,901
63	34,227	32,475	66,702
64	22,076	16,699	38,775
65	87,040	112,216	199,256
66	15,854	20,821	36,675
67	25,864	27,037	52,901
68	17,828	21,646	39,474
69	17,242	20,803	38,045
90	67,660	57,616	125,276

5.6 The table below shows the 2010 female population of Ghana for single ages. Calculate for the female population:

1) age ratio;
2) the age-ratio-type indices for 3-year and 5-year ranges;
3) the Whipple's index;
4) the Myers' index;

Comment on your results.

Age	Female Population	Age	Female Population	Age	Female Population
0	360,881	34	121,623	68	27,710
1	304,769	35	242,672	69	15,544
2	333,189	36	137,396	70	104,315
3	337,341	37	118,476	71	21,985
4	337,439	38	155,762	72	31,367
5	319,387	39	90,329	73	21,699
6	322,287	40	247,444	74	22,452
7	308,865	41	82,158	75	55,084

Age	Female Population	Age	Female Population	Age	Female Population
8	311,013	42	123,415	76	19,588
9	277,768	43	81,285	77	13,028
10	358,714	44	79,428	78	18,882
11	248,500	45	173,824	79	10,222
12	296,652	46	84,669	70	60,194
13	268,959	47	73,989	81	7,722
14	265,690	48	95,606	82	12,366
15	297,835	48	57,035	83	8,217
16	248,652	50	181,754	84	8,228
17	235,558	51	50,688	85	23,974
18	298,693	52	77,302	86	8,156
19	218,139	53	69,019	87	6,258
20	338,590	54	59,735	88	6,350
21	204,737	55	88,630	89	5,395
22	246,677	56	59,849	90	21,995
23	214,499	57	39,907	91	2,359
24	218,261	58	49,950	92	3,946
25	303,014	59	26,777	93	1,941
26	199,990	60	124,564	94	1,836
27	230,097	61	26,449	95	6,517
28	225,134	62	39,951	96	2,998
29	148,663	63	30,458	97	1,719
30	317,248	64	27,377	98	3,159
31	139,455	65	70,742	99	16432
32	178,517	66	22,097		
33	131,665	67	21,534		

5.7 The table below shows the population of Ghana and Mauritius in 2000 by age. Calculate

1) index of relative difference;
2) index of dissimiliarity;

of age distribution of the populations of Ghana and Mauritius and comment on your results.

Age-group (years)	Population of Ghana	Population of Mauritius
0–4	2,769,421	94,303
5–9	2,775,206	105,189
10–14	2,262,216	97,740
15–19	1,883,753	102,088
20–24	1,600,820	110,892
25–29	1,487,299	93,797
30–34	1,206,809	99,515
35–39	1,029,765	101,946
40–44	886,931	90,406
45–49	720,357	77,931
50–54	568,369	56,939
55–59	355,842	40,491
60–64	366,351	33,097
65–69	258,709	25,768
70–74	225,158	21,694
75–79	144,830	14,910
80–84	140,847	7,132
85+	229,396	5,010
Total	18,912,079	1,178,848

5.8 The table below shows the population of Ghana in 2000 and 2010 by regions.

Region	2000 Population	2010 Population
Western	1,924,577	2,376,021
Central	1,593,823	2,201,863
Greater Accra	2,905,726	4,010,054
Eastern	1,635,421	2,118,252
Volta	2,106,696	2,633,154
Ashanti	3,612,950	4,780,380
Brong Ahafo	1,815,408	2,310,983
Northern	1,820,806	2,479,461
Upper West	920,089	1,046,545
Upper East	576,583	702,110
Total	18,912,079	24,658,823

Calculate

1) index of relative difference;
2) index of dissimiliarity;

of population distribution of Ghana in 2000 and 2010 and comment on your results.

5.9 The table below shows the population and area of Ghana in 1984 by regions.

Region	Population	Area (sq. km.)
Western	1,157,807	23,921
Central	1,142,335	9,826
Greater Accra	1,431,099	3,245
Volta	1,211,907	20,570
Eastern	1,680,890	19,323
Ashanti	2,090,100	24,389
Brong Ahafo	1,206,608	39,557
Northern	1,164,583	70,384
Upper East	772,744	8,842
Upper West	438,008	18,476
Total	12,296,081	238,533

Calculate

1) population densities for all the regions;
2) concentration ratio;
3) index of concentration;

of Ghana in 1984 and comment on your results.

5.10 Refer to the table in Exercise 5.3. Calculate

1) the child dependency ratio;
2) the old age dependency ratio;
3) the total dependency ratio;

of Ghana in 1984 and interpret your results.

5.11 Refer to the table in Exercise 5.3. Calculate

1) index of ageing;
2) average age of population;
3) age-sex accuracy index.

and comment on the result in each case.

5.12 The table below shows the population of Ghana in 1960 by age and sex.

1) Construct the pyramid;
2) Calculate age-specific sex ratios;
3) Calculate age-sex accuracy index.

Age-group	Total	Male	Female
0–4	1,296,625	642,367	654,258
5–9	1,018,590	515,520	503,070
10 -14	681,291	357,831	323,460
15–19	541,076	275,542	265,534
20–24	590,912	268,336	322,576
25–29	584,930	278,601	306,329
30–34	488,398	242,515	245,883
35–39	377,413	198,231	179,182
40–44	311,509	165,937	145,572
45–49	218,346	122,756	95,590

Age-group	Total	Male	Female
50–54	178,490	96,775	81,715
55–59	107,719	59,307	48,412
60–64	118,039	63,467	54,572
65–69	60,958	32,377	28,581
70–74	56,529	29,796	26,733
75–79	30,961	16,183	14,778
80–84	28,809	15,022	13,787
85+	36,220	19,707	16,513
Total	6,726,815	3,400,270	3,326,545

6 MEASURES OF MORTALITY

6.1 INTRODUCTION

Mortality is the negative component in the balance of the vital process and it is the first of the demographic processes we shall consider.

Mortality may be described as that branch of demography which deals with the total process of deaths and the changes it brings about in the population. It is a continuous force of attrition, tending to reduce population having its effect countervailed by the force of fertility.

The analytical study of mortality in demography can simply be described as the study of the "risk of dying" as it varies from one population to another and within a population from one sub-group to another. Compared to other demographic phenomena, its study is relatively simple: all persons die and, therefore, the intensity of the process is always equal to unity in every cohort. With regard to the tempo of events, humanity has tried, in general, to raise the expectation of life.

6.1.1 DEFINITION OF CONCEPTS IN MORTALITY MEASUREMENT

Death

The meaning of ***death*** may seem to be obvious, at least in a physical sense, to most people. However, recent developments in the medical field have gone a long way to blur even the clinical distinction between life and death. It is, therefore, important to formalise the definition of death at least for statistical purposes.

Death is the permanent disappearance of all evidence of life at any time after birth has taken place (that is, post-natal cessation of vital functions without capability of resuscitation). This definition of "death" excludes foetal deaths (i.e. deaths prior to live birth).

Foetal death

Foetus is a product of conception which takes about nine months or 280 days to develop in the womb before live birth can take place.

Foetal death is defined as a product of conception that shows no sign of life after complete birth. Death is indicated by the fact that after such separation the foetus does not breathe or show any other evidence of life, such as beating of the heart, pulsation of the umbilical cord, or definite movement of voluntary muscles. Foetal deaths are excluded from both "live births" and deaths.

In present demographic practice the term foetal death is employed to embrace the events variously called stillbirths, miscarriages, and abortions in popular, medical, or legal usage.

Stillbirth

Although the term stillbirth is sometimes employed to refer to all foetal deaths, it is often employed to refer to foetal death of at least 28 completed weeks of gestation. A period of gestation (gestation age) is defined as the number of completed weeks which have elapsed between the first day of the last menstrual period and the date of delivery of the foetus.

Abortion

Technically, abortion is defined, with reference to the woman, as any interruption of pregnancy before 28 weeks of gestation with a dead foetus. There are two major categories of abortion – spontaneous and induced.

Spontaneous abortion is popularly employed to refer to spontaneous or accidental terminations of foetal life occurring early in pregnancy. This is popularly referred to as ***miscarriage***.

Induced abortion involves deliberate action undertaken with the intention of terminating pregnancy. The term abortion is popularly used to refer to this category, whether it was done legally or illegally.

6.1.2 USES OF STUDY OF MORTALITY

The level and pattern of mortality is a reflection of the health status of a population and is a fundamental indicator of socio-economic development. The study of mortality, therefore, is important to a wide range of people and professionals.

1) ***Demographers***

   It is an instrument for demographers

   a) to measure the growth potential of a population;
   b) to look into the probable changes in population composition;
   c) in the study of other demographic variables;
   d) in the construction of population projections.

   The concepts and methods developed for mortality analysis are needed to build the systems for the analysis of marriage, fertility and migration.

2) ***Medical Professionals***

   a) It helps the medical professionals to understand the prevailing patterns of the various causes of deaths with varying dominance
   b) It guides the Health Ministry in the formulation of policies, implementation and evaluation of public health and disease control programmes and projects.

3) ***Undertakers***

   It is of use to undertakers and to those concerned with estimating the cemetery space needed in the country.

4) ***Actuaries***

   Mortality data are used by actuarial scientists to calculate probable life span of population which insurance companies require to set premiums.

5) ***Researchers***

   It is of vital interest to researchers involved in public programmes.

6) ***Policy Makers***

   It is important to the government and other agencies who are concerned with taking policy-decisions in relation to the levels and variation of mortality.

6.2 CRUDE DEATH RATE

The simplest, commonest and crudest way of obtaining death rates is to express deaths per unit of the population. This gives a measure of totality, describing the frequency with which the deaths occur in a population in a specified period.

6.2.1 DEFINITION OF CRUDE DEATH RATE

The crude death rate is defined as the number of deaths from all causes in one calendar year per 1,000 of the mid-year population; that is:

$$d = \frac{D}{P} \times k$$

where

$$\begin{aligned} D &= \text{total number of deaths occuring during a calendar year (January 1 to December 31);} \\ P &= \text{mid-year total population (that is total population as of} \\ &= \text{July 1);} \\ k &= \text{the radix, usually chosen as } 1,000. \end{aligned}$$

The crude death rate for any period gives the rate at which the population is depleted through deaths over the course of the period. Crude death rate usually lies between 8 and 30 per thousand and female crude death rate is generally less than male crude death rate.

Example 6.1

The mid-year population of the Republic of Mauritius in 2004 was 1,233,386. The number of deaths in that year was 8,475. Calculate the crude death rate of the country.

Solution

$P = 1,233,386;\; D = 8,475$

$$\begin{aligned} d &= \frac{D}{P} \times 1,000 \\ &= \frac{8,475}{1,233,386} \times 1,000 = 6.9 \end{aligned}$$

That is, 6.9 out of every 1,000 people died in the Republic of Mauritius in 2004.

Alternatively, we may say that "the death rate of Mauritius in 2004 was about 6.9 per 1,000 people", or 69 per 10,000 people.

6.2.2 AVERAGE CRUDE DEATH RATES

We may sometimes be interested in computing the mean of crude death rates which, covering data for two or three years. This is done in order to:

1) represent the longer period with a single figure,
2) add stability to rates based on small numbers,
3) use it in extensive comparative analysis.

These annual average rates may be computed in several different ways. For lack of appropriate terminology, we have adopted expressions "mean annual rate", "annual average rate", and "mid-period rate" to describe the methods for computation.

Method 1: Mean Annual Crude Death Rate

Crude death rates are computed for each year and averaged, i.e.:

$$\begin{aligned}\bar{d} &= \frac{1}{n}\left(\frac{D_1}{P_1}\times 1,000 + \frac{D_2}{P_2}\times 1,000 + \cdots + \frac{D_n}{P_n}\times 1,000\right)\\ &= \frac{1}{n}\left(\frac{D_1}{P_1} + \frac{D_2}{P_2} + \cdots + \frac{D_n}{P_n}\right)\times 1,000\\ &= \frac{1}{n}\sum_{t=1}^{n}\frac{D_t}{P_t}\times 1,000\\ &= \frac{1}{n}\sum_{t=1}^{n} d_t \times 1,000\end{aligned}$$

where

$\bar{d}$	=	mean annual crude death rate covering data for *n* years;
n	=	number of years;
$D_1, D_2, \cdots, D_n$	=	total number of deaths of year 1, year 2, · · ·, year *n*, respectively;
$P_1, P_2, \cdots, P_n$	=	mid-year population of year 1, year 2, · · ·, year *n*, respectively;
d_t	=	crude death rate of year *t* (*t* = 1, 2, · · · , *n*).

This procedure takes into consideration the deaths in relation to the population in each year. However, in computing the composite rate (mean annual crude death rate), equal weight is given to the annual crude death rates for the n years. Consequently, the mean annual crude death rate is an "unweighted" average.

This procedure may be viewed as more exact than the subsequent procedures for measuring mortality in the period.

Example 6.2

The mid-year population of a country was 16 million in 1985, 18 million in 1986 and 19.5 million in 1987. In these years the number of people who died were 250,000; 245,000 and 240,000, respectively. Find the mean annual crude death rate for the entire period (1985–1987).

Solution

$$
\begin{aligned}
{}_{1985} &= \frac{D_{1985}}{P_{1985}} \times k \\
&= \frac{250,000}{16,000,000} \times 1000 = 15.6
\end{aligned}
$$

That is, the death rate of the country in 1985 was about 16 per 1,000 people.

$$
\begin{aligned}
d_{1986} &= \frac{D_{1986}}{P_{1986}} \times k \\
&= \frac{245,000}{18,000,000} \times 1000 = 13.6
\end{aligned}
$$

That is the death rate of the country in 1986 was about 14 per 1,000 people.

$$
d_{1987} = \frac{D_{1987}}{P_{1986}} \times k
$$

$$
= \frac{240,000}{19,500,000} \times 1,000 = 12.3
$$

That is, the death rate of the country in 1987 was about 12 per 1,000 people.

Therefore, the mean annual death rate of the country for the period 1985–1987 is:

$$
\begin{aligned}
\bar{d} &= \frac{1}{n}\sum_{t=1}^{n} d_i \\
&= \frac{1}{3}(15.6 + 13.6 + 12.3) = 13.83
\end{aligned}
$$

That is, on the average, about 13.8 per 1,000 people died each year in the country in the three-year period (1985–1987).

Method 2: Annual Mean Crude Death Rate

The mean number of deaths is divided by the mean population:

$$\begin{aligned}
\bar{d} &= \frac{\frac{1}{n}(D_1 + D_2 + \cdots + D_n)}{\frac{1}{n}(P_1 + P_2 + \cdots + P_n)} \times 1{,}000 \\
&= \frac{D_1 + D_2 + \cdots + D_n}{P_1 + P_2 + \cdots + P_n} \times 1{,}000 \\
&= \frac{\sum_{t=1}^{n} D_t}{\sum_{t=1}^{n} P_t} \times 1{,}000
\end{aligned}$$

Example 6.3

Refer to Example 6.2. Calculate the annual mean death rate for the period 1985–1987.

Solution

$$
\begin{aligned}
&= \frac{250,000+245,000+240,000}{16,000,000+18,000,000+19,500,000} \times 1,000 \\
&= \frac{735000}{53500,000} \times 1,000 = 13.73
\end{aligned}
$$

Method 3: Mid-period Crude Death Rate

Often, reliable figures for the total population are not available for each individual year and so it becomes customary to use the mean number of deaths for the *n* years and divide the results by the mid-period population:

$$\bar{d} = \frac{\frac{1}{n}\left(D_1 + D_2 + \cdots + D_n\right)}{P_{n/2}} \times 1000$$

where $P_{n/2} =$ the mid-period population (or usually a census figure).

The mid-period crude death rate is a "weighted" average because a large (or small) number of deaths in one year will have a large (or small) influence on the value of the rate. It is the most commonly used and most convenient procedure even though it is somewhat less precise.

Example 6.4

Compute the mid-period crude death rate for the data in Example 6.2.

Solution

$$
\begin{aligned}
\bar{d} &= \frac{\frac{1}{n}\left(D_1 + D_2 + D_3\right)}{P_{n/2}} \times k \\
&= \frac{\frac{1}{3}\left(250,000+245,000+240,000\right)}{18,000,000} \times 1,000 = 13.6
\end{aligned}
$$

These three procedures may be expected to give approximately the same results unless there are sharp fluctuations in the size of the population.

6.2.3 ADVANTAGES AND DISADVANTAGES OF CRUDE DEATH RATE

Advantages of Crude Death Rate

1) It is easy and quick to compute, requiring the barest minimum of data for a vital rate: it requires only the total deaths and total mid-year population in a given period of time.
2) It is an average and so a suitable measure that indicates the general level of mortality and its changes in an entire population.
3) It is easy to understand and its meaning can be communicated to the general public.

Disadvantages of Crude Death Rate

1) It is affected by many factors, such as, age, sex and other characteristics. These characteristics, especially, age and sex, are fundamental to the study of mortality because the risk of death is very different at different ages and for different sexes. It is for this reason that this measure is called "crude".
2) The crude death rates of an area for a period of time are not comparable unless the area's population characteristics, such as, age and sex remain unchanged or change slowly.
3) The crude death rate is not a reliable index for comparing mortality of two areas in the same year unless, besides the health conditions, the areas are comparable with respect to other demographic characteristics, such as, age and sex.
4) It does not measure only the rates of occurrence of events, but mixes them up together with changes in population size. For example, birth refers to the addition of one person to the total population, while a death is the removal of one person from the population.
5) The numerator is taken from registration of vital events as they occur throughout the year, but the denominator represents the total number of people with reference to a fixed date in the year, usually, the mid-year. Hence the crude death rate does not measure the risk factor in the probability sense.

6.3 SPECIFIC DEATH RATE

6.3.1 DEFINITION OF SPECIFIC DEATH RATE

The crude death rate gives only a very general indication of the level of mortality and its changes. In this wise, it is usually referred to as general crude death rate. There is also the need for measures that describe the death rate for any population group in a community such as the female population, or the rural population over a period. Such measures are referred to as specific death rates. They are of interest both in themselves and for their value in the analysis of the total number of deaths and the crude rate.

Specific death rate is defined as

$$d_j = \frac{D_j}{P_j} \times k$$

where

d_j = specific death rate for the j^{th} segment of the population;
D_j = total number of deaths in the j^{th} specified segment of the population in the given year;
P_j = population of the j^{th} specified segment as at July 1;
k = $1,000$;
j = 1, 2, ..., S (S is number of segments of the population).

Note

The formula calls for dividing the deaths in the j^{th} sub-group during a year by the mid-year population of the j^{th} sub-group. The sub-groups for which specific death rates may be computed include subdivisions of the population according to sex, age, occupation, educational level, ethnic group, cause of death, etc.

Example 6.5

The (census) population of the Republic of Mauritius in 2004 was 1,233,386, with 610,108 males and 623,278 females. In that year, 4,716 deaths were recorded in the male population while 3,759 deaths were recorded in the female population. Calculate sex-specific death rate.

Solution

Population:	Male	(P_m)	=	$610,108$
	Female	(P_f)	=	$623,278$
Deaths:	Male	(D_m)	=	$4,716$
	Female	(D_f)	=	$3,759$

Sex-specific death rate $(d_j) = \frac{D_j}{P_j} \times k$

$$\text{For male:} \quad (d_m) = \frac{D_m}{P_m} \times 1,000$$

$$= \frac{4,716}{610,108} \times 1,000 = 7.7$$

$$\begin{aligned} \text{For female:} \quad (d_f) &= \frac{D_f}{P_m} \times 1,000 \\ &= \frac{3,759}{623,278} \times 1,000 = 6.0 \end{aligned}$$

That is, for every 1,000 males who lived in Mauritius in 2004, 7.7 died during the year; for every 1,000 females, 6 died during the year.

Specific rates may be computed for two or more characteristics simultaneously. For example, we may compute the death rate for Hausa females, thus obtaining an ethnicity-sex specific rate. As pointed out earlier, there is no clear dividing line between a crude death rate and a specific death rate except in the case of age, which always defines rate as specific. For example, death rate for the rural population may be considered crude if we are interested in that population only. If interest is its relationship with the whole country's population, the death rate becomes specific.

Example 6.6

The census population of Malawi in 1977 was 5,547,460 out of which 5,076,802 lived in the rural areas and 470,658 in the urban areas. In that year, 133,705 deaths were recorded in the rural areas and 4,989 deaths in the urban areas. Calculate

1) the death rate of the rural community;
2) the death rate of the rural community as part of the whole country.

Solution

1) Consider the rural community as an entity in itself.

$$\begin{aligned} P &= 5,076,802 \\ D &= 133,705 \\ d &= \frac{D}{P} \times 1,000 \\ &= \frac{133,705}{5,076,802} \times 1,000 = 26.3 \end{aligned}$$

This is the crude death rate for the rural community of Malawi in 1977.

Consider the rural community as a component of the country. The resulting death rate becomes specific.

$$
\begin{aligned}
P_r &= 5,076,802 \\
D_r &= 133,705 \\
d_r &= \frac{D_r}{P_r} \times 1,000 \\
&= \frac{133,705}{5,076.802} \times 1,000 = 26.3.
\end{aligned}
$$

That is, on the average, for every 1,000 people living in rural areas in Malawi in 1977, 26.3 died during the year.

Note

1) The results in cases (1) and (2) above are the same though they have different names.
2) As a consequence from (1) unless specifically indicated otherwise, by specific death rate we mean the age-specific death rate.

6.3.2 AGE-SPECIFIC DEATH RATES

The crude death rate helps us to arrive at the conclusion that mortality exerts a much more greater force on some populations than on others and that its force may change quickly over time. To explore these ideas in more detail and to understand more clearly the socio-demographic implications of mortality, we must recognise that the relative frequency of deaths varies with age. It is high in the youngest and the oldest members of the population.

The amount of loss to a society, in terms of potential population growth and of financial loss is determined to a greater extent by the age at which death takes place. Thus, we must begin the intensive study of mortality by learning its age pattern, which begins by computing death rates separately for each age or age group. These are called age-specific death rates (ASDRs).

The age-specific death rate is the number of deaths per k persons of a specific age (group):

$$d_a = \frac{D_a}{P_a} \times k$$

where

d_a = specific death rate for age-group *a*;
D_a = total number of deaths in age-group *a*;
P_a = mid-year population in age-group *a*;
k = $1,000$
a = 1, 2, ..., n (where n is the number of single ages or age groups).

Note

1) The age-specific death rate is simply a crude death rate computed for each age group individually
2) Age-specific death rates may be computed for single years of age, or for any age interval. Death rates for five-year age groups are the ones most frequently computed.
3) Death rates specific for age are generally calculated separately for the two sexes, so as to exhibit the variation by sex.

Example 6.7

Table 6.1 (a) shows the mid-year population and the number of deaths in Mali in 1976 for each age group. Calculate the age-specific death rates.

Age-groups (year)	Mid-year Population (thousand)	Number of deaths (thousand)
0–19	345.8	8.4
20–39	169.0	8.8
40–59	845.8	10.4
60 or more	399.6	13.2

TABLE 6.1(a) Mid-year Population and Number of Deaths in Mali in 1976

Solution

$$d_{0-19} = \frac{D_{0-19}}{P_{0.19}} \times k$$
$$= \frac{8.4}{345.8} \times 1,000 = 24.3$$

$$d_{20-39} = \frac{D_{20-39}}{P_{20-39}} \times k$$
$$= \frac{8.8}{169.0} \times 1,000 \quad = 52.1$$

$$d_{40-59} = \frac{D_{40-59}}{P_{40-56}} \times k$$
$$= \frac{10.4}{845.8} \times 1,000 = 12.3$$

$$d_{60+} = \frac{D_{60+}}{P_{60+}} \times k$$
$$= \frac{13.2}{399.6} \times 1,000 = 33.0$$

The results are presented in Table 6.1(b).

Age-groups (year)	Mid-year Population (thousand)	Number of deaths	Age-specific death rate per 1,000 population
0–19	345.8	8.4	24.3
20–39	169.0	8.8	52.1
40–59	845.8	10.4	12.3
60 or more	399.6	13.2	33.0

TABLE 6.1(b) Age-Specific Death Rates of Example 6.7

6.3.3 AGE CURVE OF MORTALITY

The incidence of mortality by age follows a similar pattern (curve) in every human population. It shows "bath tub" -shaped curve, that is, a ∪-shape. The two sides of the ∪ represent high mortality of infancy and old-age with the base of ∪ shape widening as the reduced mortality becomes a characteristic of increased span of life. Thus, the general pattern of the age curve of mortality is as follows: mortality is high in the first year of life, it declines rapidly in the second and third years until by age 9, when it is very low. It remains low until the mid-thirties, when it gradually begins to increase at an accelerating rate. Thereafter, it rises at an ever increasing pace until by age 60 when it becomes high and by age 70 extremely high.

In countries where mortality in the past has reduced, the observation is that the risk of mortality reduces first (or to a larger extent) at the infancy and child-ages where it was extremely high and produces a nearly *J*-shaped curve.

The age-specific death rates for males and females differ from developing and developed countries. In the developed countries, where mortality is usually low, ASDRs for females are generally found to be lower at all ages than for males. On the other hand, in developing countries, where mortality is usually high, there are various typical differences in ASDRs for the two sexes. One such typical pattern is that of higher female mortality than of male's at infant and reproductive ages and vice versa at other ages.

Example 6.8

Table 6.2 shows the age-specific death rates of the Republic of Mauritius for 2004 by sex. Construct the age mortality curves and give comments.

Age-groups (year)	Male Population	Female Population	Total Population
0	16.2	12.9	14.6
1–4	0.5	0.5	0.5
5–9	0.2	0.3	0.3
10–14	0.3	0.1	0.2
15–19	0.8	0.3	0.6
20–24	0.9	0.5	0.7
25–29	1.4	0.5	0.9
30–34	2.1	0.9	1.5
35–39	3.6	1.2	2.4
40–44	5.1	1.8	3.5
45–49	8.2	3.6	5.9
50–54	10.8	5.7	8.2
55–59	15.3	8.5	11.7
60–64	26.1	15.9	20.7
65–69	37.7	24.0	30.1
70–74	54.0	34.8	43.1
75–79	85.1	52.2	65.5
80–84	120.6	81.6	96.0
85+	215.2	174.5	186.2

TABLE 6.2 Age-Specific Death Rates of Republic of Mauritius, 2004 (per 1,000 population)
Source: Central Statistical Office, *Annual Digest of Statistics of the Republic of Mauritius (excluding Agalega and St. Brandon),* Port Louis, 2004. Author's calculation

Solution

Figure 6.1 shows the age mortality curves for Mauritius in 2004. As expected, they follow the usual "bath tub" shape.

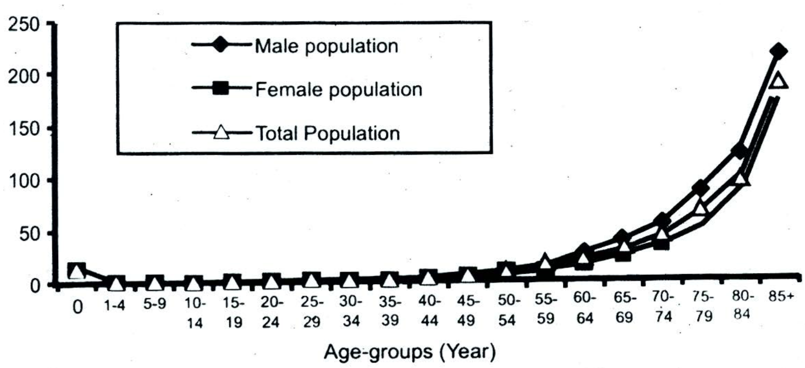

Figure 6.1 Age Mortality Curve for Mauritius, 2004

6.3.4 ADVANTAGES AND DISADVANTAGES OF SPECIFIC DEATH RATES

Advantages of Age-specific Death Rates

1) Specific death rates overcome the drawback of the crude death rates, since, to a large extent, they are computed by taking into consideration the composition of the population.
2) Specific death rates are essential because it is only through the analysis of specific rates that an accurate and detailed study can be made of the variation (of the phenomenon under study) among population classes.
3) For general analytical purposes the death rate specific for age and sex is one of the most important and widely applicable types of death rates.
4) The death rate specific for age and sex supplies one of the essential components required for the computation of net reproduction (see Chapter 9).

Disadvantages of Age-specific Death Rates

1) Specific death rates are not of much utility for overall comparison of mortality conditions prevailing in two different communities, say A and B. For certain age-groups the mortality rate for say, community A may be greater than that for community B and vice-versa for other age-groups.
2) When specific death rates are calculated for one demographic factor, say age or sex, other demographic factors (social, occupational and topographical factors) are completely ignored resulting in what is called differential mortality.
3) To examine the trend of mortality using specific rates over a series of years, one would have to look at a very large number of figures and it might become difficult to see any of the broad general trends that are in operation.

6.4 CAUSE-SPECIFIC DEATH RATIOS AND RATES

Mortality by cause of death is a matter of concern in the control of diseases. It may be analysed in terms of two observed measures, namely, cause-specific death rate and cause-specific death ratio. In both cases, they are computed by including in the numerator only those deaths due to a particular cause of death, say, hypertension, cancer, diabetes, heart disease, malaria, or accidents.

6.4.1 USES OF CAUSE-SPECIFIC DEATH RATIO AND RATE

The study of mortality by cause provides researchers, planners and the policy maker a detailed picture of the general state of health and mortality situation prevailing. Cause-specific death ratio and rate are used to:

1) Compare the causes of deaths in different areas in the same yrar, or in different years for the same area;
2) Prepare multiple decrement life tables (refer to Chapter 7) by causes of death and to describe how much sensitive the expectation of life at a specified age is for the changes in mortality due to one or more causes.

6.4.2 CAUSE-SPECIFIC DEATH RATIO

Cause-specific death ratio (C) represents the percentage of all deaths due to a particular cause or group of causes in a given year. This is defined as:

$$C = \frac{D_c}{D} \times 100$$

wheres
$D_c =$ deaths from a cause or group of causes c;
$D = \sum D_c =$ total number of deaths from all causes;
Cause-specific death ratio (C) gives the relative risk due to any particular cause of death.

Example 6.9

The mid-year population of Egypt in 1987 was 49,089,000 and in that year there were 466,161 deaths out of which 3,319 died of hyperplasia of prostate. Calculate the cause-specific death ratio.

Solution

$$\begin{aligned} D &= 466,161 \\ D_c &= 3,319 \end{aligned}$$

$$\begin{aligned} C &= \frac{D_c}{D} \times 100 \\ &= \frac{3,319}{466,161} \times 100 = 0.7 \end{aligned}$$

That is, there are 0.7 deaths of hyperplasia prostate per 100 deaths or 7 deaths of hyperplasia prostate per 1,000 deaths in Egypt in 1987.

Advantage of Cause-specific Death Ratio The cause-specific death ratio is used to measure the relative importance of a given cause of death in an area and over time.

6.4.3 CAUSE-SPECIFIC DEATH RATE

Cause-specific death rate (CSDR) is defined as the number of deaths from a given cause or group of causes during a year per 100,000 of the mid-year population:

$$CSDR = \frac{D_c}{P} \times k$$

where $k = 100,000$.

This rate employs a larger k than a crude death rate or an age-specific death rate because there are relatively few deaths from many of the causes.

Note

The cause-specific death rates in a set covering all causes sum up to the crude death rate.

Example 6.10

For data in Example 6.9, calculate

1) the crude death rate;
2) the cause-specific death rate.

Solution

Mid-year population (P)	= 49, 089, 000
Number of deaths (D)	= 466, 161
Population who died of hyperplasia of prostate (D_c)	= 3319

1) The crude death rate is:

$$\begin{aligned} d &= \frac{D}{P} \times 1,000 \\ &= \frac{466,161}{49,089,000} \times 1,000 = 9.5 \end{aligned}$$

2) The cause-specific death rate is:

$$\begin{aligned} CSDR &= \frac{D_c}{P} \times k \\ &= \frac{3319}{49,089,000} \times 100,000 = 6.8 \end{aligned}$$

That is, there were about 6.8 deaths of hyperplasia of prostate per 100,000 population in Egypt in 1987.

Advantage of Cause-specific Death Rate

The cause-specific death rate is used to compare the relative frequency of particular cause of death of two areas "adjusted for" the size of the population in each area. We can say, for example, that the death rate from heart disease in area A (50 per 1,000 population) is twice the rate in area B (25 per 1,000 population).

Disadvantages of Cause-specific Death Rate

1) Unlike other types of death rates, (for example, the age-specific death rate, where the deaths at a given age are related to the population at that age), in this rate the base population logically is not limited to the population at risk of death from the given cause.
2) Certain causes of death have relevance only to a very restricted part of the age distribution or to only a particular sex. For example, deaths due to measles and whooping cough are largely confined to childhood; deaths due to diabetes, nephritis, etc. affect largely the adult population, deaths due to complications of pregnancy are confined to women.

6.4.4 AGE AND CAUSE-SPECIFIC DEATH RATE

As we have observed, the general cause-specific death rates have no significance in disease, which are peculiar to age or sex. For such diseases, it is important to calculate cause-specific rates specific for age and/or sex.
An age and cause-specific death rate (ACSDR) is defined as:

$$ACSDR = \frac{D_{ca}}{P} \times 1,000$$

where $D_{ca}=$ deaths at age a due to cause c.

Similarly, sex and cause-specific death rates may be defined.

6.4.5 CASE-FATALITY RATE

Case-fatality rate (CFR) measures the risk that a person having contracted a particular disease, will die from it. It is defined as:

$$CFR = \frac{Dd}{Cd} \times 100$$

where Dd = number of deaths due to a particular disease;

Cd = number of cases of that disease.

Example 6.11

At the University of Ghana hospital, 955 malaria cases were reported in 1999. Out of this number 54 persons died from this disease. Calculate the case-fatality rate for malaria.

Solution

Cd = 955; Dd = 54

$$CFR = \frac{54}{955} \times 100 = 5.7$$

That is, there were 5.7 deaths per 100 malaria cases at University of Ghana in 1999.

6.4.6 PROPORTIONAL MORTALITY RATIO

The proportional mortality ratio (PMR) has been suggested as a single measure for comparing the overall health conditions of different communities over a period. It is defined as:

$$PMR = \frac{D_{50}}{D} \times 100$$

where D_{50} = number of deaths of persons 50 years of age and over.

Example 6.12

There were 230,522 deaths in Algeria in 1982 out of which 56,979 were people of 50 years and above. Calculate the proportional mortality ratio.

Solution

$$D = 230,522; \quad D_{50} = 56,979$$

$$PMR = \frac{56,979}{230,522} \times 100 = 24.7$$

That is, in Algeria in 1982, 24.7 out of 100 deaths occurred among persons 50 years or more.

6.5 MORTALITY RATES

Mortality rates are probabilities of death which are specific by age and relate to a one-year period. The most commonly computed probabilities of death are specified by the following.

1) *Age relating to a one-year period*
   An age-specific probability of dying might answer the question, for example, what is the chance that a new-born child will die before it reaches its first birthday?
2) *Age-sex relating to child-bearing age of females*
   An age-sex specific probability of dying might answer the question, for example, what is the chance that a pregnant woman will die of complications of pregnancy, childbirth, and puerperium?

6.5.1 MATERNAL MORTALITY

Maternal Death

Maternal death is defined as the death of a woman while pregnant or within 42 days of the termination of pregnancy, irrespective of the duration and the size of the pregnancy, from any cause related to or aggravated by the pregnancy or its management but not from accident or incidental causes. All maternal deaths should be subdivided into two groups:

1) *Direct Obstetric Deaths*
Direct obstetric deaths are deaths resulting from obstetric complications of the pregnant state (pregnancy, labour, and puerperium) from interventions, omissions, incorrect treatment, or from a chain of events relating to any of the above.

2) *Indirect Obstetric deaths*
Indirect obstetric Deaths are deaths resulting from previous existing diseases or diseases not developed during pregnancy and that was not due to direct obstetric causes but that was aggravated by physiologic effects of pregnancy.

Maternal Mortality Rate

Maternal mortality rate (MMR), also referred to as ***mortality of reproductive ages*** is a widely used type of cause-specific mortality rate representing approximately the risk of dying as a result of "deliveries and complications of pregnancy, childbirth, and the puerperium". This rate is conventionally defined as the number of deaths due to puerperal causes per 100,000 live births over a given period. Symbolically,

$$MMR = \frac{D_p}{B} \times k$$

where

D_p = deaths from all puerperal causes during a year;
B = total live births during the year;
k = $100,000$

Note

The preferred denominator for this rate is the number of women who were pregnant during the year. This denominator, however, is impossible to determine.

Example 6.13

There were 16,000 live births in a country in 2005. During childbirth, 300 pregnant women died; before giving birth, 200 of them died and immediately after giving birth 100 of them died. Calculate the maternal mortality rate.

Solution

$$\begin{aligned}
\text{Number of live births } B &= 16,000 \\
\text{Deaths from all puerperal causes } D_p &= 300 + 200 + 100 = 600 \\
\text{Maternal mortality rate} &= \frac{D_p}{B} \times 100,000 \\
&= \frac{600}{16,000} \times 100,000 = 3,750
\end{aligned}$$

That is, the maternal mortality rate in the country in 2005 is 3,750 per 100,000 live births.

Example 6.14

The maternal deaths of Mauritius in 1987 was 19. In that year, there were 19,152 live births. Calculate the maternal mortality rate.

Solution

$$\begin{aligned}
\text{Maternal deaths} &= 19 \\
\text{Live births} &= 19,152 \\
\text{Maternal mortality rate} &= \frac{19}{19,152} \times 100,000 = 99
\end{aligned}$$

That is, for every 100,000 women who had live births in 1987 in Mauritius, 99 of them died from puerperal causes.

Limitations of Maternal Mortality Rate

1) Foetal deaths (see Section 6.6) are not included in the denominator. This results in an inflated rate, since a mother can die from a puerperal cause without producing a live birth.
2) Under-registration of live births, which results in too small a denominator, causes the rate to be too large.
3) Maternal death may occur in a year later than the year in which the birth occurred. [26]

6.5.2 UNDER FIVE MORTALITY

If a child dies the day it is born or during the early period of life, its death has far greater demographic implications than does the death of an elderly person. In demographic and health analysis, deaths occurring during the first five years of life is of great concern. This is usually referred to as ***under five mortality*** and its rate measures the risk of dying during the first five years of life.

The under five mortality is usually decomposed into ***infant mortality*** and ***child mortality*** (see Figure 6.2 in Section 6.6.4). Infant mortality consists of deaths occurring during the first year of life while child mortality refers to death before age 5 years after surviving the first year of life. In the subsequent section, we shall discuss infant mortality because of its interest in mortality analysis.

6.6 INFANT MORTALITY RATES

6.6.1 USES OF INFANT MORTALITY

Estimates of infant mortality are useful for health planning, and fertility analysis, as well as in the forecasting of social needs for amenities for the future. In particular, infant mortality has the following uses.

1) It is one of the most sensitive indicators of the medical and health facilities in a population;
2) There is some sort of statistical relationship between the infant mortality and birth rates. Consequently, infant mortality becomes one of the important parameters to understand the mechanism of fertility change in a population;
3) Any mortality in general affects the infant mortality and it is through this that it influences the age distribution.

Infant mortality "rate" measures the risk of dying during the first year of life. i.e. from birth to exact age 1.

We shall distinguish between conventional infant mortality rates and adjusted infant mortality rates.

6.6.2 CONVENTIONAL INFANT MORTALITY RATE

Conventional Infant Mortality Rate (CIMR) is defined as the number of infant deaths per 1,000 live births during the year[27]:

$$CIMR = \frac{D_0}{B} \times k$$

where

$CIMR$ = Conventional Infant Mortality Rate;
D_0 = number of deaths of children under 1 year of age during a year;
B = total number of live births during the year;
k = base taken generally as 1,000.

Example 6.15

There were 1,902,604 live births in Egypt in 1987 out of which 25,284 children died before the age of 4 weeks and 94,044 died before their first birthday. In addition, there were 12,020 late foetal deaths in that year. Calculate the $CIMR$.

Solution

$$\begin{aligned} \text{Number of live births} \quad & B && = 1,902,604 \\ \text{Deaths before first birthday} \quad & D_0 && = 94,044 \\ & CIMR && = \frac{D_0}{B} \times 1,000 \\ & && = \frac{94,044}{1,902,604} \times 1,000 = 49.4 \end{aligned}$$

That is, 49.4 infants per 1,000 births in Egypt in 1987 died before attaining their first birthday.

Limitation of Conventional Infant Mortality Rate

The conventional infant mortality rate is not a true probability. The infants who die in a given calendar year were born either in that year or in the previous year; and similarly, some children born in a given calendar year will die during the following year. If the number of births does not fluctuate much from year to year, the conventional infant mortality rate for a year will represent rather well the probability of an infant dying during the year. If there are sharp fluctuations in the number of births between years and within years, however, the conventional infant mortality rate will give a distorted indication of the level and trend of infant mortality. In periods of rapid change, therefore, some adjustment should be made.

6.6.3 ADJUSTED INFANT MORTALITY RATES

As noted earlier, total infant deaths in a given year t, D_0, occur to births of that year, $B(t)$, and also to births of the previous year, $B(t-1)$. That is:

$$D_0 = D_0'(t-1) + D_0(t)$$

where

$$\begin{aligned} D_0 &= \text{Total infant deaths in a given year } t; \\ D_0'(t-1) &= \text{infant deaths in year } t \text{ which occurred to year } t-1 \text{ birth cohort}, B(t-1). \\ D_0(t) &= \text{infant deaths in a year } t \text{ which occurred to year } t \text{ birth cohort}, B(t); \end{aligned}$$

Again, some infant deaths occur in the following year, $D_0(t+1)$, from birth cohort of the given year, $B(t)$:

$$D_0(t+1) = D_0''(t+1) + D_0^*(t+1)$$

where

$$\begin{aligned} D_0(t+1) &= \text{Total infant deaths in year } t+1; \\ D_0''(t+1) &= \text{infant deaths which occurred in year } t+1 \text{ from year } t \text{ birth cohort, } B(t); \\ D_0^*(t+1) &= \text{infant deaths in year } t+1 \text{ which occurred to year } t+1 \text{ birth cohort.} \end{aligned}$$

Table 6.3 shows this concept to reflect the true population exposed to risk of death.

From the understanding of the idea in the table, we shall discuss three techniques used to adjust Infant Mortality Rate:

1) Cohort probability;
2) Weighted Birth Cohort probability;
3) Data by year and cohort;

Vital Event	Previous Year (Year t-1)	Given Year (Year t)			Following Year (Year t+1)		
		Year $t-1$ Cohort	Year t Cohort	Total	Year t Cohort	Year $t+1$ Cohort	Total
Births	$B(t-1)$		$B(t)$	B		$B(t+1)$	$B(t+1)$
Infant Deaths	$D_0(t-1)$	$D_0'(t-1)$	$D_0(t)$	D_0	$D_0''(t+1)$	$D_0^*(t+1)$	$D_0(t+1)$

TABLE 6.3 Data Requirement for Computing Adjusted Infant Mortality Rate

Cohort Probability

The adjusted infant mortality rate using the Cohort Probability technique is defined as:

$$AIMR_c = \frac{D_0(t) + D_0''(t+1)}{B(t)} \times k$$

Weighted Birth Cohort Probability (Separated Factors)

The adjusted infant mortality rate using the Weighted Birth Cohort Probability technique $(AIMR_w)$ is defined as:

$$AIMR_w = \frac{D_0'(t-1) + D_0(t)}{\alpha' B(t-1) + \alpha B(t)} \times k$$

where

$$\alpha = \frac{D_0(t)}{B(t)} \qquad \text{and} \qquad \alpha' = 1 - \alpha$$

When information on deaths from the previous year birth cohort is not known, we apply the assumption that one-third of the death in year t was from the previous year birth cohort. That is

$$\alpha' = \frac{1}{3} \qquad \text{and} \qquad \alpha = \frac{2}{3}$$

so that:

$$AIMR_w = \frac{D'_0(t-1) + D_0(t)}{\frac{1}{3}B(t-1) + \frac{2}{3}B(t)} \times k$$

Data by Year and Cohort of birth

The adjusted infant mortality rate using the data by year and cohort of birth $(AIMR_{yc})$ is defined as:

$$AIMR_{yc} = \left(\frac{D'_0(t-1)}{B(t-1)} + \frac{D_0(t)}{B(t)}\right) \times k$$

Example 6.16

A certain country experienced 1,600,000 births in 2011 and 1,800,000 in 2012. There were 710 infant deaths in 2011 and 800 in 2012. Out of the deaths in 2012, 580 were born in that year and 220 in the previous year. It was also reported that there were 250 deaths in 2013 from the births in 2012.

Calculate

1) the conventional (unadjusted) infant mortality rate (CIMR)
2) the adjusted infant mortality rate (AIMR) using the following techniques:
   a) Cohort probability;
   b) Weighted Birth Cohort probability;
   c) Data by year and cohort of birth;

Solution

We summarise the information in Table 6.4.

Vital Event	2011	2012			2013		
		2011 Cohort	2012 Cohort	Total	2012 Cohort	2013 Cohort	Total
Births	1, 600, 000		1,800,000	1,800,000		$B(t+1)$	$B(t+1)$
Infant Deaths	710	220	580	800	250	$D_0^*(t+1)$	$D_0(t+1)$

TABLE 6.4 Summary information of Example 6.16

1) ***The conventional (unadjusted) infant mortality rate (CIMR)***

$$\begin{aligned} CIMR &= \frac{D_0}{B} \times 1,000 \\ &= \frac{800}{18,000} \times 1,000 = 4.44 \end{aligned}$$

That is, 4.62 infants per 1,000 live births died in 2004 before reaching their first birthday.

2) ***The adjusted infant mortality rate (AIMR)***

a) AIMR using Cohort probability

$$D_0(t) = 580; \quad D_0''(t+1) = 250 \qquad B(t) = 1800$$

Hence

$$\begin{aligned} AIMR_c &= \frac{D_0(t) + D_0''(t+1)}{B(t)} \times k \\ &= \frac{580 + 250}{1800000} \times 1000 = 4.61 \end{aligned}$$

b) AIMR using Weighed Birth Cohort

$$\begin{aligned} &B(t-1) = 1,600,000; \quad B(t) = 1,800,000 \\ &D_0(t-1) = 710; \quad D_0'(t-1) = 220; \quad D_0(t) = 580; \end{aligned}$$

Therefore

$$= \frac{220}{710} = 0.309859; \quad \alpha' = 1 - 0.309859 = 0.690141$$

Hence

$$\begin{aligned} IMR_w &= \frac{D_0'(t-1) + D_0(t)}{\alpha B(t-1) + (1-\alpha)B(t)} \times k \\ &= \frac{220 + 580}{0.309859(1,600,000) + 0.690141(1,800,000)} \times 1,000 \\ &= 4.60 \end{aligned}$$

Alternatively:

$$\begin{aligned} AIMR_w &= \frac{D_{0(t)} + D_{0(t-1)}}{\frac{2}{3}B_{(t)} + \frac{1}{3}B_{(t-1)}} \times 1,000 \\ &= \frac{220 + 580}{\frac{2}{3}(1,800,000) + \frac{1}{3}(1,600,000)} \times 1,000 \\ &= \frac{800}{1,200,000 + 533,333.33} \times 1,000 = 4.62 \end{aligned}$$

c) AIMR using Data by Year and Cohort o Birth

$$B(t-1) = 1,600,000; \quad D_0'(t-1) = 220;$$
$$B(t) = 1,800,000; \quad D_0(t) = 580;$$

Therefore

$$IMR_{yc} = \left(\frac{D_0'(t-1)}{B(t-1)} + \frac{D_0(t)}{B(t)}\right) \times k$$
$$= \left(\frac{220}{1,600,000} + \frac{580}{1,800,000}\right) \times 1000 = 4.60$$

6.6.4 DECOMPOSITION OF INFANT MORTALITY

Infant mortality may usefully be broken down into a death covering the first month (called the ***neonatal mortality***) and a death for the remainder of the year (called the ***post-neonatal mortality***) (see Figure 6.2) and there are two main reasons for doing that:

1) There is a very high level of mortality in the first hours, days, and weeks of life
2) There is a difference in the causes accounting for infant deaths at the earlier and later ages of infancy.

We may also observe from Figure 6.2 that neonatal mortality can further be decomposed as ***early neonatal (EN) mortality*** and ***late neonatal (LN) mortality***. Early neonatal mortality (ENNM), denoted by $D_{<1\text{week}}$, occurs within the first week (under 7 days) of life and late neonatal mortality (LNNM), denoted by $D_{1-3\text{weeks}}$, takes place after the first week of life but before the end of the first four weeks (from 7 to 27 days) of life. Related to early neonatal mortality is perinatal mortality. This includes still births from the seventh month of pregnancy to deaths occurring within the first week of life (see details in the sequel).

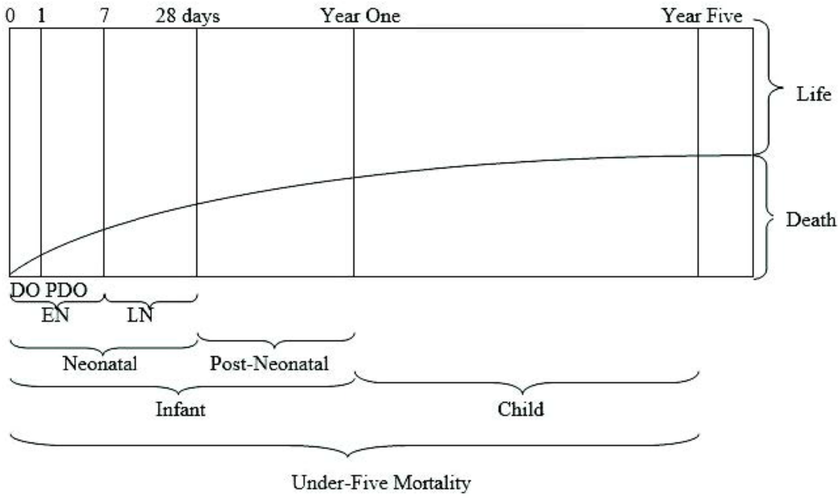

Figure 6.2 Decomposition of Infant Mortality

Early neonatal deaths may further be disaggregated into deaths recorded within the first 24 hours of birth, referred to as ***day one (DO) mortality*** $(D_{<1\text{day}})$ and deaths recorded after day one but before the seventh day, called ***post-day one (PDO) mortality***.

For each of the components of neonatal mortality, the denominator used to calculate the rate is the live birth.

Neonatal Mortality Rate

Neonatal mortality rate (NNMR) refers to the probabilities of child in a cohorts of 1,000 live births dying within the first month. It is defined as the number of deaths of infants under 4 weeks of age (28 days or under 1 month of age) during a year per 1,000 live births during the year:

$$NNMR = \frac{D_{0-3\text{ weeks}}}{B} \quad \times \quad k$$

where

$D_{0-3\,\text{weeks}}$ is the number of infants under 4 weeks;

$k \;= 1{,}000.$

Or

$$NNMR = \frac{D_{<1\text{ month}}}{B} \quad \times \quad k$$

where $D_{<1\text{ month}}$ is the number of infants under one month.

Example 6.17

For the data in Example 6.15, calculate the neonatal mortality rate (NNMR).

Solution

$$
\begin{aligned}
B &= 1,902,604 \\
D_{0-3 \text{ weeks}} &= 25,284 \\
NNMR &= \frac{D_{0-3 \text{ weeks}}}{B} \times 1,000 \\
&= \frac{25,284}{1,902,604} \times 1,000 = 13.3
\end{aligned}
$$

That is, out of 1,000 children born in 1987, 13.3 died before the age of one month.

Example 6.18

In the Republic of Mauritius in 2004, among the 19,230 live births, 55 deaths occurred within one day; 152 died within one week while 207 died within four weeks. Calculate the

1) Day one mortality rate (DOMR);
2) Neonatal mortality rate (NNMR);
3) Early neonatal mortality rate (ENNMR);
4) Late neonatal mortality rate (LNNMR);

Solution

$$B = 19,343; \quad D_{<1\text{day}} = 55; \quad D_{<1\text{week}} = 152; \quad D_{0-3\text{weeks}} = 207$$

Hence,

$$
\begin{aligned}
D_{1-3\text{weeks}} &= D_{0-3\text{weeks}} - D_{<1\text{weeks}} \\
&= 207 - 152 = 55
\end{aligned}
$$

Therefore

1) Day one mortality rate is:

$$\begin{aligned} DOMR &= \frac{D_{<1\text{day}}}{B} \times 1,000 \\ &= \frac{55}{19,230} \times 1,000 = 2.9 \end{aligned}$$

2) Neonatal mortality rate is:

$$\begin{aligned} NNMR &= \frac{D_{0-3\,\text{weeks}}}{B} \times 1,000 \\ &= \frac{207}{19.230} \times 1,000 = 10.8 \end{aligned}$$

3) Early neonatal mortality rate is:

$$\begin{aligned} ENNMR &= \frac{D_{<1\text{week}}}{B} \times 1,000 \\ &= \frac{152}{19,230} \times 1,000 = 7.9 \end{aligned}$$

4) Late neonatal mortality rate is:

$$\begin{aligned} LNNMR &= \frac{D_{1-3\text{weeks}}}{B} \times 1,000 \\ &= \frac{55}{19,230} \times 1,000 = 2.9 \end{aligned}$$

That is, in a cohort of 1,000 children born in Mauritius in 2004, 2.9 died within the first day, 7.9 within the first week, 2.9 survived the first week but died before one month. In general, 10.8 children died before reaching one year.

Post-Neonatal Mortality Rate

Post-neonatal mortality rate (PNMR) is defined as the number of infant deaths at 4 through 51 weeks of age (or 1 through 11 months of age) during a year per 1,000 live births. It is given by:

$$PNMR = \frac{D_{4-51\,\text{weeks}}}{B} \times k$$

or

$$PNMR = \frac{D_{1-11\,\text{months}}}{B} \times k$$

Example 6.19

Refer to Example 6.18. It was known that there were 277 infant deaths, calculate the post-neonatal mortality rate (PNMR).

Solution

From Examples 6.18,

$$\begin{aligned} B &= 19,230 \\ D_0 &= 277 \\ D_{0-3\text{ weeks}} &= 207 \\ D_{4-51\text{ weeks}} &= D_0 - D_{0-3\text{ weeks}} \\ &= 277 - 207 = 70 \end{aligned}$$

Therefore,

$$\begin{aligned} PNMR &= \frac{D_{4-51\text{ weeks}}}{B} \times 1,000 \\ &= \frac{70}{19,230} \times 1,000 = 3.6 \end{aligned}$$

That is, out of 1,000 live births in Mauritius in 2004, 3.6 of them died before reaching their first birthday, after surviving the first month.

Note

The formula for the post-neonatal mortality rate is quite far from expressing a true probability because more than a third of the infant deaths over one month of age in a year may occur to births of the previous year.

6.7 MEASURES OF PREGNANCY WASTAGE

If a woman becomes pregnant but the pregnancy does not result in a live birth, then pregnancy wastage or foetal mortality has occurred. The United Nations and the World Health Organisation recommend that the events, miscarriages, abortions and stillbirths, be grouped under "foetal death" and be classified as early, intermediate, and late according to months of gestation.

6.7.1 CLASSIFICATION OF FOETAL DEATHS

Foetal deaths tabulated by periods of gestation may be grouped into the following classes and illustrated in Figure 6.3:

Under 20 weeks' gestation	–	"early" foetal deaths;
20 to 27 weeks' gestation	–	"intermediate" foetal deaths;
28 weeks' gestation or more	–	"late" foetal deaths.

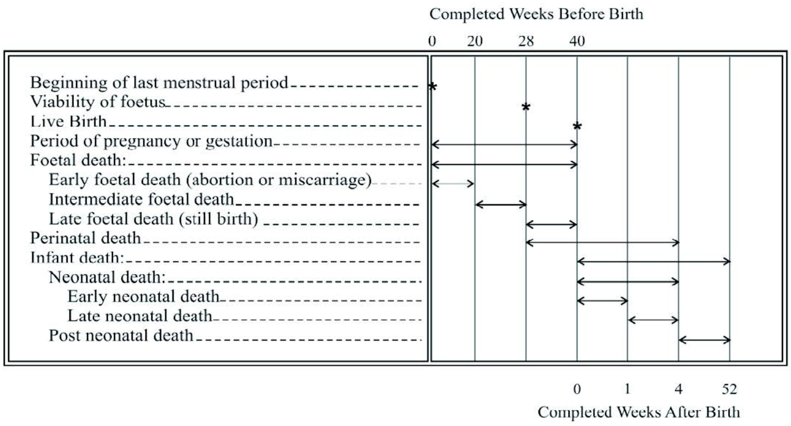

Figure 6.3 Events during Pregnancy and First Year of life

For measuring the reproductive loss due to foetal deaths we use the foetal death ratio or the foetal death rate. For the computation of these measures, we may define foetal deaths to include all foetal deaths or late foetal deaths only.

6.7.2 FOETAL DEATH RATIO

Foetal Death Ratio with Respect to All Foetal Deaths

Foetal death ratio with respect to all foetal deaths (FDr_A) is defined as:

$$FDr_A = \frac{D_f}{Del} \times k$$

where D_f = total number of all foetal deaths during a year;

Del = total deliveries during the year;

k = 1, 000.

Deliveries should not be confused with live births. Live births include only births which survived, that is, premature or full term births. Deliveries include live births and stillbirths.

Example 6.20

There were 4,000 premature births and 12,000 full term births in a country in a certain year. Table 6.3 also shows the distribution of the foetal deaths of the country in the same year, given that there were 40 stillbirths:

Period of Gestation (weeks)	Number of foetal deaths
below 20	125
20–27	75
28 or more	40

TABLE 6.3 Distribution of Foetal Deaths of a Country

Calculate the foetal death ratio.

Solution

$$D_f = 125 + 75 + 40 = 240$$
$$B = 4,000 + 12,000 = 16,000$$
$$\text{Stillbirths} = 40$$

Therefore,

$$Del = B + \text{Stillbirths} = 16,000 + 40 = 16,040$$

Hence

$$FDr_A = \frac{D_f}{Del} \times 1,000 = \frac{240}{16,040} \times 1,000 \approx 15$$

That is, for every 1,000 deliveries in the country in that year, there are approximately 15 foetal deaths.

Limitation of Foetal Death Ratio with Respect to All Foetal Deaths

The main problem associated with the use and interpretation of the foetal death ratio is the variation among reporting areas with respect to the duration of gestation. Some areas report all foetal deaths regardless of length of gestation while others have a minimum gestation period that must be reached before reporting is required.

Foetal Death Ratio with Respect to Late Foetal Deaths

To overcome the problem associated with the foetal death ratio with respect to all foetal deaths (FDr_A), the foetal death ratio is computed on the basis of late foetal deaths only (FDr_ℓ). It is defined as the number of foetal deaths reported in a year per k live births in the same year. Thus:

$$FDr_\ell = \frac{D_{\ell f}}{B} \times k$$

where $D_{\ell f}$ = total number of late foetal deaths during a year;
B = total live births in the same year;
k = 1, 000.

This ratio is preferable from the point of view of international comparability. By foetal death ratio, we shall refer to this ratio, that is, foetal death ratio with respect to late foetal deaths.

Example 6.21

Calculate the foetal death ratio for the data in Example 6.20.

Solution

From Example 6.18,

$$\begin{aligned} D_{\ell f} &= 40 \\ B &= 16,000 \\ FDr_\ell &= \frac{D_{\ell f}}{B} \times 1,000 \\ &= \frac{40}{16,000} \times 1,000 = 2.5 \end{aligned}$$

That is, in the country in that year, 2.5 per 1,000 births after 28 weeks of gestation showed no sign of life after birth.

Example 6.22

For the data in Example 6.15, calculate the foetal death ratio.

Solution

$$\begin{aligned} B &= 1,902,604; \\ D_{\ell f} &= 12,020 \\ FDr_{\ell} &= \frac{D_{\ell f}}{B} \times 1,000 \\ &= \frac{12,020}{1,902,604} \times 1,000 = 6.3 \end{aligned}$$

That is, the foetal death rate in Egypt in 1987 was 6.3 per 1,000 births.

Limitation of Foetal Death Ratio

The foetal death ratio does not closely relate to the "population at risk" because the denominator does not include the foetal deaths.

6.7.3 FOETAL DEATH RATE

To overcome the shortcoming of the foetal death ratio, the foetal death rate is calculated.

Foetal Death Rate with Respect to All Foetal Deaths

Using all foetal deaths, foetal death rate (FDR_A) is defined as:

$$FDR_A = \frac{D_f}{B + D_f}$$

where $D_{\ell f}$ and B have their usual meanings. Similarly, by foetal death rate, we shall refer to this one.

Example 6.23

Calculate the foetal death rate for the data in Example 6.20.

Solution

From Example 6.18

$$\begin{aligned} D_f &= 125 + 75 + 40 = 240 \\ B &= 16,000 \end{aligned}$$

$$\begin{aligned} FDR_A &= \frac{D_f}{B + D_f} \times 1,000 \\ &= \frac{240}{16,000 + 240} \times 1000 = 14.8 \end{aligned}$$

That is, there were 14.8 foetal deaths per 1,000 cases of pregnancy.

Foetal Death Rate with Respect to Late Foetal Deaths

Using late foetal death, the late foetal death (FDR_ℓ) is defined as

$$FDR_\ell = \frac{D_{\ell f}}{B + D_{\ell f}}$$

where D_f and B have their usual meanings.

Example 6.24

Calculate the foetal rate for data in Example 6.20.

Solution

From Example 6.20,

$$\begin{aligned} D_{\ell f} &= 40 \\ B &= 16,000 \end{aligned}$$

$$\begin{aligned} FDR_{\ell f} &= \frac{D_{\ell f}}{B + D_{\ell f}} \times 1,000 \\ &= \frac{40}{16,000 + 40} \times 1,000 = 2.49 \end{aligned}$$

That is, there were 2.5 late foetal deaths per 1,000 cases of pregnancy.

Example 6.25

For the data on Egypt in Example 6.15, calculate the foetal death rate.

Solution

$$\begin{aligned} B &= 1,902,604 \qquad D_{\ell f} = 12,020 \\ FDR_{\ell f} &= \frac{D_{\ell f}}{B + D_{\ell f}} \times 1,000 \\ &= \frac{12,020}{1,902,604 + 12,020} \times 1,000 = 6.3 \end{aligned}$$

6.7.4 RELATIONSHIP BETWEEN FOETAL DEATH RATIO AND RATE

We may calculate foetal death rate (FDR) from foetal death ratio (FDr) as:

$$FDR = \frac{FDr \times B}{\text{Late Foetal deaths + B}} \times 1,000$$

Example 6.26

For the data in Example 6.20, calculate the FDR from FDr computed from late foetal deaths.

Solution

From Examples 6.20 and 6.21,

$$\begin{aligned} B &= 16,000; \\ \text{Foetal deaths} &= 40 \\ FDr_\ell &= 2.5 \text{ per } 1,000 \\ FDr_\ell &= 0.0025 \end{aligned}$$

$$\begin{aligned} FDR_\ell &= \frac{FDr_\ell \times B}{\text{Late Foetal deaths } + \ B} \\ &= \frac{0.0025 \times 16,000}{40 + 16,000} \times 1,000 = 2.49 \end{aligned}$$

This result is the same as in Example 6.24.

Similarly, we can calculate FDR_A from FDr_A and Exercise 6.5 illustrates this case.

6.7.5 FOETAL DEATH RATIO AND RATE COMPARED

1) The foetal death rate relates the foetal deaths more closely to the population at risk than the foetal death ratio, i.e. it is more a probability than a rate.
2) The denominator in the foetal death rate makes it possible to include all pregnancies in the computation of the rate.
3) In spite of the above, the foetal death ratio may be considered preferable for international comparisons. The registration of foetal deaths is irregular and the effect of this irregularity is compounded when foetal deaths are included with the births in the denominator of the foetal death rate.

6.8 PERINATAL MORTALITY

For medical research into causes of death, the distinction made between stillbirth (foetal death which occurs late in the woman's pregnancy) and death following soon after live birth is not very meaningful because both are related to the same set of causes arising during the pregnancy. It has been suggested, therefore, that the two be combined to obtain what is known as the ***perinatal deaths***. The combination of these deaths also eliminates the conflicting judgments as to whether a foetus showed any sign of life and whether or not to classify it as foetal death, birth, and neonatal death.

6.8.1 WHAT IS PERINATAL MORTALITY?

Variations in the precise definition of the perinatal mortality exist specifically concerning the issue of inclusion or exclusion of early fetal and late neonatal fatalities. Perinatal describes the period surrounding birth, and traditionally includes the time from foetal viability from about 22 weeks of pregnancy up to either 7 or 28 days of life or deaths of foetuses weighing 1,000 grams.

The World Health Organization defines perinatal mortality as the "number of stillbirths and deaths in the first week of life per 1,000 live births, the perinatal period commences at 22 completed weeks of gestation and ends 7 completed days after birth", but other definitions have been used, such as the period between the end of the 28th week of pregnancy and the first month of life.

The combined risk of dying during the period near parturition (i.e., just before, during, and just after birth) is measured by perinatal mortality ratios and rates.

6.8.2 PERINATAL MORTALITY RATIO

Perinatal Mortality Ratio with Respect to Neonatal Deaths

Perinatal mortality ratio with respect to neonatal deaths (PMr_N) is defined as the number of late foetal deaths and neonatal deaths per 1,000 live births in a year:

$$Mr_N = \frac{D_{\ell f} + D_{0-3\,\text{weeks}}}{B} \times 1,000$$

where the symbols have the same meaning as in foetal death rate.

Example 6.27

For the data in Examples 6.15, calculate the perinatal mortality ratio with respect to neonatal deaths.

Solution

$$\begin{aligned} D_{\ell f} &= 12,020 \\ D_0 &= 94,044 \\ D_{0-3\,\text{weeks}} &= 25,284 \end{aligned}$$

$$\begin{aligned} B &= 1,902,604 \\ PMr_N &= \frac{D_{\ell f} + D_{0-3\,\text{weeks}}}{B} \times 1,000 \\ &= \frac{12,020 + 25,284}{1,902,604} \times 1,000 = 19.6 \end{aligned}$$

That is, for every 1,000 live births in Egypt in 1987, there were 19.6 late foetal and neonatal deaths.

Perinatal Mortality Ratio with Respect to Early Neonatal Deaths

Perinatal mortality ratio with respect to early neonatal deaths (PMr_{EN}) is defined as the number of late foetal deaths and deaths under one week of age per 1,000 live births in a year:

$$PMr_{EN} = \frac{D_{\ell f} + D_{<1\,\text{week}}}{B} \times 1,000$$

where the symbols have the usual meanings.

Example 6.28

In addition to data in Example 6.22, it was observed that 13,185 children died in the first week. Calculate the perinatal mortality ratio with respect to early neonatal deaths.

Solution

From Example 6.22, $D_{\ell f} = 12,020$

$$\begin{aligned} D_{<1\text{week}} &= 13,185 \\ B &= 1,902,604 \\ PMr_{EN} &= \frac{D_{\ell f} + D_{<1\,\text{week}}}{B} \times 1,000 \\ &= \frac{13,185 + 12,020}{1,902,604} \times 1,000 = 13.2 \end{aligned}$$

That is, for every 1,000 live births there were 13.3 late foetal deaths together with deaths under one week of age, that occur.

6.8.3 PERINATAL MORTALITY RATE

Similar to perinatal mortality ratio (PMr), the perinatal mortality rate (PMR) may also be defined with respect to neonatal or early neonatal deaths. Unlike PMr, PMR includes in the denominator also the late foetal deaths $(D_{\ell f})$, besides the births in a year, thus approximating the probability more closely.

Perinatal Mortality Rate with Respect to Neonatal Deaths

PMR with respect to neonatal deaths is defined as:

$$PMR_N = \frac{D_{\ell f} + D_{0-3\,\text{weeks}}}{D_{\ell f} + B}$$

Example 6.29

For the data in Example 6.15, calculate the perinatal mortality rate with respect to neonatal deaths.

Solution

From Example 6.15,

$$\begin{aligned} D_{\ell f} &= 12,020 \\ D_{0-3\,\text{weeks}} &= 25,284 \\ B &= 1,902,604 \end{aligned}$$

$$\begin{aligned} \therefore \quad PMR_N &= \frac{D_{\ell f} + D_{0-3\,\text{weeks}}}{D_{\ell f} + B} \times 1,000 \\ &= \frac{12,020 + 25,284}{12,020 + 1,902,604} \times 1,000 = 19.483 \end{aligned}$$

Perinatal Mortality Rate with Respect to Early Neonatal Deaths

Perinatal mortality rate with respect to early neonatal deaths is defined as:

$$PMR_{EN} = \frac{D_{\ell f} + D_{<1\text{week}}}{D_{\ell f} + B} \times 1,000$$

Example 6.30

Using the data in Example 6.15, calculate the perinatal mortality rate with respect to early neonatal deaths.

Solution

From Example 6.15,

$$\begin{aligned} D_{\ell f} &= 12,020 \\ D_{<1\text{week}} &= 13,185 \\ B &= 1,902,604 \end{aligned}$$

$$\begin{aligned} \therefore \quad PMR_{EN} &= \frac{D_{\ell f} + D_{<1\text{week}}}{D_{\ell f} + B} \times 1,000 \\ &= \frac{12,020 + 13,185}{12,020 + 1,902,604} \times 1,000 = 13.2 \end{aligned}$$

6.8.4 RELATIONSHIP BETWEEN PERINATAL MORTALITY RATIO AND RATE

We may calculate perinatal mortality rate (PMR) from perinatal mortality ratio (PMr) as:

$$PMR = \frac{PMr \times B}{\text{Foetal deaths + B}} \times 1,000$$

Example 6.31

For the data in Example 6.15, calculate the perinatal mortality rate from the perinatal mortality death ratio calculated from neonatal deaths.

Solution

From Examples 6.15,

$$\begin{aligned} B &= 1,902,604; \\ FDr_{\ell f} &= 19.6 \text{ per } 1,000 \\ \text{or} \quad FDr_{\ell f} &= 0.0196 \\ \text{Foetal deaths } (D_{\ell f}) &= 12,020 \\ \therefore \quad PMR_N &= \frac{PMr \times B}{\text{Foetal deaths + B}} \times 1,000 \\ &= \frac{0.0196 \times 1,902,604}{12,020 + 1,902,604} \times 1,000 = 19.477 \end{aligned}$$

This result is similar to that in Example 6.29 except for rounding error. Similarly, PMR_{EN} can be computed from PMR_{EN} and Exercise 6.6 illustrates this case.

6.9 STANDARDISED DEATH RATE

In Chapter 3, standardisation has been defined as the process of adjusting rates to eliminate from them the effect of differences in population composition with respect to age, sex, marital status, race, or other categories of the population. The reasons for standardisation have also been given in that chapter.

Death rates may, therefore, be standardised (adjusted) with respect to say, age, sex, marital status. However the commonest is the age-adjusted. Hence standardised death rate and the age-adjusted death rate will be used interchangeably in this text.

6.9.1 DIRECT STANDARDISED DEATH RATE

The simplest and most straightforward measure of the standardised death rate is the one derived by the direct method, defined as:

$$DSDR = \frac{\sum d_a P_a^s}{\sum P_a^s} \times k$$

where

$DSDR$ = direct standardised death rate;
d_a = age-specific death rate for the age group *a* for the study population;
P_a^s = mid-year population of the age-group *a* for the standard population;
$d_a P_a^s$ = number of expected deaths in the standard population for each age group *a*.

The suffix s refers to the standard population[28]; otherwise it is the population under study.

This formula indicates that $DSDR$ is the weighted mean of the age-specific death rate, d_a, of the study population, where the weight is the mid-year standard population by age, P_a^s. That is, the age distribution of the standard population is used as a base for adjustment. Using another population as standard may change our conclusions.

Note

The sum of the weights of each age group, $\sum P_a^s$, is equal to the total standard population; that is,

$$\sum P_a^s = P^s$$

We may re-write DSDR as

$$DSDR = \frac{D_E}{P^s} \times k$$

where $D_E = \sum d_a P_a^s$.

The physical meaning of a directly standardised death rate of a population becomes clearer. It is a crude death rate that would result if the age-specific death rates of the population under study (d_a) apply to a population with the age distribution as that of the standard population.

Data Requirement for Computing DSDR

The data necessary for the implementation of direct standardised rate are the following:

1) Age-specific death rates for the populations being studied, (d_a);
2) Population distribution across the various segments for a standard population (P_a^s).

Assumption

It is assumed that the study population has the same age structure as the standard population.

Note

The term direct refers to working directly with the specific rates of the population being studied.

Example 6.32

Table 6.4 (a) shows the 1982 and 1984 populations and 1982 deaths for Algeria.

Age Group (Years)	1982 Population ('000)	Deaths in 1982	1984 Census Population as Standard ('000)
0–4	3,526.6	113,762	3,698.0
5–9	3,028.2	5,955	3,138.0
10–14	2,605.2	3,195	2,752.0
15–19	2,214.1	3,998	2,329.0
20–24	1,729.5	4,748	1,814.0
25–29	1,439.6	4,248	1,516.0
30–34	1,102.1	4,015	1,142.0
35–39	763.7	3,837	766.0
40–44	701.0	4,032	754.0
45–49	667.2	4,258	707.0
50–54	552.0	4,969	579.0
55–59	424.9	5,721	448.0
60–64	357.4	7,003	380.0
65–69	270.1	8,245	286.0
70–74	234.3	9,546	247.0
75 +	268.1	21,495	285.0
Total	19,884.0	209,027	20,841.0

TABLE 6.4(a) Distribution of Population and Deaths in Algeria by Age, 1982–1984
Source: *United Nations Demographic Yearbook* (1980–1991),
(New York: Statistics Office, Department of Economic and Social Affairs)

1) Calculate the crude birth rate for Algeria in 1982.
2) Use the direct method to obtain the 1982 adjusted death rate for Algeria if her 1984 population was used as the standard. Interpret your result.
3) Comment on your results in (1) and (2).

Solution

1) The crude death rate of Algeria in 1982:

$$d = \frac{D}{P} \times k$$

$$= \frac{209,027}{19,884,000} \times 1000$$

$$= 10.5 \quad \text{deaths per 1,000 population.}$$

2) The steps for calculating a standardised death rate by the direct method are illustrated in Table 6.4 (b).

Age Group (years)	1982 Population ('000) P	Deaths in 1982 D	Age-Specific Death Rate for Algeria for 1982 d_a	1984 Population Census As Standard ('000) P_a^s	Number of Expected Deaths in Standard Population $d_a P_a^s$
1	2	3	$4 = (3)/(2)$	5	$6 = (4) \times (5)$
0–4	3,526.6	113,762	0.03226	3,698.0	119,291
5–9	3,028.2	5,955	0.00295	3,138.0	6,171
10–14	2,605.2	3,195	0.00197	2,752.0	3,375
15–19	2,214.1	3,998	0.00123	2,329.0	4,205
20–24	1,729.5	4,748	0.00181	1,814.0	4,980
25–29	1,439.6	4,248	0.00275	1,516.0	4,473
30–34	1,102.1	4,015	0.00364	1,142.0	4,160
35–39	736.7	3,837	0.00502	766.0	3,849
40–44	701.1	4,032	0.00575	754.0	4,337
45–49	667.2	4,258	0.00638	707.0	4,512
50–54	552.0	4,969	0.00900	579.0	5,212
55–59	424.9	5,721	0.01346	448.0	6,032
60–64	357.4	7,003	0.01959	380.0	7,446
65–69	270.1	8,245	0.03053	286.0	8,730
70–74	234.3	9,546	0.04074	247.0	10,063
75+	268.1	21,495	0.08018	285.0	22,687
Total	19,884.0	209,027		20,841.0	219,828

TABLE 6.4(b) Preliminary Calculation of Direct Standardised Death Rate

a) The study population (population of interest) is listed (column 2) according to age group (column 1).
b) The deaths in the study population are listed (column 3) by age group.
c) The age-specific death rates (column 4) for each age group are calculated by dividing column 3 by column 2.
d) The standard population (column 5) is listed by age group.
e) The expected number of deaths in the standard population for each age group (column 6) is computed by multiplying column 4 by column 5.
f) The entries in column 6 are summed to obtain the total number of expected deaths in the standard population.

Note

The entries in column 6 indicate the number of deaths per 1,000 population that would have been expected if the death rates had prevailed for one year in the standard population.

The study population death rate by direct method is:

$$\begin{aligned} DSDR &= \frac{D_E}{P^s} \times k \\ &= \frac{219,687}{20,841,000} \times 1,000 = 10.54 \end{aligned}$$

Interpretation

The death rate for Algeria that could have been expected in 1982 if the age composition of the 1982 population had remained the same as that of the 1984, was 10.5 per 1,000 population.

(3) *Comment*

The crude death rate of Algeria has remained the same at about 10.5 per 1,000 population by adjusting its 1982 population to the age distribution of her 1984 (standard) population. This stability in the death rate following adjustment reflects the fact that the age structure of the population in both years remained almost the same. For example, in both years, 5.7 per 10,000 of the country's population was 60 years of age or older.

Alternative Formula For Direct Standardised Death Rate

An alternative formula for computing direct standardised death rate is:

$$DSDR = \sum_a d_a p_a^s \times k$$

where p_a^s = the proportion of population for each age group a in the standard population, defined as:

$$p_a^s = \frac{P_a^s}{P^s}$$

with

$$\sum_a p_a^s = 1$$

$d_a p_a^s$ = expected age-specific death rate for the standard population.

Example 6.33

Re-work Example 6.32 using the alternative formula for computing direct standardised death rate.

Solution

The preliminary calculations are done in Table 6.4(c).

$$\begin{aligned} DSDR &= \sum_{a=1}^{n} d_a p_a^s \times k \\ &= 0.01054 \times 1,000 = 10.54 \end{aligned}$$

Direct Standardised Death Rate for Two Variables

All along we have assumed that standardisation is done for only one variable, age. However, it is possible to do standardisation for two or more variables. For example, the age-sex direct standardised death rate (ASDSDR) may be defined as:

$$ASDSDR = \frac{\sum d_a^m P_a^{sm} + \sum d_a^f P_a^{sf}}{\sum P_a^{sm} + \sum P_a^{sf}} \times k$$

where the superscripts *m* and *f* refer to male and female populations, respectively; the other notations have the usual meaning; and in particular,

$\sum d_a^m P_a^{sm}$ = the expected number of male deaths in the standard population;

$\sum d_a^f P_a^{sf}$ = the expected number of female deaths in the standard population.

Age Group (years)	Age-Specific Death Rate for Algeria for 1970	1984 Population Census As Standard ('000)	Proportion of members of Standard Population	Proportion of Expected deaths in Standard Population
	d_a	P_a^s	p_a^s	$d_a p_a^s$
1	2	3	4	$5 = (2) \times (4)$
0–4	0.03226	3,698	0.17744	0.00572
5–9	0.00197	3,138	0.15057	0.00030
10–14	0.00123	2,752	0.13205	0.00016
15–19	0.00181	2,329	0.11175	0.00020
20–24	0.00275	1,814	0.08704	0.00024
25–29	0.00295	1,516	0.07274	0.00021
30–34	0.00364	1,142	0.05480	0.00020
35–39	0.00502	766	0.03675	0.00028
40–44	0.00575	754	0.03618	0.00021
45–49	0.00638	707	0.03392	0.00022
50–54	0.00900	579	0.02778	0.00025
55–59	0.01346	448	0.02150	0.00029
60–64	0.01959	380	0.01823	0.00036
65–69	0.03053	286	0.01372	0.00042
70–74	0.04074	247	0.01185	0.00048
75+	0.08018	285	0.01367	0.00110
Total		20,841	1.0000	0.010639

TABLE 6.4(c) Calculation of Direct Standardised Death Rate

Example 6.34

Table 6.5(a) shows the distribution of population and deaths in the Republic of Mauritius in 2000 and 2004 by age and sex.

1) Calculate the crude death rate for 2004 for the
   a) Male population,
   b) Female population.
2) Calculate the DSDR for 2004 for the
   a) Male population,
   b) Female population.
3) Calculate the age-sex DSDR for the 2004 population, using the census male and female populations of 2000 as standard. Interpret the result.
4) Comment on the results in (1) to (3).

Age-group (Years)	2000 Population (Standard)		2004 Population (Study)		2004 Deaths (Study)	
	Male	Female	Male	Female	Male	Female
0	9,574	9,341	9,708	9,289	157	120
1–4	38,066	37,322	39,994	38,657	20	20
5–9	53,037	52,152	50,440	49,357	11	14
10–14	49,428	48,312	55,251	53,698	14	6
15–19	51,671	50,417	48,008	46,955	37	16
20–24	55,108	55,784	54,276	53,308	49	27
25–29	46,749	47,048	52,798	53,955	73	26
30–34	49,964	49,551	46,256	46,591	96	43
35–39	51,621	50,325	49,955	49,954	178	59
40–44	45,798	44,608	49,907	49,072	257	89
45–49	39,133	38,798	42,880	42,597	352	152
50–54	27,790	29,149	36,847	37,382	397	212
55–59	19,228	21,263	24,029	26,521	367	226
60–64	15,301	17,796	16,375	18,826	428	300

Age-group (Years)	2000 Population (Standard)		2004 Population (Study)		2004 Deaths (Study)	
	Male	Female	Male	Female	Male	Female
65–69	11,758	14,010	12,714	15,851	479	380
70–74	9,491	12,203	9,035	11,734	488	408
75–79	6,047	8,863	6,675	9,882	568	516
80–84	2,584	4,548	3,408	5,799	411	473
85+	1,408	3,602	1,552	3,850	334	672
Total	583,756	595,092	610,108	623,278	4,716	3,759

TABLE 6.5(a) Age-sex Distribution of Population and Deaths in Republic of Mauritius
Source: Central Statistical Office, *Annual Digest of Statistics of the Republic of Mauritius (excluding Agalega and St. Brandon)*, Port Loius, 2004.

Solution

The preliminary calculations are done in Table 6.5(b).

1) Calculation of crude death rate

a) *Male population*
From column 2, the male population in 2004 is:

$$P^m = \sum P_a^m = 610,108$$

From column 6, number of male deaths in 2004 is:

$$D^m = 4,716$$

Hence, the crude death rate for the male population is:

$$d^m = \frac{4,716}{610,108} \times 1,000 = 7.730$$

Age group	2000 Population (Standard)		2004 Population (Study)		2004 Deaths (Study)		Age-specific Death Rate (Study)		Expected Deaths	
(Years)	Male	Female	Male	Female	Male	Female	Male	Female	Male	Female
1	2	3	4	5	6	7	8=(6)/(4)	9=(7)/(5)	10=(8)(2)	11=(9)(3)
0	9,574	9,341	9,708	9,289	157	120	0.01617	0.0.01292	154.834	120.672
1–4	38,066	37,322	39,994	38,657	20	20	0.00050	0.0.00052	19.036	19.309
5–9	53,037	52,152	50,440	49,357	11	14	0.00022	0.0.00028	11.566	14.793
10–14	49,428	48,312	55,251	53,698	14	6	0.00025	0.0.00011	12.525	5.3982
15–19	51,671	50,417	48,008	46,955	37	16	0.00077	0.0.00034	39.823	17.180
20–24	55,108	55,784	54,276	53,308	49	27	0.00090	0.0.00051	49.751	28.254
25–29	46,749	47,048	52,798	53,955	73	26	0.00138	0.0.00048	64.636	22.672
30–34	49,964	49,551	46,256	46,591	96	43	0.00208	0.0.00092	103.696	45.732
35–39	51,621	50,325	49,955	49,954	178	59	0.00356	0.0.00118	183.936	59.438
40–44	45,798	44,608	49,907	49,072	257	89	0.00515	0.0.00181	235.840	80.904
45–49	39,133	38,798	42,880	42,597	352	152	0.00821	0.0.00357	321.241	138.444
50–54	27,790	29,149	36,847	37,382	397	212	0.01077	0.0.00567	299.417	165.309

Age group (Years)	2000 Population (Standard)		2004 Population (Study)		2004 Deaths (Study)		Age-specific Death Rate (Study)		Expected Deaths	
	Male	Female	Male	Female	Male	Female	Male	Female	Male	Female
55–59	19,228	21,263	24,029	26,521	367	226	0.01527	0.0.00852	293.673	181.194
60–64	15,301	17,796	16,375	18,826	428	300	0.02614	0.0.01594	399.928	283.587
65–69	11,758	14,010	12,714	15,851	479	380	0.03768	0.0.02397	442.983	335.865
70–74	9,491	12,203	9,035	11,734	488	408	0.05401	0.0.03477	512.630	424.307
75–79	6,047	8,863	6,675	9,882	568	516	0.08509	0.0.05222	514.561	462.792
80–84	2,584	4,548	3,408	5,799	411	473	0.12060	0.0.08157	311.627	370.962
85+	1,408	3,602	1,552	3,850	334	672	0.21521	0.0.17455	303.010	628.713
Total	583,756	595,092	610,108	623,278	4,716	3,759			4,274.713	3,405.523

TABLE 6.5(b) Age-sex Distribution of Population and Deaths in Republic of Mauritius

b) *Female population*

From column 3, the female population in 2004 is:

$$P^f = \sum P_a^f = 623,278$$

From column 7, the number of female deaths in 2004 is:

$$D^f = 3,759$$

Hence, the crude death rate for the female population is:

$$d^f = \frac{3,759}{623,278} \times 1,000 = 6.031$$

2) Calculation of direct standardised death rate

a) *Male population:*

From column 10, the expected number of male deaths is:

$$D_E^m = \sum d_a^m P_a^{sm} = 4,274.71326$$

From column 2, the male population in 2000 (standard):

$$P^{2000(m)} = 583,756$$

Hence, the direct standardised death rate for the male population is:

$$\frac{D_E^m}{P^{2000(m)}} = \frac{4,274.71326}{583,756} \times 1,000 = 7.323$$

b) *Female population:*
From column 7, the expected number of female deaths is:

$$D_E^f = \sum d_a^f P_a^{sf} = 3,405.52304$$

From column 3, the female population in 2000 (standard):

$$P^{2000(f)} = 595,092$$

Hence, the direct standardised death rate for the female population is:

$$\frac{D_E^f}{P^{2000(f)}} = \frac{3,405.52304}{595,092} \times 1,000 = 5.723$$

3) Calculation of age-sex standardised death rate (ASDSDR)
From previous calculations, we have:

$$\begin{aligned} ASDSDR &= \frac{\sum d_a^m P_a^{sm} + \sum d_a^f P_a^{sf}}{\sum P_a^{sm} + \sum P_a^{sf}} \times 1,000 \\ &= \frac{\sum d_a^m P_a^{2000(m)} + \sum d_a^f P_a^{2000(f)}}{\sum P_a^{2000(m)} + \sum P_a^{2000(f)}} \times 1,000 \\ &= \frac{4,274.71326 + 3405.52304}{583,756 + 595,092} \times 1,000 = 6.5 \end{aligned}$$

That is, the death rate for Mauritius that could have been expected in 2004 if the age-sex composition of the census population in 2000 had remained the same in 2004, was 6.5 per 1,000 people.

4) Comment
Table 6.5 (c) summarises the results for Example 6.34.

Death Rate	Male	Female	Age-sex
CDR	7.7	6.0	
DSDR	7.3	5.7	
ASDSDR			6.5

TABLE 6.5(c) Crude and Standardised Death Rates for Mauritius, 2004

We observe that the crude death rates (CDR) in the male and female populations differ very much. The DSDRs for the male and female populations still remain different even when the effect of the differences in the age distribution in both sexes were supposed to have been removed after standardisation. This can be explained by th fact that both sexes have similar population structure in 2000 (standard population).

If the age distribution in the male and female populations was quite different, the effect of standardisation would have been noticed and the DSDR for both sexes would have then been close to each other.

Advantages of Direct Standardised Death Rate

1) It is easy to compute; in the same manner as a crude death rate.
2) It is easy to explain. It is the weighted average of the age-specific death rates in a given community, using as weights the age distribution of the standard population.
3) If the same standard population is employed, as required, all the rates are directly comparable.

Disadvantage of Direct Standardised Death Rate

The choice of the standard population is subjective and may influence the comparison of adjusted rates.

6.9.2 INDIRECT STANDARDISED DEATH RATE

In practice, not all the data required for direct standardisation are always available. Often only the crude total rate for the study population is available with its age-specific death rates unknown. Such a situation permits only indirect standardisation to be carried out.

Data Requirement

For indirect standardised death rate (ISDR) to be computed, the following information must be available.

1) *For study population:*
   a) The population distribution across the various segments (in this case across ages);
   b) Crude death rate (or total number of deaths).

2) *For standard population:*
   a) Age specific death rates;
   b) Crude death rate.

   or population and death by age distribution.

Assumption

It is assumed that the age-specific death rate for the study population experiences the specific conditions as for the standard population.

Method

The technique is to first compute the ratio of actual deaths in the study population (D) to the number of deaths expected in the study population (D_E) on the basis of age-specific death rates in a standard population (d_a^s). This ratio multiplied by the crude death rate of the standard population yields the adjusted death rate by indirect method:

$$\begin{aligned} ISDR &= \frac{D}{D_E} \times d^s \\ &= \frac{\text{Number of actual deaths in the study population}}{\text{Expected number of deaths in the study population}} \end{aligned}$$

where d^s is the crude death rate of the standard population, which may be defined as:

$$d^s = \frac{\sum d_a^s P_a^s}{\sum P^s} \times k = \frac{D^s}{P^s} \times k$$

Noting that in defining the ISDR,

$$\begin{aligned} D &= \sum d_a P_a \\ D_E &= \sum d_a^s P_a \end{aligned}$$

we may rewrite the formula of ISDR as:

$$ISDR = \frac{\sum d_a P_a}{\sum d_a^s P_a} \times d^s$$

Example 6.35

Table 6.6(a) shows the 1989 populations of Mauritius and Tunisia and the distribution of deaths by age in Mauritius in 1989. The total number of deaths in Tunisia in 1989 was 20,650.

Calculate the indirect standardised rate for Tunisia using the age-specific death rates for Mauritius as standard.

Solution

1) For the standard population, i.e, for Mauritius, we calculate:

a) Crude death rate for Mauritius (d^M):

$$\begin{aligned} d^M &= \frac{D^M}{P^M} \times k \\ &= \frac{6,946}{1,026,813} \times 1,000 \\ &= 6.76462 \text{ per } 1000 \end{aligned}$$

b) Age-specific death rate (d_a^M), see column 4 of Table 6.6(b).

Age Group	Population of Mauritius in 1989	Population of Tunisia in 1989	Number of Deaths in Mauritius in 1989	Number of Deaths in Tunisia in 1989
0–4	93,983	1,014,143	516	Not
5–9	107,047	1,022,505	39	available
10–14	107,128	963,423	41	
15–19	93,887	832,231	66	
20–24	104,312	769,184	92	
25–29	104,450	645,086	150	
30–34	88,918	533,030	164	
35–39	76,563	418,015	215	
40–44	54,048	305,403	252	
45–49	43,483	273,963	299	
50–54	39,327	288,823	374	
55–59	31,206	256,087	473	
60–64	30,558	200,605	779	
65–69	22,268	150,564	858	
70–74	14,314	100,199	807	
75 +	15,321	136,294	1,821	
Total	1,026,813	7,909,555	6,946	20,650

TABLE 6.6(a) Distribution of Population Mauritius and Tunisia and of Deaths by Age in Mauritius, 1989
Source: United Nations Demographic Yearbook (1980–1993), (New York: Statistics Office, Department of Economic and Social Affairs)

2) For the study population, i.e, Tunisia, we calculate its expected death for each age group, D_{aE} using age-specific death rate d_a^M of Mauritius:

$$D_{aE} = d_a^M P_a$$

These are given in column 6 of Table 6.6 (b). The sum of these values gives the total expected deaths in Tunisia:

$$D_E = \sum d_a^M P_a = 53,555.5$$

3) Finally, we calculate the indirect standardised death rate
From column 6 of Table 6.6(b), $D_E = 53,346.4$. Hence,

$$\begin{aligned} ISDR &= \frac{\text{Total observed deaths for Tunisia}}{\text{Total expected deaths in Tunisia}} \times d^M \\ &= \frac{D}{D_E} \times d^M \\ &= \frac{20,650}{53,346.4} \times 6.76462 = 2.62 \end{aligned}$$

Age Group (Years)	Standard Country (Mauritius)			Study Country (Tunisia)	
	Standard Population in 1989 (P_a^M)	Number of Deaths in 1989 (D_a^M)	Age Specific Deaths in 1989	Study Population in 1989 (P_a)	Expected Deaths in 1989 $(D_E = d_a^M P_a)$
1	2	3	4 = (3)/(2)	5	6 = (4) × (5)
0–4	93,983	516	0.00549	1,014,143	5,568.00
5–9	107,047	39	0.00036	1,022,505	372.53
10–14	107,128	41	0.00038	963,423	368.72
15–19	93,887	66	0.00070	832,231	585.04
20–24	104,312	92	0.00088	769,184	678.40
25–29	104,450	150	0.00144	645,086	926.40
30–34	88,918	164	0.00184	533,030	983.12
35–39	76,563	215	0.00281	418,015	1,173.85
40–44	54,048	252	0.00466	305,403	1,423.95
45–49	43,483	299	0.00688	273,963	1,883.84
50–54	39,327	374	0.00951	288,823	2,746.71
55–59	31,206	473	0.01516	256,087	3,881.60
60–64	30,588	779	0.02549	200,605	5,113.92
65–69	22,260	858	0.03853	150,564	5,801.33
70–74	14,314	807	0.05638	100,199	5,649.06
75 +	15,320	1,821	0.11886	136,294	16,199.90
Total	1,026,813	6,946	0.11886	7,909,555	53,346.42

TABLE 6.6(b) Calculation of Indirect Standardised Death Rate for Example 6.35

Interpretation

The death rate for Tunisia that could have been expected if its age-specific death rate had been the same as that of Mauritius is 2.62 per 1,000 population.

Alternative Formula for Indirect Standardised Death Rate

Suppose that the data available are:

1) *For study population:*
   a) the population distribution across the various segments (in this case across ages);
   b) the crude death rate, d.

2) *For standard population:*
   a) age-specific death rates, d_a^s;
   b) the crude death rate, d^s.

Then the indirect standardised death rate is given by:

$$ISDR = \frac{d}{\sum \frac{P_a}{P} \times d_a^s} \times d^s \times k$$

or

$$ISDR = \frac{d}{\sum p_a \times d_a^s} \times d^s \times k$$

where p_a is as defined above.

Example 6.36

For the data in Table 6.6(a), use the alternative formula to calculate the indirect standardised death rate for Tunisia using the age-specific death rate of Mauritius as standard.

Solution

Death rate of Mauritius, $d^M = 6.76$ per 1,000 population;;
Number of deaths in Tunisia, $D = 20,650$.

The death rate of Tunisia is:

$$\begin{aligned} d &= \frac{D}{\sum P_a} \times k \\ &= \frac{20,650}{7,909,555} \times 1000 = 2.61 \end{aligned}$$

The preliminary calculations are done in Table 6.7.

The indirect standardised death rate, ISDR, is

$$ISDR = \frac{d}{\sum \frac{P_a}{P} \times d_a^M} \times d^M$$
$$= \frac{2.61}{6.75} \times 6.76 = 2.61$$

The result is the same as that in Example 6.35. This is because it was just a reformulation of that example in another way to enable us use the alternative formula.

Age Group (Years)	Age-Specific Death rate of Mauritius in 1989 d_a^M	Population of Tunisia in 1989 P_a	Proportion of Population of Tunisia for Tunisia for a $p_a = \frac{P_a}{P}$	Expected Age-specific Death Rate for Tunisia $d_a^M p_a^s$
	2	3	4	$5 = (2) \times (4)$
0–4	0.00549	1,014,143	0.12822	0.00070
5–9	0.00036	1,022,505	0.12927	0.00005
10–14	0.00038	963,423	0.12180	0.00005
15–19	0.00070	832,231	0.10522	0.00007
20–24	0.00088	769,184	0.09725	0.00009
25–29	0.00144	645,086	0.08156	0.00012
30–34	0.00184	533,030	0.06739	0.00012
35–39	0.00281	418,015	0.05285	0.00015
40–44	0.00466	305,403	0.03861	0.00018
45–49	0.00688	273,963	0.03464	0.00024
50–54	0.00951	288,823	0.03652	0.00035
55–59	0.01516	256,087	0.03238	0.00049
60–64	0.02547	200,605	0.02536	0.00065

Age Group (Years)	Age-Specific Death rate of Mauritius in 1989 d_a^M	Population of Tunisia in 1989 P_a	Proportion of Population of Tunisia for Tunisia for a $p_a = \frac{P_a}{P}$	Expected Age-specific Death Rate for Tunisia $d_a^M p_a^s$
65–69	0.03854	150,564	0.01904	0.00073
70–74	0.05638	100,199	0.01267	0.00071
75 +	0.11886	136,294	0.01723	0.00205
Total		7,909,555	1.00000	0.00675

TABLE 6.7 Calculation of ISDR for Example 6.35

Advantages of Indirect Standardised Death Rate

1) ISDR for comparison is more easily employed than DSDR because
   a) it requires only knowledge of the population by age and the total number of deaths in the study population;
   b) it involves fewer computations;
   c) it does not require schedules of specific rates for all the population involved in the comparison;
   d) its results may have smaller sampling errors.

2) The indirect method of standardisation is particularly appropriate when we wish to locate the relative level of mortality of quite a small population for which calculation of age-specific rates would make no sense because of the small number involved.

Disadvantages of Indirect Standardised Death Rate

1) Unlike DSDR, the ISDR is dependent upon the age distribution of the study population. As a consequence, the rates for a number of study populations are not comparable among themselves since the study populations will usually differ in their age distributions.
2) To compute rates for specific causes of death, it is necessary to have available age-specific death rates for each such cause in the standard population. This is not a requirement for age adjustment by the direct method.

6.10 OTHER TYPES OF STANDARDISED MORTALITY MEASURES

There are other types of standardised death measures which are used to compare mortality between two populations. These include the standardised mortality ratio, the comparative mortality figure and the comparative mortality index.

6.10.1 STANDARDISED MORTALITY RATIO

If we look at the definition of ISDR, we may observe that the first term alone is sufficient for purposes of comparison. This gives what is called the standardised mortality ratio. Thus, the standardised mortality ratio (SMR) is defined as the ratio of the observed and expected deaths in the study population:

$$SMR = \frac{\text{Observed (actual) deaths in the study population}}{\text{Expected deaths in the study population}} \times k$$

That is,

$$\begin{aligned} SMR &= \frac{D}{D_E} \times k \\ &= \frac{\sum d_a P_a}{\sum d_a^s P_a} \times k \end{aligned}$$

where the symbols have the usual meaning and $k = 100$.

Example 6.37

For the data of Example 6.35, calculate SMR.

Solution

From Example 6.35,

$$D = 34,930 \quad D_E = 53,36.42$$

Hence

$$\begin{aligned} SMR &= \frac{D}{D_E} \times 100 \\ &= \frac{34,930}{53,346.42} \times 100 = 65.5 \end{aligned}$$

That is, the actual deaths in Tunisia in 1989 was 65.5 percent of its expected deaths based on Mauritius's age-specific death rates in 1989.

Note

The indirect standardised death rate is simply the SMR multiplied by the crude death rate of the standard population:

$$ISDR = SMR \times d^s$$

Uses of Standardised Mortality Ratio

The standardised mortality ratio (SMR) may be used to:

1) Compare the general level of mortality in an occupation or industry with the general level of mortality in the total population;
2) Compare mortality due to a particular cause in an occupation or industry with the mortality due to a particular cause in the total population.

SMR for Particular Occupation

For a particular occupation o, the SMR is defined as:

$$SMR^o = \frac{D^o}{\sum d_a^s P_a^o} \times 100$$

where

D^o = the observed deaths in occupation o;
P_a^o = the population at age a in occupation o;
d_a^s = the age-specific death rate at age a in the total population (standard population).

Taking note that $\sum d_a^s P_a^o$ is expected number of deaths in an occupation or industry o, we may write SMR^o as:

$$SMR^o = \frac{D^o}{D_E^o} = \frac{\text{Number of observed deaths in occupation } o}{\text{Expcted number of deaths in occupation } o} \times 100$$

Thus, SMR^o may be interpreted as the ratio of observed deaths to the expected number of deaths in an occupation or industry o had the age-specific death rate for the entire population prevailed in that occupation or industry.

Example 6.38

Table 6.8(a) gives the data on a particular province of a country. Suppose that 42 civil servants died in that year, calculate SMR for the civil servants.

Age Group (Years)	Total Population	Population of Civil Servants	Number of Deaths in Total Population
15–19	94,963	29,128	53
20–24	107,584	26,921	76
25–29	106,753	22,578	99
30–34	92,847	18,656	139
35–39	99,909	14,630	237
40–44	98,979	10,689	346
45–49	85,477	9,588	504
50–54	74,229	10,108	609
55–59	50,550	8,963	593
Total	811,291	151,261	2656

TABLE 6.8(a) Age Distribution of Total Population, Civil Servants' Population and Deaths of Civil Servants in an Area

Solution

Total number of observed deaths of the Civil Servants $D^o = 42$. The preliminary calculations are in Table 6.8(b).

Age Group (Years)	Total Population P_a^s	Population of Civil Servants P_a^o	Deaths in Total Population D	Age-specific Death Rate in Total Population d_a^s	Expected Number of Deaths of Civil Servants $d_a^s P_a^o$
(1)	(2)	(3)	(4)	(5)=(4)/(2)	(6)=(6) × (3)
15–19	94,963	29,128	53	0.00056	16.25669
20–24	107,584	26,921	76	0.00071	19.01766
25–29	106,753	22,578	99	0.00093	20.93826
30–34	92,847	18,656	139	0.00150	27.92965
35–39	99,909	14,630	237	0.00237	34.70468

Age Group (Years)	Total Population P_a^s	Population of Civil Servants P_a^o	Deaths in Total Population D	Age-specific Death Rate in Total Population d_a^s	Expected Number of Deaths of Civil Servants $d_a^s P_a^o$
40–44	98,979	10,689	346	0.00350	37.36544
45–49	85,477	9,588	504	0.00590	56.53394
50–54	74,229	10,108	609	0.00820	82.92948
55–59	50,550	8,963	593	0.01173	105.14459
Total	811,291	151,261	2656		400.832039

TABLE 6.8(b) Calculation of SMR^o for Example 6.38

The expected deaths of the civil servants is:

$$\sum d_a^s P_a^o = 400.82039$$

That is, if deaths of civil servants were to follow the prevailing age-specific death rates of the entire population, about 400.8 deaths would occur among the civil servants. However, this was not the case and only 42 civil servants died in that year.

Hence,

$$\begin{aligned} SMR^o &= \frac{D^o}{\sum d_a^s P_a^o} \times 100 \\ &= \frac{42}{400.82039} \times 100 = 10.48 \end{aligned}$$

That is, the observed deaths of civil servants in the province was about 10.5 percent of its expected deaths, based on the age-specific death rates prevailing in the province.

SMR for Particular Cause of Death

For a particular cause of death *c* in occupation o, SMR is expressed as:

$$SMR^{oc} = \frac{D^{oc}}{\sum d_a^{sc} P_a^o} \times 100$$

where

D^{oc} = total number of observed deaths in occupation o due to cause *c*;
d_a^{sc} = age-cause-specific death rate at age *a* in total population (standard population).

Example 6.39

Table 6.9(a) gives the data on a particular province of a country. Suppose that 23 drivers died from malaria in that year, calculate SMR of drivers who died from malaria.

Solution

Total number of observed deaths of drivers from malaria, $D^{oc} = 23$.

From the preliminary calculations in Table 6.9(b), expected number of deaths of drivers from malaria is:

$$\sum d_a^{sc} P_a^o = 16.8$$

Age Group (Years)	Total Population	Population of Drivers	Number of Deaths in Total Population
15–19	4,963	111	16
20–24	7,584	1,269	8
25–29	6,753	1,225	5
30–34	9,284	2134	10
35–39	9,990	2231	11
40–44	9,897	1106	23
45–49	8,547	927	34
50–54	7,422	897	9
55–59	5,055	890	12
Total	69,495	10,790	128

TABLE 6.9(a) Age Distribution of Total Population, Drivers' Population and Deaths of Drivers in an Area

Age Group (Years)	Total Population P_a^s	Population of Drivers P_a^o	Deaths due Malaria in Total Population D	Cause-specific Death Rate in Total Population d_a^{sc}	Expected Number of Deaths of Drivers $d_a^{sc} P_a^o$
(1)	(2)	(3)	(4)	(5)=(4)/(2)	(6)=(6) × (3)
15–19	4,963	111	16	0.00322	0.35785
20–24	7,584	1,269	8	0.00105	1.33861
25–29	6,753	1,225	5	0.00074	0.90700
30–34	9,284	2,134	10	0.00108	2.29858
35–39	9,990	2,231	11	0.00110	2.45656
40–44	9,897	1,106	23	0.00232	2.57027
45–49	8,547	927	34	0.00398	3.68761
50–54	7,422	897	9	0.00121	1.08771
55–59	5,055	890	12	0.00237	2.11276
Total	69,495	10,790	128		16.81695

TABLE 6.9(b) Calculation of SMR^o for Example 6.39

That is, if deaths of drivers from malaria followed the distribution by age of deaths from malaria of the entire population, only 16.8 drivers would die from the disease. But this was not so. The number of deaths from malaria that occurred among drivers was 23.

Hence,

$$\begin{aligned} SMR^o &= \frac{D^o}{\sum d_a^{sc} P_a^{oc}} \times 100 \\ &= \frac{23}{16.81695} \times 100 = 136.8 \end{aligned}$$

That is, the observed deaths of drivers from malaria was about 136.8 percent of the expected deaths of drivers from malaria, based on the age-specific death rates of the population in the province.

6.10.2 COMPARATIVE MORTALITY FIGURE

The comparative mortality figure (CMF) is defined as the ratio of the directly standardised death rate of the study population to the crude death rate of the standard population. That is,

$$CMF = \frac{\sum d_a P_a^s}{\sum P_a^s} / \frac{\sum d_a^s P_a^s}{\sum P_a^s} \times k$$

This definition can be simplified to:

$$CMF = \frac{\sum d_a P_a^s}{\sum d_a^s P_a^s} \times k$$

which can be expressed as:

$$CMF = \frac{D_E}{D^s} \times k = \frac{\text{Expected deaths in standard population}}{\text{Observed deaths in standard population}} \times k$$

Similarly, we define the comparative mortality figure for the study population as:

$$\begin{aligned} CMF &= \frac{\sum d_a^s P_a}{\sum d_a P_a} \times k \\ &= \frac{D_E^{study}}{D^{study}} \times k = \frac{\text{Expected deaths in study population}}{\text{Observed deaths in study population}} \times k \end{aligned}$$

Example 6.40

For the data in Table 6.10 (a), calculate the CMF for Mauritius.

Solution

Preliminary calculations are given in columns 6 and 7 in Table 10 (b).
The observed deaths in Mauritius $D^s = 8{,}475$;
The expected deaths in Mauritius $D_E = 13{,}015$.

Age Group (Years)	South Africa (Study)		Mauritius (Standard)	
	Populatio*	Deaths	Population	Deaths
0–4	4,436,683	5,957	94,303	317
5–9	4,898,700	4,344	105,189	25
10–14	5,190,005	3,336	97,740	20
15–19	5,263,274	8,977	102,088	53
20–24	4,392,357	21,883	110,892	76
25–29	4,100,416	41,571	93,797	99
30–34	3,422,110	46,982	99,515	139
35–39	3,216,513	43,177	101,946	237
40–44	2,794,291	36,771	90,406	346
45–49	2,241,976	31,733	77,931	504
50–54	1,779,225	29,674	56,939	609
55–59	1,249,427	25,290	40,491	593
60–64	1,127,147	28,751	33,097	728
65–69	795,652	26,936	25,768	859
70–74	691,433	29,150	21,694	896
75+	830,614	23,862	14,910	1,084
Total	46,429,823	499,268	1,178,848	8,475

TABLE 6.10(a) Age-sex Distribution of Population and Deaths in South Africa and Republic of Mauritius in 2002

Age Group (Years)	South Africa (Study)			Mauritius (Standard)			South Africa
	Population	Deaths	Age-specific death rates	Population	Deaths	Age-specific death rates	Expected Deaths in Mauritius population
1	2	3	4=(3)/(2)	5	6	7=(6)/(5)	8=(4) × (5)
0–4	4,436,683	45,957	0.01036	94,303	317	0.00336	976.83
5–9	4,898,700	4,344	0.00089	105,189	25	0.00024	93.28
10–14	5,190,005	3,336	0.00064	97,740	20	0.00020	62.82
15–19	5,263,274	8,977	0.00171	102,088	53	0.00052	174.12
20–24	4,392,357	21,883	0.00498	110,892	76	0.00069	552.47
25–29	4,100,416	41,571	0.01014	93,797	99	0.00106	950.94
30–34	3,422,110	46,982	0.01373	99,515	139	0.00140	1366.24
35–39	3,216,513	43,177	0.01342	101,946	237	0.00232	1368.48
40–44	2,794,291	36,771	0.01316	90,406	346	0.00383	1189.68

Age Group (Years)	South Africa (Study)			Mauritius (Standard)			South Africa
	Population	Deaths	Age-specific death rates	Population	Deaths	Age-specific death rates	Expected Deaths in Mauritius population
45–49	2,241,976	31,733	0.01415	77,931	504	0.00647	1103.04
50–54	1,779,225	29,674	0.01668	56,939	609	0.01070	949.63
55–59	1,249,427	25,290	0.02024	40,491	593	0.01465	819.59
60–64	1,127,147	28,751	0.02551	33,097	728	0.02200	844.23
65–69	795,652	26,936	0.03385	25,768	859	0.03334	872.35
70–74	691,433	29,150	0.04216	21,694	896	0.04130	914.59
75+	830,614	23,862	0.02873	27,052	2,974	0.10994	777.15238.81
Total	46,429,823	448,394		1,178,848	8,475		13,015

TABLE 6.10(b) Preliminary calculations of Comparative Mortality Figure for Example 6.40

Then,

$$\begin{aligned} CMF &= \frac{D_E}{D^s} \times 100 \\ &= \frac{13,015}{8,475} \times 100 = 153.6 \end{aligned}$$

That is, the death rate in Mauritius in 2002 that could be expected if South Africa age-specific death rates were applied to the age composition of the population of Mauritius in 2002 was 153.6 percent of the actual death rate in Mauritius.

6.11 STANDARDISED DEATH INDICES

6.11.1 TYPES OF STANDARDISED DEATH INDICES

Another measure that is used to directly summarize the differences between two schedules of specific rates is the standardized index. The standardized death index is used to

1) Monitor changes in mortality levels over time;
2) Compare different sets of mortality over time;
3) Compare sets of mortality among different populations.

Following the theory of price index in economic statistics, we may define various form of standardised death indices. Suppose in the calculation of CMF, the study and standard populations are taken to be the populations at two time points of the same area. Then CMF provides a suitable measure to study relative change in mortality between the two periods. We shall refer to this measure as comparative mortality index (CMI), in line with index numbers as defined in economic statistics. It measures the relative level of mortality in an area in current (later) year, as compared with the level of mortality in same area in some past specified (earlier) year. A value greater than 100 indicates an increasing trend in mortality between the two periods.

Suppose, the age-specific death rate d_a is used as price and the population P_a as quantity in the current (study) year. Suppose also that P_a^s and d_a^s are population and age-specific death rates respectively of earlier (base) year, which is considered the standard year. When in the calculation of CMI,

1) the current year of the population is the study population,
2) the base year of the population is the standard population,

then we shall obtain the Laspeyre-type mortality[29] and the Paasche-type[30], depending which period of the population is used as weight.

6.11.2 LASPEYRE-TYPE COMPARATIVE MORTALITY INDEX

When the base year of the population is used to weight the age specific death rate then we have the Laspeyre-type CMI (CMI^L, also refered to as "base-weighted" or "fixed-weighted" mortality index, defined as:

$$CMI^L = \frac{\sum d_a P_a^s}{\sum d_a^s P_a^s} \times 100$$

where

d_a = the age-specific death rate of the current (study) year;
d_a^s = the age-specific death rate of the base (standard) year;
P_a^s = the population of the base (standard) year.

That is, the Laspeyre-type CMI is the ratio of the weighted sum of the age-specific death rates in current (study) year to the weighted sum of the age-specific death rates in base (standard) year, where the weight is the base (standard) year population.

The distinctive feature of the Laspeyres-type CMI is that it uses the population in the base period as the basis for comparison as against the Paasche-type CMI which uses the population in the current period as the basis for comparison (discussed in the sequel). In other words, in computing the CMI, the relative age specific death rate (the ratio of the current age specific death rate to the base-period age specific death rate) is weighted by the population's relative importance to all deaths during the base period. Similar to the Laspeyre's price index in economic statistics, we may write CMI^L as:

$$CMI^L = \frac{\sum d_1 P_0}{\sum d_0 P_0} \times 100$$

where

d_0 and d_1	the age-specific death rates for base (standard) year and current (study) year, respectively;
P_0	the population of the base (standard) year.

Example 6.41

For the data of Table 6.11 (a), calculate the Laspeyre-type CMI.

Age Group (years)	1989 (Standard)		1999 (Study)	
	Population	Deaths	Population	Deaths
0–4	93,983	516	99,949	440
5–9	107,047	39	108,571	27
10–14	107,128	41	95,478	29
15–19	93,887	66	109,075	56
20–24	104,312	92	108,841	92
25–29	104,450	150	94,045	116
30–34	88,918	164	101,212	147
35–39	76,563	215	100,482	253
40–44	54,048	252	86,804	347
45–49	43,483	299	76,031	468
50–54	39,327	374	53,125	462

Age Group (years)	1989 (Standard)		1999 (Study)	
	Population	Deaths	Population	Deaths
55–59	31,206	473	38,056	509
60–64	30,558	779	31,691	710
65–69	22,268	858	24,836	780
70–74	14,314	807	22,085	1,012
75+	15,321	1,821	24,136	2,496
Total	1,026,813	6,946	1,174,417	7,944

TABLE 6.11(a) Population and Deaths in Republic of Mauritius (1989–1999)
Source: *United Nations Demographic Yearbook* (1992–2002), (New York: Statistics Office)

Solution

The preliminary calculations are given in Table 6.11 (b).

Age Group	Standard Year (1989)		Study Year (1999)		Age-specific Death Rate		Actual deaths in standard year	Expected deaths in study year
(Years)	Population (P_0)	Deaths (D_0)	Population (P_1)	Deaths (D_1)	1989 (d_0)	1999 $(d_1$	d_0P_0	d_1P_0
1	2	3	4	5	6	7	8=(6) × (2)	9=(7) × (2)
0–4	93,983	516	99,949	440	0.00549	0.00440	516.00	413.74
5–9	107,047	39	108,571	27	0.00036	0.00025	39.00	26.62
10–14	107,128	41	95,478	29	0.00038	0.00030	41.00	32.54
15–19	93,887	66	109,075	56	0.00070	0.00051	66.00	48.20
20–24	104,312	92	108,841	92	0.00088	0.00085	92.00	88.17
25–29	104,450	150	94,045	116	0.00144	0.00123	150.00	128.83
30–34	88,918	164	101,212	147	0.00184	0.00145	164.00	129.14
35–39	76,563	215	100,482	253	0.00281	0.00252	215.00	192.78
40–44	54,048	252	86,804	347	0.00466	0.00400	252.00	216.06

Age Group	Standard Year (1989)		Study Year (1999)		Age-specific Death Rate		Actual deaths in standard year	Expected deaths in study year
(Years)	Population (P_0)	Deaths (D_0)	Population (P_1)	Deaths (D_1)	1989 (d_0)	1999 $(d_1$	d_0P_0	d_1P_0
45–49	43,483	299	76,031	468	0.00688	0.00616	299.00	267.65
50–54	39,327	374	53,125	462	0.00951	0.00870	374.00	342.01
55–59	31,206	473	38,056	509	0.01516	0.01338	473.00	417.38
60–64	30,558	779	31,691	710	0.02549	0.02240	779.00	684.62
65–69	22,268	858	24,836	780	0.03853	0.03141	858.00	699.35
70–74	14,314	807	22,085	1,012	0.05638	0.04582	807.00	655.91
75+	15,321	1,821	24,136	2,496	0.11886	0.10341	1821.00	1584.41
Total	1,026,813	6,946	1,174,417	7,944			6946.00	5927.40

TABLE 6.11 (b) Preliminary calculations of Laspeyre-type Comparative Mortality Index for Example 6.41

Thus, the Laspeyre-type CMI is:

$$
\begin{aligned}
CMI^L &= \frac{\sum d_1 P_0}{\sum d_0 P_0} \times 100 \\
&= \frac{5927.40}{6946.00} \times 100 = 85.3
\end{aligned}
$$

Interpretation

If the deaths in Mauritius in 1999 were adjusted by the age composition by the country's 1989 population, the expected number of death rate in 1999 would be only 85.3 percent of the observed number of deaths in 1989.

Comment

We have observed that the number of deaths in 1999 was 14.4 percent (7,944/6,946=1.144) higher than that in 1989. However, when the 1999 deaths were adjusted by the age composition of the 1989 population, the expected number of deaths in 1999 was 14.7 percent lower than in 1989. The observation is supported also by the pattern of the age-specific death rates which were consistently lower in 1999 than in 1989.

This result further re-emphsises the importance of adjusted death rate.

Limitation of Laspeyre-type Comparative Mortality Index

As the tendency is for population to increase and mortality to decrease, Laspeyre-type CMI will tend to under-estimate the index. This is because if mortality is declining, we will have $P_a^s \leq P_a$ and $d_a^s > d_a$ at least for a large number of age groups particularly of the younger brackets. The reverse is also true. That is, if mortality is rising, Laspeyre-type CMI would tend to over-estimate the differences in mortality.

In Example 6.41, mortality declined in all age groups except in the last one. It is, therefore, not surprising that the Laspeyre-type CMI is lower than the Paache-type (see Example 6.42).

6.11.3 PAASCHE-TYPE COMPARATIVE MORTALITY INDEX

Suppose in the calculation of CMI, we use as weight the current (study) year population, then we have the Paasche-type CMI (CMI^P), also called a "current weighted index", defined as:

$$
\frac{\sum d_a P_a}{\sum d_a^s P_a} \times 100
$$

where

d_a	=	the age-specific death rate of the current (study) year;
d_a^s	=	the age-specific death rate of the base (standard) year;
P_a	=	the population of the current (study) year.

That is, the Paasche-type *CMI* is the ratio of the weighted sum of the age specific death rates in current (study) year to the weighted sum of the age specific death rates in a base (standard) year, where the weight is the current (study) year population.
Similar to the Paasche's price index in economic statistics, we may

$$CMI^P = \frac{\sum d_1 P_1}{\sum d_0 P_1} \times 100$$

where

d_0 and d_1 = as defined above;
P_1 = the population of the current (study) year.

Example 6.42

For the data of Example 6.41, calculate Paasche-type CMI.

Solution

The preliminary calculations are given in Table 12. Thus the Paasche-type CMI is:

$$\begin{aligned} CMI^P &= \frac{\sum d_1 P_1}{\sum d_0 P_1} \times 100 \\ &= \frac{9289.67}{7944.00} \times 100 = 85.5 \end{aligned}$$

Note that this index, as expected, is higher than CMI^L.

Age Group (Years)	Standard Year (1989)		Study Year (1999)		Age-specific Death Rate		Actual deaths in standard year	Expected deaths in study year
	Population (P_0)	Deaths (D_0)	Population (P_1)	Deaths (D_1)	1989 (d_0)	1999 (d_1	d_0P_1	d_1P_1
1	2	3	4	5	6	7	8=(6) × (2)	9=(7) × (2)
0–4	93,983	516	99,949	440	0.00549	0.00440	548.76	440.00
5–9	107,047	39	108,571	27	0.00036	0.00025	39.56	27.00
10–14	107,128	41	95,478	29	0.00038	0.00030	36.54	29.00
15–19	93,887	66	109,075	56	0.00070	0.00051	76.68	56.00
20–24	104,312	92	108,841	92	0.00088	0.00085	95.99	92.00
25–29	104,450	150	94,045	116	0.00144	0.00123	135.06	116.00
30–34	88,918	164	101,212	147	0.00184	0.00145	186.68	147.00
35–39	76,563	215	100,482	253	0.00281	0.00252	282.17	253.00
40–44	54,048	252	86,804	347	0.00466	0.00400	404.73	347.00
45–49	43,483	299	76,031	468	0.00688	0.00616	522.81	468.00
50–54	39,327	374	53,125	462	0.00951	0.00870	505.22	462.00
55–59	31,206	473	38,056	509	0.01516	0.01338	576.83	509.00
60–64	30,558	779	31,691	710	0.02549	0.02240	807.88	710.00
65–69	22,268	858	24,836	780	0.03853	0.03141	956.95	780.00
70–74	14,314	807	22,085	1,012	0.05638	0.04582	1245.12	1012.00
75+	15,321	1,821	24,136	2,496	0.11886	0.10341	2868.72	2496.00
Total	1,026,813	6,946	1,174,417	7,944			9289.67	7944.00

TABLE 6.12 Preliminary calculations of Paasche-type Comparative Mortality Index for Example 6.41

Interpretation

If deaths in Mauritius in 1989 was adjusted by the age composition of the 1999 population, then actual number of deaths in Mauritius in 1999 was 85.5 percent of the expected number of deaths of the country in 1989.

Limitation of Paasche-type Comparative Mortality Index

When mortality is declining in a country, Paasche-type CMI would tend to over-estimate the index. The reverse is also true.

Marshall-Edgeworth-type Comparative Mortality Index

Marshall-Edgeworth-type CMI uses, as weight (w_a), and is defined as the average of the proportions of persons of corresponding age groups in the base (standard) year (P_a^s) and current (study) year (P_a). Thus,

$$w_a = \frac{1}{2}\left(\frac{P_a^s}{\sum P_a^s} + \frac{P_a}{\sum P_a}\right)$$

where

$\sum P_a^s = P^s$ is total standard population;
$\sum P_a = P$ is total study population.

Thus,

$$w_a = \frac{1}{2}\left(p_a^s + p_a\right)$$

where

p_a^s is the proportion of population in age group *a* in current (study) year;
p_a^s is the proportion of population in age group *a* in base (standard year.

When we replace P_a^s or P_a in Laspeyre- and Paasche-type CMI formulas by (w_a), we obtain the Marshall-Edgeworth-type CMI (CMI^{ME}):

$$CMI^{ME} = \frac{\sum d_a w_a}{\sum d_a^s w_a} \times 100$$

where d_a and d_a^s are as earlier defined.

Example 6.43

For the data of Example 6.41, calculate Marshall-Edgeworth-type CMI.

Solution

The preliminary calculations are given in Table 13.

Age Group (Years)	Age-specific Death Rate		$p_a^s = \frac{P_a^s}{\sum P^s}$	$p_a = \frac{P_a}{\sum P}$	$w = \frac{1}{2}(p_a^s + p_a)$	$d_a^s w_a$	$d_a w_a$
	1989 (d_0)	1999 (d_1)					
1	2	3	4	5	6	7=(1) × (6)	8=(2) × (6)
0–4	0.00549	0.00440	0.09153	0.08511	0.08832	0.00048	0.00039
5–9	0.00036	0.00025	0.10425	0.09245	0.09835	0.00004	0.00002
10–14	0.00038	0.00030	0.10433	0.08130	0.09281	0.00004	0.00003
15–19	0.00070	0.00051	0.09144	0.09288	0.09216	0.00006	0.00005
20–24	0.00088	0.00085	0.10159	0.09268	0.09713	0.00009	0.00008
25–29	0.00144	0.00123	0.10172	0.08008	0.09090	0.00013	0.00011
30–34	0.00184	0.00145	0.08660	0.08618	0.08639	0.00016	0.00013

Age Group (Years)	Age-specific Death Rate		$p_a^s = \frac{P_a^s}{\sum P^s}$	$p_a = \frac{P_a}{\sum P}$	$w = \frac{1}{2}(p_a^s + p_a)$	$d_a^s w_a$	$d_a w_a$
	1989 (d_0)	1999 (d_1)					
35–39	0.00281	0.00252	0.07456	0.08556	0.08006	0.00022	0.00020
40–44	0.00466	0.00400	0.05264	0.07391	0.06327	0.00030	0.00025
45–49	0.00688	0.00616	0.04235	0.06474	0.05354	0.00037	0.00033
50–54	0.00951	0.00870	0.03830	0.04524	0.04177	0.00040	0.00036
55–59	0.01516	0.01338	0.03039	0.03240	0.03140	0.00048	0.00042
60–64	0.02549	0.02240	0.02976	0.02698	0.02837	0.00072	0.00064
65–69	0.03853	0.03141	0.02169	0.02115	0.02142	0.00083	0.00067
70–74	0.05638	0.04582	0.01394	0.01881	0.01637	0.00092	0.00075
75+	0.11886	0.10341	0.01492	0.02055	0.01774	0.00211	0.00183
Total						0.00734	0.00627

TABLE 6.13 Preliminary calculations of Marshall-Edgeworth-type Comparative Mortality Index for Example 6.41

Columns 2 and 3 are taken from columns 6 and 7 of Table 6.11 (b). Thus the composite CMI is:

$$\begin{aligned} CMI^{ME} &= \frac{\sum d_a w_a}{\sum d_a^s w_a} \times 100 \\ &= \frac{0.00627}{0.00734} \times 100 = 85.4 \end{aligned}$$

Fisher's Ideal-type Comparative Mortality Index

Fisher's Ideal-type CMI (CMI^F) is defined as the geometric mean of Laspeyre-type and Paasche-type CMI. Thus,

$$CMI^F = \sqrt{CMI^L \times CMI^P}$$

Or

$$CMI^F = \sqrt{\frac{\sum d_1 P_0}{\sum d_0 P_0} \times \frac{\sum d_1 P_1}{\sum d_0 P_1}} \times 100$$

The CMI^F gives a close approximation of the unknown (CMI) and will be between the Laspeyres and Paasche indices. The CMI^L is normally the upper bound and the CMI^P the lower bound indices.

Example 6.44

For the data of Example 6.41, calculate Fisher's Ideal-type CMI.

Solution

From Example 6.41, $CMI^L = 85.3$.
From Example 6.42, $CMI^P = 85.5$.

Hence,

$$\begin{aligned} CMI^F &= \sqrt{CMI^L \times CMI^P} \\ &= \sqrt{85.3 \times 85.5} = 85.4 \end{aligned}$$

Alternatively,
From Example 6.41,

$$\frac{\sum d_1 P_0}{\sum d_0 P_0} = \frac{6946.00}{5927.40}$$

From Example 6.42,

$$\frac{\sum d_1 P_1}{\sum d_0 P_1} = \frac{9289.67}{7944.00}$$

Hence,

$$\begin{aligned} CMI^F &= \sqrt{\frac{\sum d_1 P_0}{\sum d_0 P_0} \times \frac{\sum d_1 P_1}{\sum d_0 P_1}} \times 100 \\ &= \sqrt{\frac{6946.00}{5927.40} \times \frac{9289.67}{7944.00}} \times 100 = 85.4 \end{aligned}$$

EXERCISES

6.1 The following table (from Central Statistics Office of Mauritius) shows the mid-year population and the number of deaths in the Republic of Mauritius for the period 2000–2004.

Year	Mid-year Population	Number of Deaths
2000	1,186,873	7,982
2001	1,199,881	7,983
2002	1,210,196	8,310
2003	1,222,811	8,520
2004	1,233,86	8,475

Compute the:

1) Crude death rate for each year;
2) Mean annual crude death rate for the period;
3) Annual mean crude death rate for the period;
4) Mid-period crude death rate for the period.

6.2 Distinguish between crude death and standardised death rates? Why is this distinction necessary?

6.3 The table below shows the distribution of the mid-year population and the number of deaths in a certain country in 2002.

1) Calculate the age-specific death rates for the country.
2) Construct an age curve of mortality and comment on the shape.

Age Group (year)	Population	Number of Deaths
0–4	109,900	572
5–9	118,500	39
10–14	106,300	33
15–19	97,950	77
20–24	74,400	83
25–29	50,400	90
30–34	44,500	158
35–39	37,600	252
40–44	40,100	374
45–49	35,350	600
50–54	26,150	613
55–59	23,800	670
60–64	16,150	724
65–69	14,300	762
70–74	11,500	825
75–79	10,000	925
80–84	9,600	635
85+	8,200	975

6.4 The following were data on a certain community in 2000. The live births for the previous year was 2,200 including 1,500 males and 700 females. In that year 150 children under one year of age died out of which 85 were males and 65, females. Compute for the total population and by sex the following:

1) Crude death rate;
2) Specific death rates;
3) Maternal mortality rate;
4) Conventional infant mortality rate;
5) Adjusted infant mortality rate;

6) Neonatal mortality rate;
7) Post-neonatal mortality rate;
8) Foetal death ratio with respect to:
   i) all foetal deaths,
   ii) late foetal deaths;

Vital Events	Number		
	Total	Male	Female
Estimated population as of July 1	100,000	60,000	40,000
Births			
Full term births	2,000	2,000	5,000
Pre-mature births	500	360	240
Total	2,500	2,300	5,240
Foetal deaths			
Under 20 weeks gestation	55	30	25
20 to 27 weeks gestation	34	28	6
28 weeks and more	11	18	3
Total	100	66	34
Deaths			
Under 1 year	160	100	60
Under 28 days	120	75	55
50 years or older	25	16	9
Maternal Deaths	8	6	2
Total of all ages	1,500	1,000	500
Cause of death			
AIDS	50	40	10
Heart diseases	1,100	850	250

9) Foetal death rate with respect to:
   i) all foetal deaths,
   ii) late foetal deaths,
   iii) all foetal deaths from foetal death ratio,
   iv) late foetal deaths from foetal death ratio;
10) Perinatal mortality ratio;
11) Perinatal mortality rate;

12) Perinatal mortality rate from perinatal mortality ratio;
13) Cause-of-death rates for AIDS and heart diseases;
14) Cause-of-death ratios for AIDS and heart diseases;

6.5 For data in Example 6.20, calculate foetal death rate from foetal death ratio computed for all foetal deaths.

6.6 For data in Example 6.22, calculate perinatal mortality rate from the perinatal mortality ratio computed for all foetal deaths.

6.7 The table below shows the population of an area in 2000 and 2003 and the number of deaths of the same area in 2003.

1) Use the direct method to obtain the 2003 adjusted death rate for the area if its 2000 population is used as the standard population.
2) Comment on your result.

Age Group (years)	Population in 2000	Deaths in 2003	Population in 2003
0–4	109,900	734	216,787
5–9	118,500	133	219,206
10–14	106,300	77	152,665
15–19	97,950	80	138,274
20–24	74,400	110	111,318
25–29	50,400	95	89,029
30–34	44,500	108	73,724
35–39	37,600	131	61,709
40–44	40,100	208	68,686
45–49	35,350	171	47,843
50–54	26,150	289	47,120
55–59	23,800	414	34,398
60–64	16,150	500	27,262
65–69	13,300	571	14,572
70–74	8,500	604	18,414
75+	9,230	1,003	17,823

6.8 The table below shows the population and deaths in Mauritius in 2000 and the popultion of Ghana in the same year.

1) Calculate the age-specific death rate for Mauritius in 2000;
2) Construct the mortality curve for the data and comment on the shape;
3) Using the direct method of standardisation,
   a) Obtain the standardised death rate for Mauritius for 2000 if the 2000 population of Ghana is used as the standard population.
   b) Comment on your result;
4) Construct the age-specific mortality curve for Mauritius based on the expected deaths. Compare it with the one constructed in (2) and comment.

Age Group (years)	Population of Mauritius in 2000	Deaths in Mauritius in 2000	Population of Ghana in 2000
0–4	94,303	368	2,769,421
5–9	105,189	26	2,775,206
10–14	97,740	32	2,262,216
15–19	102,088	61	1,883,753
20–24	110,892	87	1,600,820
25–29	93,797	124	1,487,299
30–34	99,515	152	1,206,809
35–39	101,946	220	1,029,765
40–44	90,406	353	886,931
45–49	77,931	424	720,357
50–54	56,939	503	568,369
55–59	40,491	521	355,842
60–64	33,097	681	366,351
65–69	25,768	840	258,709
70–74	21,694	1,033	225,158
75–79	14,910	991	144,830
80–84	7,132	706	140,847
85+	5,010	860	229,396

6.9 The following table shows the distribution of mid-year population and deaths in South Africa in 2003 and 2004 by age and sex.

1) Calculate crude death rate for South Africa in both years for:
   a) Male population,
   b) Female population.
2) Using its 2003 population as standard, calculate, for 2004, the direct standardised death rate for South Africa for the
   a) Male population,
   b) Female population.
3) Calculate the age-sex direct standardised death rate for 2004, using its male and female populations in 2003 as standard.
4) Comment on the results in (1) to (3).

Age Group (Years)	2003 Population of South Africa (Standard)		2004 Population of South Africa (Study)		2004 Deaths in South Africa (Study)	
	Male	Female	Male	Female	Male	Female
0–4	2,216,840	2,219,843	2,616,514	2,559,951	29,454	26,280
5–9	2,446,939	2,451,761	2,583,940	2,535,525	3,129	2,765
10–14	2,584,438	2,605,567	2,545,484	2,506,130	2,105	1,744
15–19	2,594,731	2,668,543	2,475,651	2,448,341	4,602	4,533
20–24	2,156,680	2,235,677	2,358,355	2,320,855	10,188	14,759
25–29	1,979,090	2,121,326	2,179,953	2,111,636	19,479	26,978
30–34	1,633,298	1,788,812	1,852,780	1,843,665	27,977	30,040
35–39	1,492,181	1,724,332	1,401,549	1,449,763	27,751	24,638
40–44	1,305,199	1,489,092	1,226,582	1,312,067	26,034	20,134
45–49	1,022,352	1,219,624	1,056,100	1,157,891	22,702	15,955
50–54	833,726	945,499	834,806	927,175	20,778	13,840
55–59	574,985	674,442	636,046	733,172	17,756	11,844
60–64	478,705	648,442	475,193	575,222	16,712	13,185
65–69	304,126	491,526	329,787	437,665	14,973	13,611
70–74	245,710	445,723	206,198	308,319	13,277	15,199
75+	281,308	549,306	325,280	567,263	18,558	29,447

6.10 The following table shows the distribution of population and deaths in the Republic of Mauritius in 2000 by age and sex and the distribution of population of Ghana by sex in the same year.

1) Calculate the crude death rate for the Republic of Mauritius for the
   a) Male population,
   b) Female population.

2) Calculate the direct standardised death rate for the Republic of Mauritius for the:
   a) Male population,
   b) Female population,
   using the male and female population of Ghana in 2000 as standard.

3) Calculate the age-sex direct standardised death rate for the the Republic of Mauritius population, using the population of Mauritius as standard. Interpret the result.

4) Comment on the results in (1) to (3).

Age Group (Years)	Population of Ghana by Sex in 2000		Population of Mauritius by Sex in 2000		Deaths by Sex in Mauritius in 2000	
	Male	Female	Male	Female	Male	Female
0–4	1,379,770	1,389,651	47,640	46,663	228	139
5–9	1,390,652	1,384,554	53,037	52,152	17	9
10– 14	1,151,131	1,111,085	49,428	48,312	21	11
15–19	961,162	922,591	51,671	50,417	37	24
20–24	763,051	837,769	55,108	55,784	66	21
25–29	695,494	791,805	46,749	47,048	80	44
30–34	566,439	640,370	49,964	49,551	106	46
35–39	490,864	538,901	51,621	50,325	162	58
40–44	443,284	443,647	45,798	44,608	254	99
45–49	377,315	343,042	39,133	38,798	295	129
50–54	279,950	288,419	27,790	29,149	324	179
55–59	182,843	172,999	19,228	21,263	310	211
60–64	177,347	189,004	15,301	17,796	404	277
65–69	129,090	129,619	11,758	14,010	521	319
70–74	106,513	118,645	9,491	12,203	555	478
75–79	74,268	70,562	6,047	8,863	499	492
80–84	66,941	73,906	2,584	4,548	311	395
85+	121,268	108,128	1,408	3,602	298	562

6.11 What is an age-standardised death rate and why is it necessary in demographic analysis? Discuss the various methods for obtaining such a rate and under what conditions is each method applicable?

6.12 How is it different from the comparative mortality figure? Why would you choose the Marshall-Edgeworth-type over the other types.

6.13 The table below shows the distribution by age, the populations of Country A and Country B in 2000 and the number of deaths in Country A in 2000. Assuming the total number of deaths in Country B in 2000 was 98,000, calculate the indirect standardised death rate for Country B using the age-specific death rates of Country A as standard.

Age Group (year)	Population of Country A	Population of Country A	Number of Deaths in Country A	Number of Deaths in Country A
0–4	93,983	2,769,421	516	Not
5–9	107,047	2,775,206	39	available
10–14	107,128	2,262,216	41	
15–19	93,887	1,883,753	66	
20–24	104,312	1,600,820	92	
25–29	104,450	1,487,299	150	
30–34	88,918	1,206,809	164	
35–39	76,563	1,029,765	215	
40–44	54,048	886,931	252	
45–49	43,483	720,357	299	
50–54	39,327	568,369	374	
55–59	31,206	355,842	473	
60–64	30,558	366,351	779	
65–69	22,268	258,709	858	
70–74	14,314	225,158	807	
75–79	15,321	144,830	1,821	
Total	1,026,813	18,912,079	6,946	98,000

6.14 For the data in Example 6.40, calculate the comparative mortality figure for South Africa and interpret the result.

6.15 The table below shows the distribution of population and deaths by age in 2004 for South Africa and Mauritius.

Age Group (Years)	South Africa (Study)		Mauritius (Standard)	
	Population in 2004	Deaths in 2004	Population in 2004	Deaths in 2004
0–4	5,176,465	55,734	97,648	317
5–9	5,119,465	5,894	99,797	25
10–14	5,051,614	3,849	108,949	20
15–19	4,923,992	9,135	94,963	53
20–24	4,679,210	24,947	107,584	76
25–29	4,291,589	46,457	106,753	99
30–34	3,696,445	58,017	92,847	139
35–39	2,851,312	52,389	99,909	237
40–44	2,538,649	46,168	98,979	346
45–49	2,213,991	38,657	85,477	504
50–54	1,761,981	34,618	74,229	609
55–59	1,369,218	29,600	50,550	593
60–64	1,050,415	29,897	35,201	728
65–69	767,452	28,584	28,565	859
70–74	514,517	28,476	20,769	896
75–79	312,251	25,527	16,557	1,084
80–84	580,292	20,331	9,207	884
85+	24,101	27,674	5,402	1,006
Total	46,586,607	565,954	1,233,386	8,475

Calculate the comparative mortality figure for:

1) South Africa;
2) Mauritius.

6.16 The table below gives data on a particular area in 2003. Suppose that 50 teachers died from cholera in that year, calculate the standard mortality ratio for teachers who died from cholera in that area.

Age Group (Years)	Total Population	Population of Teachers	Number of Deaths in Total Population
20–24	17,584	269	4
25–29	16,753	225	6
30–34	19,284	134	10
35–39	19,990	231	15
40–44	19,897	136	19
45–49	18,547	109	28
50–54	17,422	108	17
55–59	15,055	90	15
Total	69,495	10,790	128

6.17 The following table shows the age distribution of population and deaths in South Africa in 2003 and 2004.

Age Group (years)	2003 (Standard)		2004 (Study)	
	Population	Deaths	Population	Deaths
0–4	4,436,683	51,123	5,176,465	55,734
5–9	4,898,700	4,948	5,119,465	5,894
10–14	5,190,005	3,613	5,051,614	3,849
15–19	5,263,274	9,309	4,923,992	9,135
20–24	4,392,357	24,246	4,679,210	24,947
25–29	4,100,416	45,709	4,291,589	46,457
30–34	3,422,110	55,015	3,696,445	58,017
35–39	3,216,513	48,613	2,851,312	52,389
40–44	2,794,291	42,718	2,538,649	46,168
45–49	2,241,976	36,195	2,213,991	38,657
50–54	1,779,225	33,198	1,761,981	34,618

55–59	1,249,427	27,986	1,369,218	29,600
60–64	1,127,147	30,481	1,050,415	29,897
65–69	795,652	28,397	767,452	28,584
70–74	691,433	30,686	514,517	28,476
75+	830,614	78,677	892,543	48,005
Total	46,429,823	540,427	46,586,607	550,914

Calculate and interpret:

1) Laspeyre-type CMI
2) Paasche-type CMI
3) Marshall-Edgeworth-type CMI
4) Fisher's Ideal-type CMI.

7 LIFE TABLE

7.1 INTRODUCTION

7.1.1 WHAT IS LIFE TABLE?

The life table is another and effective way of expressing the death rates experienced by a population during a chosen period of time.

The death rates we have discussed so far are "observed rates" and "standardised rates". The problem in using these rates as levels of death rates is that they reflect the effects of the age distribution of an actual population (as in the case of central rates) or requires the adoption of a standard population for acceptable comparisons of levels of mortality in different populations (as in the case of standardised rates). Besides, both are not true probabilities. The mortality rates (see Sections 6.5–6.7) though are probabilities and are specific by age, relate to a one-year period, but as it will be indicated in the next chapter, reproductivity is expressed in terms of a generation. Again, the discussions in Chapter 6 show the mortality conditions of individual groups or of total populations. To overcome the problems discussed above, the life table is used. It best expresses the pattern of human mortality. It presents a detailed picture of a population that systematically gets depleted through deaths at each age.

Keyfitz (1968) defines the life table as "…a scheme for expressing the form of mortality in terms of probabilities". He further states "It is also a population model, covering the simplest case which is worth discussing: a cohort or group of people born at the same moment, closed to migration and followed through successive ages until they die. Like other successful models, the life table has given its shape to the natural world; we are incapable of thinking of population change and mortality from any other starting point."

The life table (sometimes referred to as the ***mortality table***), is one form of combining mortality rates of a population at different ages into a single statistical model. Ramakumar (1986) defines a life table as "a statistical history of a hypothetical cohort of persons born at an instant of time, followed through till all the members of the cohort are no more".

Mathematically, suppose P_0 individuals are born at an instant of time. Suppose that these individuals are observed at the end of each year. What would happen is that the number in the cohort would decrease to P_1, $P_2, \cdots, P_\omega$, where P_ω is zero, if ω is the highest number of years the last person lived. The sequence P_0, P_1, $P_2, \cdots, P_\omega$ describes the death process in a cohort. That is, the life table model conceptually traces a cohort of new-born babies through their entire life under the assumption that they are subject to the current observed schedule of age-specific mortality rates. The cohort of new-born babies, called the radix of the table, is usually taken to be 100,000.

7.1.2 HISTORICAL BACKGROUND OF LIFE TABLE

The first published table of mortality or life table expectancies is attributed to the Roman jurisconsulate Æ milius Macer, who authored a table to capitalise annuities. The table dates to about the year 225 A.D. Macer's table, though used for its simplicity, was considered arbitrary. A more correct schedule was developed by Ulpian. Ulpian's table was considered more correct than that of Macer because it reflected actual life expectancies. In 1570, Girolamo Cardano proposed that the expectancy of life is a linear decreasing function of age x. In 1662, John Graunt tabulated the number of deaths based on the study "Bill of Mortality" on the city of London in 1658. This was a rudimentary life table which became the origin of the concept of life table as we know it today. However, Graunt's life table was defective because it was based on mortality experience alone.

The first 'modern' mortality table is usually credited to Edmund Halley who in 1693 constructed a life table based on data from the city of Breslau in Poland. Halley's Table was thought to be "…the best as well as the first of its kind". It was based on birth and death data during the years 1687 to 1691 and contained most of the columns of the modern life table. It set the stage for a new era of applications which extended to annuities, pensions and insurance, and had replaced Ulpian's Table after 1600 years of use. Halley's Table became the accepted standard of its time. However, Halley's life table could not be correct because it was based on the assumption that the population had remained stationary[31] which was not entirely correct.

Various attempts at constructing life tables were made in the 17th and 18th centuries on the basis of limited data. Antoine de Parcieux (1746) is well known for his development and calculations of life expectancies. The first to prepare a scientifically correct life table based on both population and death data classified by age was by Milne and was published in 1815.

7.1.3 USES OF LIFE TABLE

Although the basic objectives of life table is to give a clear picture of the age distribution of mortality in a given population group, it has been used widely in a large number of spheres. Today, life table is widely accepted as an important basic material in demographic and public health studies. In the words of William Farr, life table is the 'Biometer' of the population. Among the uses of the life table are the following.

1) The life table is designed essentially for the purpose of measuring mortality, survivorship, and life expectation. It has proved to be the most useful and reliable model to date in the study of these phenomena even though the assumptions underlying its use may not be reasonable in real life situation. There has been no superior method suggested so far. The life table provides answers to questions such as the following:
   a) What is the probability of dying within one year of persons at each age?
   b) What is the probability that persons of specified age will survive a specified number of years?
   c) How many persons, out of selected number of persons living at some initial age, survive on the average to each attained age?

2) The life table is used by demographers, actuaries and many others to:
   a) study longevity, fertility, reproductivity, migration, and population growth,
   b) make projections of population size and characteristics,
   c) make relative comparison of various measures of mortality such as, death rate, expectation of life for two or more different groups of populations,

3) The life table readily permits making mortality allowances for age cohort, eliminating the burden of compiling data on death for age cohorts from annual death statistics by age even when the latter are available.

4. The life table, based on the scientific use of statistical methods, is the pivot on which the whole science of life assurance hinges to:
   a) form the basis for determining the rates of premium necessary to various amount of life assurance;
   b) provide the actuarial scientist with a sound foundation, for converting the insurance business from a mere gambling in human lives to the ability to offer well calculated safeguard in the event of death.

7.1.4 TYPES OF LIFE TABLE

Life Table Classified According to Reference Year

Life tables may be classified as cohort or current according to the reference year of the table.

Cohort Life Table

The cohort life table, also called ***longitudinal life table*** or ***generation life table***, takes a real cohort (group of people) that began life during a specified interval and follows them in subsequent years, until all have died. Thus, the cohort life table is constructed on basis of the sequence $P_0, P_1, P_2, \cdots, P_\omega$.

The main ***advantage*** of the cohort life tables is its conceptual simplicity. In fact it is the cohort life table that agrees with the definition of a life table. Its main ***disadvantage*** is the very long time for which data are required and to which mortality risks are referred. The upper limit of human life is about 100 years, that is, the life span of a cohort represented by ω can be anywhere near 100 years or more. Therefore, the study of the sequence of death process till the last person in the cohort is dead requires long period of waiting. This makes the cohort life table limited in use and only suitable to study mortality of insects and plants that have short life span. Furthermore, even when such tables can be constructed, they represent an amalgam of the mortality experience over a very long period.

Current Life Table

A current life table, also referred to as ***cross sectional life table***, ***period life table*** or ***time-specific life table***, employs data for a single cross section of time to represent an entire generation. In current life table, a sequence $l_0, l_1, l_2, \cdots, l_\omega$ depicting current mortality pattern of a given population is constructed to represent the death process $P_0, P_1, P_2, \cdots, P_\omega$ in the cohort.

The main ***advantage*** of the current life tables is that it provides measures localised in time. For example, it helps us answer a question such as: "What is the change in expectation of life at birth from one year to the next". Its main ***disadvantage*** is its conceptual complexity. In fact we need to make several adaptations to get current life table approximated to the definition of a life table. One approximation is to consider a population as a synthetic cohort (see Section 5.3). The approximations, however, do not belittle the usefulness of the current life table and, in practice, for human populations it is the current life tables that mostly are constructed. Consequently, in this text, by life table, unless otherwise specified, we mean current life table.

Life Table Classified According to Length of Age Interval

Life tables may also be classified according to the length of the age interval in which the data are presented. By this classification, there are two types of life tables: complete (unabridged) and abridged.

Complete Life Table

A ***complete life table*** contains data for every single year of age from birth to the last applicable age. Sometimes, to economise on space, the basic values from complete life tables are presented only for every fifth age.

Abridged Life Table

An ***abridged life table***, on the other hand, contains data by intervals of 5 or 10 years, except in the initial years. It is the abridge life tables that most users frequently encounter.

Life Table Classified According to Number of Characteristics Considered

Life tables may be classified as single decrement or multiple decrement according to the number of characteristics considered in the table.

Single Decrement Table

In a single decrement life table, only one cause of death and only one characteristic are considered at a time. This table is concerned with the general experience of a cohort by age.

Multiple Decrement Table

In a multiple decrement life table, the separate and combined effects of more than one characteristic are described. This table may consider more than one causes of death and/or more than one characteristics at a time.

7.1.5 STATIONARY AND STABLE POPULATIONS

The concepts of stationary and stable populations are important in the discussion of life tables and we introduce them now.

Stationary Population

A population is said to be stationary if it is of constant size and constant age-sex composition over time. Such a population may be conceived of under the following conditions.

1) If every year, the number of births is exactly l_0 (say) and is equal to the number of deaths and these are distributed uniformly throughout the year, and
2) if the population is not affected by emigration or immigration.

That is, for a stationary population, the growth rate is zero. This condition cannot remain the same for a long period of time and so no human population remains stationary, either in its size or in its structure. Populations continue to grow because birth and death rates tend to differ and consequently the age structure too will be constantly changing.

Stable Population

The concept of a stable population is due to A.J. Lotka. A population is said to be stable

1. if it has a fixed age and sex distribution[32],
2. if constant but different death and birth rates are experienced at each age, and
3. if the population is closed to emigration or immigration.

In other words, for a stable population, the overall rates of births and deaths remain constant but different from each other and consequently such a population increases at a constant rate, thus supporting the Malthus Law (Compound Interest Law) of population growth. In particular, if the constant overall birth and death rates are equal, then the population size remains fixed and in this case stable population becomes stationary population. That is, a stationary population is a special case of stable population when the rate of growth of stable population is zero.

7.2 LIFE TABLE FUNCTIONS, DEFINITIONS AND THEIR RELATIONSHIPS

Consider a single individual, aged 0, who lives through age x to an ultimate possible age ω. We define the following life table functions and establish their relationships.

Age: x
Age x means exact age, however, in other tables, it may refer to age interval.

Radix: l_0
It is an assumed number of births at age 0. It is usually called the cohort or radix of the life table, and usually taken as 100,000. This is to make a convenient comparison of various life tables. For mathematical purposes, l_0 is taken as one[33].

Survivors at Age x: l_x
It is the number of persons living at any specified age x in any year out of an assumed number of births l_0. It is referred to as number of survivors.

Survivors at Age $x+1$: l_{x+1}
It is the number of persons living at age $(x+1)$ in any year;

Number of Deaths[34]: d_x
It is the number of persons among the l_x persons (attaining a precise age x) who die before reaching the age $(x+1)$, that is,

$$d_x = l_x - l_{x+1}$$

Probability of Survivorship: p_x
It is the probability that a person aged x survives up to his/her next birthday $x+1$. This is computed as

$$p_x = \frac{l_{x+1}}{l_x}$$

Mortality Rate: q_x
It is the probability that a person of exact age x will die within one year following the attainment of that age. This is obtained as:

$$q_x = \frac{l_x - l_{x+1}}{l_x} = 1 - p_x$$

Thus, q_x is the complementary to the probability of survival, p_x.

It may also be defined as:

$$q_x = \frac{d_x}{l_x}$$

so that

$$d_x = l_x q_x$$

That is, d_x is equal to the number of people who reach age x, multiplied by the probability of dying before reaching the next higher age $x+1$. Since

$$d_x = l_x - l_{x+1}$$

by definition

$$\begin{aligned} l_{x+1} &= l_x - d_x \\ &= l_x - l_x q_x \\ &= l_x(1 - q_x) \\ &= l_x p_x \end{aligned}$$

so that $l_x = l_0 p_x$

Person Years: L_x

It is the number of years lived in the aggregate by the cohort of l_x persons between exact age x and exact age $(x+1)$ of persons alive at age x. It is referred to as person-years lived and is equivalent to the population. It is, therefore, called the ***life-table population***.[35]

Under the stationary conditions, L_x in each year will always be the same. If l_x is assumed linear, that is, when deaths at any age x (except first few years) are assumed to be uniformly distributed, L_x becomes the mid-year population and is given by:

$$L_x = 1.l_x + 0.5(l_x - l_{x+1})$$

Hence,

$$L_x = \frac{1}{2}(l_x + l_{x+1}), \quad \text{for } x \geq 2$$

That is, it is assumed that a person dying between the age x and $(x+1)$, on an average, lives 0.5 years.

We may also define L_x as:

$$L_x = l_x - \frac{1}{2}d_x$$

For ages 0 and 1, the linearity assumption is not valid and in these cases, other approximations are applied.

For age 0, the approximate value of L_x is given by:

$$\begin{aligned} L_0 &= 1.l_1 + 0.3(l_0 - l_1) \\ &= 0.3l_0 + 0.7l_1 \end{aligned}$$

That is, it is assumed that a person dying before the age of one year, on an average, lives 0.3 years.

For age 1, the approximate value of L_x is given by:

$$\begin{aligned} L_1 &= 1.l_1 + 0.4(l_1 - l_2) \\ &= 0.4l_1 + 0.6l_2 \end{aligned}$$

That is, it is assumed that a person dying between the age one year and two, on an average, lives 0.4 years.

It is important to note that the weights adopted for ages 0 and 1 are not to be taken as universally applicable. Even though these may be used typically for developing countries, it is important to derive one for each individual case.

For the older ages, the probabilities of dying tend to be erratic, since they are computed from small populations. Generally, therefore, the probabilities of dying at an old age are obtained by graduation method.[36] However, it is also quite adequate to end the life table with an open age interval, say 75 years and over (75+), without estimating the probabilities of dying, and other life table functions at high ages (over 75 years).

Person Years Lived After x: T_x

It is the number of years lived by the cohort l_x (persons alive at age x) after attaining the age x, that is, the total number of years remaining to the cohort when it reaches age x until the last person dies at age ω. It is usually considered as the total future life time of the l_x persons who reach age x.

The value of T_x for any age x is obtained by cumulating L_x from x to ω. Thus,

$$T_x = \sum_{y=x}^{w} L_y$$

where ω is the highest age attainable.

That is,

$$T_x = L_x + L_{x+1} + L_{x+2} + \cdots + L_w$$

We also have the following relation:

$$L_x = T_x - T_{x+1}$$
$$T_x = L_x + T_{x+1}$$

If we consider the approximation $L_x = \frac{1}{2}(l_x + l_{x+1})$, T_x can also be written as:

$$T_x = \sum_{y=x}^{w-1} \frac{1}{2}(l_x + l_{x+1})$$

Survival "Rate": P_x

The survival rate, is in fact, the probability of surviving between two completed years. It is defined as

$$P_x = \frac{L_{x+1}}{L_x}$$

where L_x, as noted earlier, are the survivors at completed age x.

Using L_x equal to $\frac{1}{2}(l_x + l_{x+1})$, we can write

$$P_x = \frac{l_{x+1} + l_{x+2}}{l_x + l_{x+1}}$$

The probability of surviving from birth (P_b) is

$$P_b = \frac{L_0}{l_0}$$

Expectation of Life:

The expectation of life, which is commonly referred to as the *life expectancy*, is one of the indicators of the health status of a country. It is the expected number (in statistical sense) of years of life remaining at a given age, assuming constant mortality rates..

Curtate Expectation of Life: e_x

The curtate expectation of life, which may be referred to as *curtate life expectancy*, is the average number of complete years of life lived by the cohort l_0 after age x by each of l_x persons attaining that age. It is defined as

$$e_x = \frac{l_{x+l} + l_{x+2} + \cdots + l_w}{l_x}$$

It is called curate expectation of life in the sense that it ignores the actual or average years really lived.

The curtate expectation of life is related to the probability of survivorship by the following relation:

$$p_x = \frac{e_x}{1 + e_x}$$

Complete Expectation of Life: e_x^0

The complete expectation of life, which may be referred to as *complete life expectancy*, is the average number of additional years a person aged x is expected to live under the prevailing mortality conditions. It is defined as

$$e_x^0 = \frac{T_x}{l_x}$$

It is called the complete expectation of life in the sense that it gives the number of years of life entirely completed under the prevailing mortality conditions thus including the fraction of the year survived in the year in which death occurs, which on the average can be taken to be $\frac{1}{2}$ year. Hence

$$e_x^0 = \frac{T_x}{l_x}$$

In general, the expectation of life at age x decreases with age x. However, it is often observed in populations with high infant mortality that the expectation of life at age one year is higher than that at birth. This is due to the fact that in such populations, the risk of dying is extremely high in the first year of life and once the child attains the age of one year, the chance of survival increases.

Expectation of Life at Birth: e_x^0

The expectation of life at birth, also called ***life expectancy at age zero*** or the ***mean length of life***, is the number of years of life a new born child can expect to live. It is defined as:

$$e_0^0 = \frac{T_0}{l_0}$$

Note

1) The higher the expectation of life, the better the health of the country.
2) Expectation of life is higher in developed countries than in developing countries.
3) Women usually have higher expectation of life than men.

The mean length of life is of special significance because it is used to measure

1) the average longetivity of a person in a given community;
2) the average age at death, an index of mortality conditions.

Note

1) Life expectancy is different from longevity. Life expectancy is a statistical concept as defined above. Problems such as war, diseases, famine and poor health can reduce expectation of life while improvements in health and welfare increases it. Besides, life expectancy is an average, and so a particular person may die many years before or many years after their "expected" survival. Longevity refers to the characteristics of the relatively long life span of some members of a population.
2) In actual population observed during a certain period of time, the average age at death is not equal to the mean length of life because of
   a) the impact of migration;
   b) differential mortality experiences;
   c) differential cohort population numbers.

   It is only in a stationary population that the two measures are identical.

Life Table Birth and Death Rates

In a stationary population, where the number of births (B) and the number of deaths (D) are equal, the life table crude birth rate (b^*) is also equal to the life table death rate (d^*) and computed as:

$$b^* = d^* = \frac{l_0}{T_0}$$

where $l_0 = \sum_{x=0}^{\omega} d_x$ and $T_0 = \sum_{x=0}^{\omega} L_x$.

Thus, in a stationary population, the crude death and birth rates are the reciprocal of the expectation of life at birth.

7.3 CONSTRUCTION OF LIFE TABLES

The life table is constructed from census data (which provides the mid-year population) and death registration data from vital registration or retrospective surveys. They are generally constructed for age or for different population subgroups. For example, life tables may be constructed for different sexes, ethnic groups and occupational groups, etc.

7.3.1 ASSUMPTIONS IN CONSTRUCTION OF LIFE TABLE

The following are a few simplified assumptions which are used in the construction of the life tables.

1) The cohort is closed to migration. In other words, death is the only factor causing number of members in the cohort at various ages to reduce.
2) Individuals die at each age according to pre-determined mortality schedule which is fixed and unchanged.
3) the cohort originates from some standard number of births, say 10,000 or 1,000,000 which is called the radix of the life table.
4) Except for the first few years, the deaths are distributed uniformly over the period $(x, x+1)$ for each x. In other words, deaths are uniformly distributed between one birthday and the next.

7.3.2 CONSTRUCTING LIFE TABLE

A typical life table has generally the following columns:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
x	l_x	d_x	q_x	L_x	T_x	e_x°

The various symbols entering in this table are as defined above.

Data on l_x are Available

Suppose we have age x and l_x alone, we can use the relations in Section 7.2 to construct the entire life table.

Example 7.1

The following is the part of a life table for the young population of a community.

x	0	1	2	3	4	5	6
l_x	100	95	80	75	55	40	0

Complete the columns for d_x, q_x, L_x, T_x, and e_x^0.

Solution

Computation of d_x:

$$\begin{array}{lllllll} d_x & = & l_x - l_{x+1} & & & & \\ d_0 & = & l_0 - l_1 & = & 100 - 95 & = & 5 \\ \vdots & & \vdots & & \vdots & & \vdots \\ d_6 & = & l_6 - l_7 & = & 40 - 0 & = & 40 \end{array}$$

The remaining d_x are given in column (3) of Table 7.1.

Computation of q_x:

$$\begin{array}{lllllll} q_x & = & \frac{d_x}{l_x} & & & & \\ q_0 & = & \frac{d_0}{l_0} & = & \frac{5}{100} & = & 0.05 \\ \vdots & & \vdots & & \vdots & & \vdots \\ q_5 & = & \frac{d_5}{l_5} & = & \frac{40}{40} & = & 1.00 \end{array}$$

The remaining q_x are given in column (4) of Table 7.1.

Computation of L_x:

$$\begin{array}{lllllll} L_x & = & \frac{1}{2}(l_x + l_{x+1}) & & & & \\ L_0 & = & \frac{1}{2}(l_0 + l_1) & = & \frac{1}{2}(100 + 95) & = & 97.5 \\ \vdots & & \vdots & & \vdots & & \vdots \\ L_5 & = & \frac{1}{2}(l_5 + l_6) & = & \frac{1}{2}(40 + 0) & = & 20.0 \end{array}$$

Computation of T_x:

$$
\begin{aligned}
T_x &= L_x + L_{x+1} + L_{x+2} + \cdots \\
T_0 &= L_x + L_{x+1} + L_{x+2} + L_{x+3} + L_{x+4} + L_{x+5} \\
&= 97.5 + 87.5 + 77.5 + 65 + 47.5 + 20 &&= 395.0 \\
T_1 &= L_{x+1} + L_{x+2} + L_{x+3} + L_{x+4} + L_{x+5} \\
&= 87.5 + 77.5 + 65 + 47.5 + 20 &&= 297.5 \\
T_2 &= L_{x+2} + L_{x+3} + L_{x+4} + L_{x+5} \\
&= 77.5 + 65 + 47.5 + 20 &&= 210.0 \\
\vdots & \quad \vdots && \quad \vdots \\
T_5 &= L_{x+5} &&= 20.0
\end{aligned}
$$

Alternatively, after obtaining T_0, we can calculate the remaining values of T_{x+1} as follows:

$$
\begin{aligned}
T_{x+1} &= T_x - L_x \\
T_1 &= T_0 - L_0 = 395 - 97.5 = 297.5 \\
T_2 &= T_1 - L_1 = 297.5 - 87.5 = 210.0 \\
\vdots & \quad \vdots \quad \vdots \quad \vdots \\
T_5 &= T_4 - L_4 = 67.5 - 47.5 = 20.0
\end{aligned}
$$

Computation of e_x°:

$$
\begin{aligned}
e_x^\circ &= \frac{T_x}{l_x} \\
e_0^\circ &= \frac{T_0}{l_0} = \frac{395}{100} = 3.95 \\
e_1^\circ &= \frac{T_1}{l_1} = \frac{297.5}{95} = 3.13 \\
\vdots & \quad \vdots \quad \vdots \quad \vdots \\
e_5^\circ &= \frac{T_5}{l_5} = \frac{20}{40} = 0.5
\end{aligned}
$$

The remaining e_x° are given in column (7) of Table 7.1.

The complete life table for the above data is given in Table 7.1.

Age x	l_x	$d_x = l_x - l_{x+1}$	q_x	$L_x = \frac{l_x + l_{x+1}}{2}$	T_x	$e_x^\circ = \frac{T_x}{l_x}$
(1)	(2)	(3)	(4)=(3)/(2)	(5)	(6)	(7)=(6)/(2)
0	100	5	0.05	97.5	395.0	3.95
1	95	15	0.16	87.5	297.5	3.13
2	85	5	0.06	77.5	210.0	2.63
3	75	20	0.27	65.0	132.5	1.77
4	55	15	0.27	47.5	67.5	1.23
5	40	40	1.00	20.0	20.0	0.50
6	0	–	–	–	–	–

TABLE 7.1 Construction of Complete Life Table for Example 7.1

Data on l_x are Not Available

Except l_0 which is arbitrarily chosen, the problem is to obtain the l_x, the reason for which we resort to current life table. In general, the mortality rate q_x is the basic function in the table through which a current life table establishes its connection with real life. It, therefore, serves as the initial function from which all other life table functions are derived. The only other data which are needed are, of course, the radix l_0. The q_x column is thus called the ***pivotal column*** of the life table. Starting with radix l_0 and $q_x, (x = 0, 1, 2 \cdots)$ we have

$$\begin{aligned} d_0 &= l_0 q_0 \quad \text{and} \quad l_1 = l_0 - d_0 \\ d_1 &= l_1 q_1 \quad \text{and} \quad l_2 = l_1 - d_1 \\ \vdots &= \vdots \\ d_x &= l_x q_x \quad \text{and} \quad l_x = l_{x-1} - d_{x-1} \end{aligned}$$

and so on. From these values of $l_x, (x = 0, 1, 2 \cdots)$ the column L_x, T_x and e_x° of the table can now be completed using the relationship in Section 7.2.

Example 7.2

Fill in the blanks in a portion of life table given below:

Age in years	l_x	d_x	p_x	q_x	L_x	T_x	$\mathring{e}_x$
4	92,000	480	?	?	?	3,500,300	?
5	?	400	?	?	?	?	?

Solution

In the usual notations, we have

$$
\begin{aligned}
l_5 &= l_4 - d_4 = 92,000 - 480 = 91,520 \\
p_4 &= \frac{l_5}{l_4} = \frac{91,520}{92,000} = 0.9948 \\
q_4 &= 1 - p_4 = 1 - 0.9948 = 0.0052 \\
l_6 &= l_5 - d_5 = 91,520 - 400 = 91,120 \\
p_5 &= \frac{l_6}{l_5} = \frac{91,120}{91,520} = 0.9956 \\
q_5 &= 1 - p_5 = 1 - 0.9956 = 0.0044 \\
L_4 &= \frac{l_4 + l_5}{2} = \frac{92,000 + 91,520}{2} = 91,760 \\
L_5 &= \frac{l_5 + l_6}{2} = \frac{91,520 + 91,120}{2} = 91,320 \\
e_4^\circ &= \frac{T_4}{l_4} = \frac{3,500,300}{92000} = 38.05 \\
T_5 &= T_4 - L_4 = 3,500,300 - 91,760 = 3,408,540 \\
e_5^\circ &= \frac{T_5}{l_5} = \frac{3,408,540}{91,520} = 37.24
\end{aligned}
$$

There results are presented in Table 7.2.

Age in years	l_x	d_x	p_x	q_x	L_x	T_x	e_x°
4	92,000	480	0.9947	0.0052	91760	3,500,300	38.05
5	91,520	400	0.9956	0.0044	91320	3,408,540	37.24

TABLE 7.2 Construction of a Portion of Complete Life Table for Example 7.2

Example 7.3

Complete the life table of the population of a certain community for the first 9 years ($l_0 = 1,000$).

x	0	1	2	3	4	5	6	7	8
q_x	0.118	0.016	0.030	0.080	0.150	0.700	0.900	0.920	0.980

Solution

Since we are given the values of q_x in order to complete the life table, first of all we shall find the values of l_x $(x = 0, 1, 2, \cdots, 8)$ by using the relations

$$q_x = \frac{d_x}{l_x} \text{ and } l_{x+1} = l_x - d_x$$

We are given $l_0 = 1000$.

$$\begin{array}{lllllll} d_0 & = & l_0 q_0 & = & 1000 \times 0.118 & = & 118.00 \\ l_1 & = & l_0 - d_0 & = & 1000 - 118.000 & = & 882.00 \\ d_1 & = & l_1 q_1 & = & 882 \times 0.016 & = & 14.11 \\ l_2 & = & l_1 - d_1 & = & 882 - 14.110 & = & 867.89 \\ d_2 & = & l_2 q_2 & = & 867.89 \times 0.030 & = & 26.04 \\ \vdots & = & \vdots & & & & \\ l_8 & = & l_7 - d_7 & = & 19.75 - 18.170 & = & 1.58 \\ d_8 & = & l_8 q_8 & = & 1.58 \times 0.980 & = & 1.55 \end{array}$$

The values of l_x and d_x $(x = 0, 1, 2, \cdots 8)$ obtained are given in Table 7.3. The remaining columns of the life table, namely, L_x, T_x and e_x° can now be completed as discussed in Example 7.1. This is given as an exercise.

We have seen above that the complete life table can be constructed if we know the values of l_0 and $q_x, (x = 0, 1, 2, \cdots)$. The values of q_x are obtained from the central mortality rate. The central mortality or death rate is the probability that a person whose exact age is not known but lies in between x and $(x+1)$ will die within one year following the attainment of that age.

x	q_x	l_x	$d_x = l_x q_x$
0	0.118	1000.000	118.000
1	0.016	882.000	14.112
2	0.030	867.888	26.037
3	0.080	841.851	67.348
4	0.150	774.503	116.175
5	0.700	658.328	460.829
6	0.900	197.498	177.748
7	0.920	19.750	18.170
8	0.980	1.580	1.548

TABLE 7.3 Calculation of the Functions l_x and d_x of Example 7.3

Age-specific Death Rates are Available

The age specific death rate is denoted by m_x and is given by the expression

$$m_x = \frac{d_x}{L_x} = \frac{\text{Numer of deaths within age interval } x \text{ to (x +1)}}{\text{Average } l_x \text{ of the cohort in that interval}}$$

where the mean population L_x replaces the population exposed to risk in the denominator L_x in the definition $q_x = \frac{d_x}{l_x}$.

The m_x are computed on the basis of census record and death registration data. From m_x, we obtain q_x as:

$$q_x = \frac{2m_x}{2 + m_x}$$

It should be borne in mind that the construction of the life table from the death registers as outlined above will yield reliable results only if the population has been stationary over a period at least equal to the age of the oldest survivor.

In practice, m_x is not easy to obtain and in its place we use the age specific death rates M_x defined as[37]

$$M_x = \frac{\text{Deaths in a year at age } x}{\text{Mid-year population at age } x}$$

The formula rests on the assumption that deaths between ages x and $x+1$ are evenly distribution. This assumption is not satisfied for early ages especially for $x=0,$ since the risk of dying is generally very high in the first few weeks after birth and continues to be high in the early ages, say, $x=0,1,2$ but declines sharply thereafter.

7.3.3 LIMITATIONS OF COMPLETE LIFE TABLE

The discussions so far concern complete life tables, that is, life tables constructed for every single year of age from birth to the last applicable age. Complete life tables

1) are unwieldy and their computations are tedious;
2) can be constructed at the expense of reliability when single year data are inaccurate or are unavailable.

7.4 ABRIDGED LIFE TABLE

7.4.1 WHY ABRIDGED LIFE TABLE?

We have discussed so far what is called a *complete life table* in which the age interval is a year throughout the table and life table functions such as l_x, q_x, m_x etc., are given for all integral values of x. We construct the abridged life table to overcome the problems faced in the construction of complete life table. Abridge life tables

1) Are less burdensome to prepare and are often more convenient to use. Few people have either the time or the skill for the detailed computation of the complete life table
2) Are found to be adequate and useful in most of the situations encountered in demographic analysis.
3) Require less exact data. Most often data are not adequate to compute a complete life table. The data usually available to us are the specific death rates for sub-groups of the population;
4) Reduce the systematic and random errors by suitably grouping the values of x;
5) Are alternative ways of overcoming graduation using Gompertz or Makehams Law or by some other summation formula.

In an *abridged life table*, as the name suggest, the values of these functions are given either for

1) some integral values of x which are at some distance apart, usually 5 years or 10 years; or
2) age groups of values of x, usually of width 5 years or 10 years.

7.4.2 RELATIONSHIPS AMONG VARIOUS ABRIDGED LIFE TABLE FUNCTIONS

In an abridged life table, the functions carry two subscripts, for example, ${}_nd_x$. The first subscript, n, indicates the number of completed years over which the interval extends. The second subscript, x, indicates the exact age at which the interval commences. Thus ${}_nd_x$ indicates the deaths experienced within the interval extending from x to $x+n$. Similar to the complete life table, a typical abridged life table consist of the following columns.[38]

1) x to $(x+n)$ – exact age interval;
2) l_{x+n} – the number of persons out of a cohort of l_x persons; living at the beginning of the interval x to $x+n$.
3) ${}_np_x$ – the probability that a person aged x survives up to age $x+n$ and is given by

$$ {}_np_x = \frac{l_{x+n}}{l_x} $$

implying that

$$ l_{x+n} = l_x \times {}_n p_x $$

4) ${}_nq_x$ – the probability of the person dying in the age interval $(x \,\text{to}\, x+n)$ and is given by

$$ {}_nq_x = 1 - {}_n p_x $$

or

$$ {}_nq_x = 1 - \frac{l_{x+n}}{l_x} $$

From the age specific death rate of the age group, assuming that l_x is linear,

$$ {}_nq_x = \frac{2n \cdot {}_n m_x}{2 + n \cdot {}_n m_x} $$

where ${}_nm_x$ is the death rate of age group $(x \,\text{to}\, x+n)$

5) ${}_nd_x$ – the number of deaths in the interval $(x \,\text{to}\, x+n)$ and is given by

$$ {}_nd_x = l_x - l_{x+n} $$

We may also deduce that

$$ {}_nq_x = \frac{{}_nd_x}{l_x} $$

so that

$$ {}_nd_x = l_x \times {}_n q_x $$

6) ${}_nL_x$ – the number of person-years lived by the cohort during the interval between x and $x+n$ and is given by

$$ {}_nL_x = \frac{n}{2}(l_x + l_{x+n}) $$

under the condition of linearity of l_x.
If we assume that those who died during an n-year interval live on the average $n/2$ years, then

$$ {}_nL_x = nl_x - \frac{1}{2}({}_nd_x) $$

or

$$ {}_nL_x = nl_{x+1} - \frac{n}{2}({}_nd_x) $$

7) T_x – the number of persons lived after age x, defined as:

$$T_x =_n L_x +_n L_{x+n} +_n L_{x+2n} + \cdots +_n L_y$$

8) e_x^o – complete expectation of life at age x and is given by

$$e_x^o = \frac{T_x}{l_x}$$

or

$$e_x^o = \frac{n}{2}\frac{n(l_{x+n} + l_{x+2n} + l_{x+3n} + \cdots)}{l_x}$$

9) ${}_nP_x$ – probability of surviving between two groups of completed years is given by

$${}_nP_x = \frac{l_{x+n} + l_{x+2n}}{l_x + l_{x+n}}$$

The probability of surviving from birth $({}_nP_b)$ is

$${}_nP_b = \frac{{}_nL_0}{{}_nl_0}$$

7.4.2 METHODS FOR CONSTRUCTING ABRIDGED LIFE TABLE

The method of constructing the abridge life table discussed above assumes that l_x is linear. It has been verified that when ${}_nm_x > 0.4$ for $n = 5$, ${}_nq_x > 1$, which is clearly incompatible since ${}_nq_x$ is a probability. Other methods suggested among them are:

1) Reed-Merrell's Method;
2) Greville's Method;
3) King's Method;
4) Keyfitz-Frauenthal's Method;
5) Chieng's.

These have not been discussed here. Refer to other text (Pressat, 1972; Shryock, *et al*, 1976; Ramakumar, 1986).

All these methods suffer from one defect or another. The Reed – Merrel's method on the whole, gives fairly accurate results and to date, it is widely adopted for constructing abridge life tables. King's method is old and has practically no use in the present day context.

EXERCISES

7.1 Refer to Example 7.2. Compute L_x, T_x and e^o_x.

7.2 The following is part of a life table. Complete d_x, q_x, L_x, T_x and e^o_x.

x	0	1	2	3	4	5	6
l_x	100	95	85	70	65	35	0

7.3 Fill in the blanks in a portion of the life table given below:

Age in years	l_x	d_x	p_x	q_x	L_x	T_x	e°_x
4	954,445	?	?	?	?	48,500,256	?
5	920,491		?	?	?	?	?

7.4 Fill in the blanks which are marked with question marks in the following table:

Age x	l_x	d_x	q_x	p_x	L_x	T_x	e°_x
34	450,000	?	?	?	?	?	26
35	420,000	-	-	-	-	?	

7.5 Show that

1) $e^o_x = e_x + \frac{1}{2}$
$e^o_x = \frac{1}{2} + \sum_{n=1}^{w} {}_np_x$

2) $e^o_x = e_x + \frac{1}{2}$
$e^o_x = \frac{1}{2} + \sum_{n=1}^{w} {}_np_x$

where ${}_np_x$ is the probability of survival from age x to $n + x$

7.6 Show that

$$m_x = \frac{2m_x}{2 + m_x}$$

7.7 The complete expectation of life at age 25 and 26 for a particular group of people are, respectively, 20.54 and 21.44 years and that the number living at age 25 is 32,345. Find the number that attains the age 26.

7.8 Complete the life table of the population of a certain community for the first 9 years $(l_0 = 1,000)$:

x	0	1	2	3	4	5	6	7	8
q_x	0.236	0.032	0.060	0.160	0.300	0.800	0.500	0.710	0.850

8 MEASURES OF FERTILITY

8.1 INTRODUCTION

The term "fertility", "natality", and "birth", are often used synonymously even though "natality" is more general referring to the demographic phenomena in connection with birth. In particular, it may be considered from the point of view of the individuals who are born or from the point of view of mothers who give birth to a child (or of couples who conceive). On the other hand, we use the term "fertility" when we are studying the circumstances of human procreation in more detail. In fertility analysis, we often refer to total births, which include live births and "stillbirths". However, since it is only a live birth that augments the existing population, only live births are considered in measuring fertility, thus excluding stillbirths. Unless otherwise specified, in this and subsequent chapters, the word "births" will be used to mean "live births".

8.1.1 DEFINITION OF CONCEPTS IN FERTILITY MEASUREMENT

We shall give definitions of certain vital events as are recommended by the Statistical Commission of the United Nations (1955) and World Health Organisation (1950).

Live Birth

Live birth is the complete expulsion or extraction from its mother of a product of conception, irrespective of the duration of pregnancy, which, after such separation, breathes or shows any other evidence of life, such as beating of the heart, pulsation of the umbilical cord or definite movement of voluntary muscles, whether or not the umbilical cord has been cut or the placenta is attached.

In order to have live birth, a woman has to pass through three stages, namely, sexual union, conception and parturition.

Sexual Union

In order to become pregnant, a woman must first have sexual intercourse with a man[39]. An arrangement between a man and a woman where sexual intercourse takes place is called a *sexual union*. The frequency of sexual intercourse is called *coital frequency*.

Conception

In order to give birth to a child, a woman must become pregnant. The state of pregnancy is what is called ***conception***.

Parturition

A woman must successfully complete the period of pregnancy (or gestation) and give birth to the child. The desire to bring forth or the process of giving birth is referred to as parturition.

Fertility

Fertility refers to the actual bearing of children or occurrence of live births. It is more easily measured for women because they and not men actually have the babies. Fertility rate, therefore, refers to the relative frequency with which births actually occur within a given population.

Natality

Natality is the study of the total process of fertility and the way it affects population change.

Fecundity

Fecundity refers to the physiological capacity to bear children, irrespective of whether or not children have been brought forth. It is, therefore, a measure of the probability that a woman will conceive within a menstrual cycle in the absence of contraceptives and postpartum abstinence. In fact, fecundity is a biological maximum value and, therefore, provides an upper bound for fertility. A partial reduction in fecundity means that the potential for offspring has declined but is not eliminated. A fecund woman may not necessarily be fertile, for example, if she is regulating her fertility by abstaining from sexual intercourse, or using contraceptive.

Sterility

Sterility or infecundity is the lack of ability by a man, woman, or a couple to conceive or procreate. In such a situation no offsprings or further offsprings are possible. Sterility may be voluntary or involuntary.

Involuntary sterility is mainly caused by age. Fecundity, as well as age-specific fertility rates, reach a peak for women aged 20–29 years, then fecundity declines until menopause, around age 50, when the woman becomes permanently sterile. Involuntary sterility may also be caused by impairment of the reproductive system. At the beginning of the reproductive period women may be less fecund because their ovulation is irregular, and this contributes towards adolescent sterility. Involuntary sterility can also be related to extreme hunger and to venereal diseases.
Voluntary sterility is largely the result of a contraceptive or medical operation on one of the marital partners. Amongst the kinds of operations usually performed for contraceptive reasons are the vasectomy for men and tubal ligation for women.

Sterility may also be classified as primary or secondary. *Primary sterility* is when a person has never been able to produce a child. *Secondary sterility* is when a person, after one or more children have been born, is not able to produce a child.

Puberty

Puberty is the period of life when the sexual organs of a girl or a boy mature so that he or she becomes able to have children.

Menstruation

Menstruation is a period of physiologic bleeding of a woman, occurring at approximately 4-week intervals, and having its source from the uterine inner lining.

Menarche

Menarche is the first menstrual period of a girl. The age of menarche is the age at which a girl has her first menstrual period.

Menopause

A woman is said to have reached menopause when she stops having menstrual periods.

Reproductivity

Reproductivity is the extent to which a group is replacing itself by natural processes. The ***reproductive period*** (also called ***childbearing age*** or ***fertility age***) of a woman is the period from menarche to menopause. For purpose of demographic analysis, the reproductive period is generally defined as beginning at age 15 and ending at age 50 or completed age 49.

Gravidity

Gravidity is the number of pregnancies a woman has had whether or not they resulted in live births.

8.2 FERTILITY MEASUREMENT

Fertility is ascertained from the data collected by registration of births and is measured as the frequency of births in a population.

8.2.1 USES OF FERTILITY MEASUREMENTS

The measures of fertility are necessarily the devices to quantify the birth performance of a population. They are important in determining the size and structure of the population. Information on fertility is, therefore, critical for the management of the population for social and economic development.
Fertility measures are used to:

1) compare the fertility levels of a number of populations during a particular time interval;
2) exhibit a time trend in fertility in a population.

8.2.2 COMPARISON OF FERTILITY AND MORTALITY MEASUREMENTS

The measurement of fertility is not as direct as in the case of mortality.

1) Fertility deals with only a section of the population.
   i) It deals with only the female population. Mortality deals with both males and females.
   ii) Even among the female population, fertility affects only those in the reproductive age. Mortality affects population of all ages.

3) Fertility does not deal with all women in the reproductive age group because fecundity is a varying factor. With regard to mortality, all sections of the population are subject to mortality, that is, everybody succumbs to mortality.
4) Fertility may be "controlled" or "natural". ***Controlled fertility*** is when the woman consciously adopts contraception or other means to prevent births while ***natural fertility*** is fertility in the absence of any form of contraception. With regard to mortality, lifestyle helps to control it only to a limited extent. Death will definitely take place no matter what.
5) Unlike mortality where it is easy to establish a one-to-one correspondence between deaths and persons, it may not be possible to do so between births and women. This is because
   i) a conception may result in foetal wastage (abortion or still birth) or in single or multiple live births;
   ii) a woman can give birth to more than one child during her life time.

3) Unlike mortality which can occur at any age, although the force of mortality is greater at the very young and older ages, fertility causes an increase of population at only one point, that is, births of persons aged zero.

8.2.3 TYPES OF FERTILITY MEASURES

Fertility measures can be classified into "period" measures and "cohort" measures.

Period measures are based on a cross-section of the population in one year. They include Crude birth rate (CBR), General fertility rate (GFR) and Child-Woman Ratio (CWR). ***Cohort measures*** are based on data which follow the same people over a period of time. They include Age-specific fertility rate (ASFR), Total fertility rate (TFR), Gross Reproduction Rate (GRR) and Net Reproduction Rate (NRR).

The details are discussed in this and the next chapters.

8.3 CRUDE BIRTH RATE

The crude birth rate (CBR) is one measures used to calculate the frequency or speed by which population is increasing. It is the direct measure of population change due to fertility. It is the simplest and commonest measure of fertility.

8.3.1 DEFINITION OF CRUDE BIRTH RATE

The crude birth rate measures the number of births in a year per 1,000 population. It is defined as:

$$b = \frac{B}{P} \quad \times \quad k$$

where

b = crude birth rate;
B = total number of live births during a year;
P = mid-year population;
k = 1, 000

The CBR usually lies between 10 and 55 per thousand population.

Example 8.1

The mid-year population of Mauritius in 1989 was 1,064,000. In that year 20,875 children were born. Calculate the crude birth rate.

Solution

$$P = 1,064,000 \quad \text{and} \quad B = 20,875$$

$$b = \frac{B}{P} \times 1,000$$

$$= \frac{20,875}{1,064,000} \times 1000 = 19.6$$

Interpretation

There were 19.6 births per 1,000 mid-year population in Mauritius.

8.3.2 AVERAGE CRUDE BIRTH RATE

There are variations in the number of births from year to year even when the potential risk remains the same. For example, if, on one hand, a few number of births take place in a particular year, there is the tendency that there will be more births in the following year. On the other hand, if in a year, a large number of births is experienced in the population, many women will not be able to give birth in the following year because the exposed population gets reduced owing to biological causes such as postpartum amenorrhea, secondary sterility and gestation period. Consequently, the number of births will be reduced even when the size of the population might not have changed much. To overcome this difficulty in measurement, we calculate the average birth rate. Similar to the crude death rate (Section 6.2), we may calculate the following averages:

Method 1: Mean Annual Crude Birth Rate

$$\bar{b} = \frac{1}{n}\left(\frac{B_1}{P_1} + \frac{B_2}{P_2} + \cdots + \frac{B_n}{P_n}\right) \times 1,000$$

Method 2: Annual Average Birth Rate

$$\bar{b} = \frac{B_1 + B_2 + \cdots + B_n}{P_1 + P_2 + \cdots + P_n} \times 1,000$$

Method 3: Mid-period Crude Birth Rate

$$\bar{b} = \frac{\frac{1}{n}\left(B_1 + B_2 + \cdots + B_n\right)}{P_{n/2}} \times 1000$$

where $P_{n/2} =$ the mid-period population (or usually a census figure).

Exercise 8.1 is meant to illustrate these three methods and the reader is asked to work through it based on Examples 6.2, 6.3 and 6.4.

8.3.3 ADVANTAGES AND DISADVANTAGES OF CRUDE BIRTH RATE

The crude birth rate is analogous to the crude death rate (subsection 6.2.1), and shares the same advantages and shortcomings.

Advantages of Crude Birth Rate

1) It is simple to calculate.
2) It is a simple concept and can easily be explained to laymen.
3) It provides an index of the relative speed at which additions are being made through child birth.

Disadvantages of Crude Birth Rate

The crude birth rate suffers from similar limitations as those of the crude death rate.

1) It is a crude indicator of fertility because the denominator (mid-year population) is not really the population at risk as it includes men, children or women who are not in the reproductive ages.
2) It is not a good index for comparing levels of fertility because it is affected by the composition of the population with regard to age, sex and marital status. Thus,
   i) If there is another unusually large proportion of the population at the childbearing ages, we would expect birth rate to be high, even though each mother was having relatively few children. Such a high birth rate would be deceptive, because as the structure of the population changes and the proportion at the childbearing ages drops, the birth rate will fall, even though the same number of children born per mother remains the same;
   ii) All things being equal, a population with an unusual concentration of women in the childbearing ages will have higher crude birth rates than another population that may have the same rate of childbearing per woman but that has small proportion of women in the childbearing ages.

3) Crude birth rate would tend to underplay changes in fertility because:
   i) It does not tell whether changes are due to a tendency for people to have more or less children or merely reflects a temporary change in the ratio of the childbearing population to the total;
   ii) Every birth is counted directly as an event and therefore as an addition to the population; consequently, both the numerator and the denomerator of the crude birth rate get increased and the rate will not reflect the actual chance of childbearing.

In the subsequent sections we shall discuss the term "fertility rates" to refer to the incidence of births within the female population aged 15–49.

8.4 GENERAL FERTILITY RATE

The crude birth rate, as indicated earlier, suffers from the limitation that it relates the total number of live births to the total mid-year population. But we know that the total number of live births depends on the population of women of the fertile age and not on the entire population.

To overcome this limitation, we use the term "fertility rates" to refer to the incidence of births within the female population aged 15–49. One of such rates is the general fertility rate.

8.4.1 DEFINITION OF GENERAL FERTILITY RATE

General fertility rate (GFR), simply referred to as "fertility rate", *f*, is a single ratio of all births to the number of women of fertile age (childbearing age):

$$GFR = \frac{B}{W_x} \times k$$

where B = as defined above;
W_x = mid-year population of women of childbearing age ($x = 15$ to 44 or $x = 15$ to 49);
k = 1,000.

In this text, unless otherwise stated, by women of fertile age (childbearing age) we mean women from 15 to 49 years of age.

GFR is a "general" ratio in the sense that it attributes all births to all women within these age limits, without further distinctions among them. This places it somewhere between crude birth rate and age-specific fertility rate (see below).

It is important to note that GFR can be computed in situations in which the registration of births and the enumeration of population are satisfactory.

Example 8.2

Using the data in Example 8.1, it was observed that the mid-year population of women between the ages 15 and 49 in Mauritius in 1989 was 284,229. Calculate the general fertility rate.

Solution

$$\begin{aligned} GFR &= \frac{B}{W_{15-49}} \times 1,000 \\ &= \frac{20,875}{284,229} \times 1,000 = 73.4 \end{aligned}$$

There were 73.4 births per 1,000 women of childbearing age in Mauritius in 1989.

8.4.2 ADVANTAGES AND DISADVANTAGES OF GENERAL FERTILITY RATE

Advantages of General Fertility Rate

1) The denominator of the GFR, when compared to the crude birth rate, approximates the number of persons actually exposed to the risk of bearing a child.
2) GFR is a typical type of standardised rate, standardised for the proportion of women in reproductive ages.
3) GFR is useful in situations in which direct evidence of births by age of parents is lacking.
4) GFR is mainly useful in comparing populations with quite dissimilar age compositions.
5) GFR is particularly useful in the case of studies and comparisons of small human groups (for example, small geographic units).

Disadvantages of General Fertility Rate

1) GFR is not adequate for a very detailed examination of fertility patterns because it ignores the age distribution of women in the reproductive age-group. It, therefore, conceals a large amount of variation according to age. Among the various populations with similar pattern of fertility, a population with higher proportion of fertile age (women in the ages of fertility) peak will produce a higher value of GFR and vice versa.
2) It is rather crude because all unmarried women are included in the denominator. Of course, in most part of the world it is not uncommon to have unmarried women giving birth to children.

8.5 AGE-SPECIFIC FERTILITY RATE

Although the GFR is an improvement over CBR, it gives only a general view of the fertility of the childbearing age group (15 to 49 years). But the rate of childbearing is not uniform throughout childbearing ages. A detailed and more meaningful analysis of the pattern of fertility in a population is provided by the fertility rates calculated for different childbearing ages. This gives what is being referred to as age-specific fertility rates (ASFR).

The ASFR may be calculated separately for single ages x or for age groups a.

8.5.1 DEFINITION OF AGE-SPECIFIC FERTILITY RATES FOR SINGLE AGES

The ASFR, calculated separately for single ages x, are given as:

$$f_x = \frac{B_x}{W_x} \times k$$

where

f_x	=	age-specific fertility rate for single age x;
B_x	=	number of births to women of age x;
W_x	=	mid-year population of women in age x;
k	=	1,000.

Limitation of Age-specific Fertility Rates for Single Ages

A set of fertility rates calculated by single years is:

1) too detailed and clumsy for comparative purposes, and
2) affected by the misreporting of ages by mothers.

8.5.2 DEFINITION OF AGE-SPECIFIC FERTILITY RATES FOR AGE GROUPS

The 5–year age groups are more commonly used in demographic literature in calculating age-specific rates and hence ASFR is defined as the number of births to women of a given age group per k women in that age group a[40]:

$$f_a = \frac{B_a}{W_a} \times k$$

where
f_a = age-specific fertility rate for age group a;
B_a = number of births to women of age group a;
W_a = mid-year population of women in age group a;
k = 1,000.

Example 8.3

Table 8.1(a) shows the mid-year population and the number of births by age of women in Mauritius in 1989. Calculate the age-specific fertility rates.

Age group	Mid-year Women Population	Number of Births
15–19	46,417	2,028
20–24	51,462	6,927
25–29	51,580	6,408
30–34	44,906	3,460
35–39	39,286	1,393
40–44	27,741	303
45–49	22,841	19

TABLE 8.1(a) Number of Births by Age of Women in Mauritius, 1989

Solution

$$\begin{aligned} f_{15-19} &= \frac{B_{15-19}}{W_{15-19}} \times 1,000 = \frac{2,028}{46,417} \times 1,000 = 43.7 \\ f_{20-24} &= \frac{B_{20-24}}{W_{20-24}} \times 1,000 = \frac{6,927}{51,462} \times 1,000 = 134.6 \\ \vdots & \quad \vdots \\ f_{40-44} &= \frac{B_{40-44}}{W_{40-44}} \times 1,000 = \frac{303}{27,741} \times 1,000 = 10.9 \\ f_{45-49} &= \frac{B_{45-49}}{W_{45-49}} \times 1,000 = \frac{19}{22,841} \times 1,000 = 0.8 \end{aligned}$$

The results are presented in Table 8.1(b).

Age-group (Years)	Mid-year women Population	Number of births	Age-specific fertility rate per 1,000
1	2	3	4 = (3)/(2) × 1, 000
15–19	46,417	2,028	43.7
20–24	51,462	6,927	134.6
25–29	51,580	6,408	124.2
30–34	44,906	3,460	77.0
35–39	39,286	1,393	35.5
40–44	27,741	303	10.9
45–49	22,841	19	0.8

TABLE 8.1(b) Age-specific Fertility Rates for Example 8.3

Interpretation

If 1,000 women started their childbearing periods together at age 15, among them they would produce 43.7 babies every year for the five years until they were 20, then 134.6 babies every year for the five years until they reached 25 and so forth.

8.5.3 PATTERN OF AGE-SPECIFIC FERTILITY RATES

In human populations, the fertility rate appears to have a standard pattern. It is an asymmetrical bell-shape. It varies markedly with age, indicating zero value before age 10, low but increasing positive values between 15–19 but increasing sharply to a peak mostly in the 25–29 age group and sometimes in age group 20–24 and ultimately declining, thereafter, to zero at age around 50 years.

Example 8.4

For the age-specific fertility rates for Mauritius computed in Example 8.3, construct a line graph (fertility curve) and comment on the shape.

Solution

The fertility curve is depicted in the Figure 8.1.

The fertility curve for Mauritius follows the standard pattern described above. It is comparatively low before age 20 but increases faster thereafter and declines after age group 30–34 till it gets to almost zero at age group 45–49.

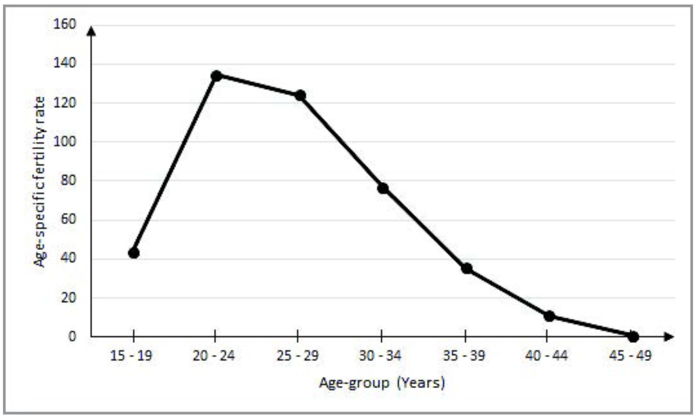

Figure 8.1 Age-specific Fertility Curve for Data in Example 8.3

8.5.4 ADVANTAGES AND DISADVANTAGES OF AGE-SPECIFIC FERTILITY RATES

Advantages of Age-specific Fertility Rates

1) For a given data and population group, the age-specific fertility rates serve as a basis for a detailed comparison, with corresponding rates for other population groups that are unaffected by difference between the groups in age-sex compositions.
2) ASFRs reveal the distribution of frequencies of births among women according to age.
3) ASFRs represent the most useful single step in analysing the fertility performance of a calendar year. It permits the study of fertility in terms of real cohorts of women, tracing out their fertility behaviour as they passed through life.
4) It is a large improvement in precision, because these rates are not significantly distorted by variations of age composition either in the total population or among the women of childbearing ages.
5) ASFRs are the raw materials for computing other important measures, such as the total fertility rate (Section 8.6), the average age of childbearing (Section 8.9), the standardised fertility rates (Section 8.11), and the net reproduction rate (Chapter 9).

Disadvantages of Age-specific Fertility Rates

There are the same objections to the use of specific fertility rates as there are to the use of specific mortality rates.

1) ASFR are difficult to interpret as they involve referring to a different stage for each age group of women.
2) One has to look at several number of figures to make comparisons over a series of years.

8.6 TOTAL FERTILITY RATE

The total fertility rate (TFR) is one of the most important fertility measures in understanding a population. It summarises the pattern of fertility exhibited by the age-specific fertility rates and presents a single index of total fertility which answers as nearly as possible the question: "How many children are women having nowadays"? It assumes that

1) mortality is constant, that is, women will survive through their childbearing years;
2) fertility is constant, that is, women will have the same birth rates over their lifetimes as women in different age cohorts in that same population have had in that year; and
3) there is no migration.

TFR may be computed using single years x or age groups a and may be expressed as per woman or per a cohort of 1,000 women.

8.6.1 DEFINITION OF TFR FOR SINGLE AGES

TFR for single ages x is defined simply as the sum of the age-specific fertility rates of women over their reproductive span (i.e., all ages of the childbearing period):

$$TFR = \sum \frac{B_x}{W_x}$$

where $B_x =$ the number of births to women aged x, denoted as W_x in a given year $(x = 15, 16, 17, \cdots, 49)$.

Total fertility rate, as defined above, indicates "the average number of children a woman would have at the end of the reproductive years if she were to bear children at prevailing age-specific fertility rates while living through the entire reproductive period". To put it in another way, the TFR of a given year indicates the total number of children who would be born by a woman, if she were to behave throughout her childbearing age the way all or other women did in that year (i.e., if she experienced that year's fertility rates for women aged 15, for women aged 16, for women aged 17, etc., all the way up to age 49.

For a cohort of 1,000 women, the TFR for single age x may be computed as:

$$TFR = \sum \frac{B_x}{W_x} \times 1,000$$

or

$$TFR = \sum_{x=15}^{49} f_x$$

where f_x is as defined above.

This definition of TFR gives an estimate of the number of children a cohort of 1,000 women would bear in their life time at the prevailing age-specific fertility rates, if none of them dies before crossing the childbearing age.

8.6.2 DEFINITION OF TFR FOR EQUAL INTERVAL OF AGE-GROUPS

Usually TFR is calculated for age groups. If the age groups have equal intervals $(a = 15-19, 20-24, \cdots, 44-49)$, then TFR per woman is computed as:

$$TFR = n \sum_{a=15-19}^{m} \frac{B_a}{W_a}$$

where n = common interval width of the age-groups,
m = age-group 45–49;
B_a = number of births in age group a;
W_a = mid-year population of women of childbearing age 15–49.

For a cohort of 1,000 women, the TFR for equal interval of age groups may be computed as:

$$TFR = n \sum_{a=15-19}^{m} \frac{B_a}{W_a} \times 1,000$$

or

$$TFR = n \sum_{a=15-19}^{m} f_a$$

where f_a = age-specific fertility rate for age group a.

Example 8.5

For the data in Example 8.3, calculate the total fertility rate.

Solution

Since the age-specific fertility rates are for 5-year age groups

$$TFR = 5 \sum_{a=15-19}^{m} f_a$$

But from Example 8.3,

$$\begin{aligned} \sum_{a=15-19}^{m} f_a &= 43.7 + 134.6 + 124.2 + 77.0 + 35.5 + 10.9 + 0.8 = 426.7 \\ \therefore \quad TFR &= 5(426.7) = 2,134 \end{aligned}$$

Interpretation

If 1,000 women in Mauritius started their childbearing periods together at age 15 they would produce 2,134 babies before they reached age 50 if the 1989 age-specific fertility rates were to continue.

Alternative Interpretation

If the 1989 age-specific fertility rates in Mauritius were to continue, a woman in Mauritius on the average would give birth to about 2 children during her childbearing years.

8.6.3 DEFINITION OF TFR FOR UNEQUAL INTERVAL OF AGE-GROUPS

If the age groups are not of equal intervals then

$$TFR = \sum_{a=15-19}^{m=45-49} n_a f_a$$

where n_a = interval width of the age-group a.
That is, for unequal intervals, the TFR is the sum of the age-specific fertility rates for all ages multiplied by the size of the class interval into which the ages were grouped.

Example 8.6

Table 8.2 (a) shows the data on the age-specific fertility rates of Ghana as of 1983–1988. (For illustration purposes, some age groups have been combined so as to have unequal class intervals). Calculate the total fertility rate.

Age group	Age-specific fertility rate per 1,000 women, f_a
1	2
15–19	124.0
20–24	258.0
25–34	263.0
35–39	195.0
40–49	118.5

TABLE 8.2 (a) Age-specific Fertility Rates of Ghana, 1983–88

Solution

Table 8.2 (b) shows the preliminary calculations of the total fertility rate.
From column 4, we obtain TFR as the sum of the age-specific fertility rates:

$$TFR = \sum_{a=15-19}^{m=45-49} n_a f_a = 6,700$$

That is, if the age-specific fertility rates observed in Ghana in 1983–1988, were to continue, a hypothetical group of 1,000 women would have a total of 6,700 children by the time they reached the end of the reproductive period taken as 49 years, assuming all of them survived to that age.

Age-group (Years)	Age-specific fertility rate per 1,000 women, f_a	Class width n_a	Weighted age-specific fertility rate per 1,000 women
1	2	3	4 = (2) × (3)
15–19	124.0	5	620
20–24	258.0	5	1,290
25–34	263.0	10	2,630
35–39	195.0	5	975
40–49	118.5	10	1,185
Total			6,700

8.6.4 ADVANTAGES AND DISADVANTAGES OF TFR

Advantages of TFR

TFR is generally regarded as the best single cross-sectional measure of fertility, because

1) it is rather closely restricted to the childbearing population; and
2) it is not influenced by differences in the age composition between childbearing populations.

Disadvantage of TFR

TFR is constructed to be independent of the age distribution both of women in the reproduction age and the births. It assumes the number of women to be one ($k = 1$) for each age x. This implies that the measure ignores the impact of mortality as well as growth of population which would have persisted in an unequal number of persons at each age x, usually as a decreasing function of age.

8.6.5 RELATIONSHIP BETWEEN TFR, GFR AND CBR

We have defined B_a as the number of births to women in age group a, W_a as the number of women in age group a and P_a as the population in age group a. We may then define:

$$CBR = \frac{B}{P} = \frac{\sum B_a}{\sum P_a}$$

and

$$GFR = \frac{B}{W} = \frac{\sum B_a}{\sum W_a}$$

compared with

$$TFR = \sum \frac{B_a}{W_a}$$

Though these rates differ in their construction, they represent similar concepts which we now discuss.

Relationship between TFR and GFR

Similarity between TFR and GFR

Both GFR and TFR represent similar concepts, namely, the average number of children per woman. GFR is the average number of births per woman per year while TFR is the total number of births per woman over the interval of the reproduction period.

Difference between TFR and GFR

While GFR ignores the age distribution of women in the age group 15–49, the TFR is constructed to be independent of the age distribution.

Relationship between TFR and CBR

Similarity between TFR and CBR

1) Conceptually, both TFR and CBR are similar.
2) Both TFR and CBR ignore the age distribution.

Difference between TFR and CBR

CBR refers to the rate of incidence in the total population but TFR refers to the rate of incidence in the female population.

Relationship between GFR and CBR

While CBR defines the denominator as total population, GFR defines the denominator as the number of women in fertile age group which is the actual population exposed to the risk.

8.6.6 CURRENT FAMILY SIZE

We have observed that in defining TFR, we applied a uniform weight of one to f_x. If we, however, apply appropriate weights at each age x to represent decrease (or increase) in the number of persons, we could obtain a more realist picture of the current status of family size, referred to as the current family size (CFS). Thus,

$$CFS = \sum f_x \frac{W_x}{W_0}$$

where

W_x represents the reproductive age x or age group a;
W_0 represents women in the beginning of the reproductive age $(x = 15)$ or age group $(a = 15 - 19)$.

Instead of W_x, we may use the actual population (P_x) or the life table population (L_x). CFS will always be different from TFR, and depending on the weights used, would give a better approximation of completed family size (discussed in the sequel) of the current, stationary or stable population.

8.7 CUMULATIVE FERTILITY RATE

The fertility rates for cohort l may be cumulated from the earliest reproductive age to any higher age. This sum is the cumulative fertility rate (CFR). It is defined as:

$$CFR = \sum_{h}^{l} n_a f_a$$

where l is the desired age group, $(l \leq m)$, and m is as defined above.
The cumulative fertility rate represents the total past fertility (or children ever born) to the survivors in the cohort up to that age.

Note

1) The cumulative fertility rate is computed in the same manner as the total fertility rate except that the adding process can terminate at the end of any desired age group.
2) The final entry in the CFR column is termed the completed fertility rate for the cohort and it corresponds to TFR based on age-specific fertility rates for a calender year or period.

Example 8.7

For the data in Example 8.6, calculate the cumulative fertility rate for the

1) age group 25–34;
2) age group 35–49.

Solution

i) The age-specific fertility rates for Example 8.5 are given in column 2 of the table of that example.

Age-group (years)	Weighted Age-specific fertility rate per 1,000 women	Cumulative fertility rate
1	2	3
15–19	620	620 = 620
20–24	1,290	620 + 1, 290 = 1, 910
25–34	2,630	2, 630 + 1, 910 = 4, 540

The cumulative fertility rate for the age group 25–34 is 4,540.

Age-group (years)	Weighted Age-specific fertility rate per 1,000 women	Cumulative fertility rate
1	2	3
15–19	620	620 = 620
20–24	1,290	620 + 1, 290 = 1, 910
25–34	2,630	2, 630 + 1, 910 = 4, 540
35–39	975	4, 540 + 975 = 5, 515

The cumulative fertility rate for the age group 35–39 is 5,515.

Completed Cumulative Fertility

We have indicated that the cumulative fertility may be computed up to any age. For ages 45 to 49 and over, it refers to ***completed fertility***, for beyond this age there is no more childbearing.

The completed cumulative fertility states the average size of family (including deceased children) that would result from a given schedule of age-specific fertility rates.

8.8 MARITAL FERTILITY RATE

Marriage is the act, ceremony or process, by which the legal relationship of husband and wife is constituted. The legality of the union may be established by civil, religious or other means as recognised by the laws of the given country. The final legal dissolution of a marriage is *divorce*, that is, the separation of husband and wife which confers on the parties the right to *remarriage* under civil, religious and/or other provisions, according to the laws of the country.

Marriage plays an important role in fertility analysis. In fact, it is next to age as a differentiating factor in fertility. In most societies these days, childbearing occurs mostly within the marital union. However some women do not enter into it at all yet a number of them give birth. Of the married women some become widows or divorcees and do not marry again. Others though married, do not give birth at all.

In this section we shall discuss fertility measures that take into account the marital status of women.

8.8.1 GENERAL MARITAL FERTILITY RATE

The general marital fertility rate (GMFR) is defined as the total number of live births in a year per 1,000 population of ever married women of childbearing age:

$$GMFR = \frac{B}{W^{em}} \times 1,000$$

where W^{em} = women, ever married, of childbearing age as at July 1.

By "ever married women", we mean women currently in active marriage and those previously married (including widowed, separated and divorced ones).

Example 8.8

The 1985 mid-year population of ever married women of childbearing age of a community was 8,000. There were 130 births in that year. Calculate the general marital fertility rate.

Solution

$$\begin{aligned} W^{em} &= 8,000 \\ B &= 130 \\ GMFR &= \frac{B}{W^{em}} \times 1,000 \\ &= \frac{130}{8,000} \times 1,000 = 16.3 \end{aligned}$$

That is, in that community in 1985, there were 16.3 births per 1,000 women ever married of childbearing age.

8.8.2 TOTAL MARITAL FERTILITY RATE

The total marital fertility rate (TMFR) is defined as the total number of live births in a year per 1,000 population of married women of childbearing age:

$$TMFR = \frac{B^m}{W^m} \times 1,000$$

where

W^m = married women of childbearing age as at 1st July;
B^m = total number of births among currently married women.

The TMFR[41] indicates the number of births a woman would have had at the end of the reproductive years if she were to bear children at prevailing age specific marital fertility rates and to remain married during the entire reproductive period.

Example 8.9

For the data of Example 8.8, calculate the total marital fertility rate if it was known that only 7,110 women were then in active marriage.

Solution

$$\begin{aligned} W^m &= 7,110 \\ B &= 130 \\ TMFR &= \frac{B}{W^m} \times 1,000 \\ &= \frac{130}{7,110} \times 1,000 = 18.3 \end{aligned}$$

That is, there were 18.3 births per 1,000 married women of childbearing age in 1985 in that community.

8.8.3 AGE-SPECIFIC MARITAL FERTILITY RATE

The proportion of married women within each age group is a deciding factor on the level of fertility. More unmarried women in the younger ages and more separated, divorced and widowed women in the older age brackets is characteristic of any society.

The age-specific marital fertility rate is defined as:

$$ASMFR_x = \frac{B_x^m}{W_x^m} \times k$$

where

$B_x^m =$ total number of births among currently married women aged x;
$W_x^m =$ mid-year (average) population of married women age x.

The age-specific marital fertility rate may also be defined as:

$$ASMFR_x = f_x m_x$$

where

$f_x =$ age-specific fertility rate for single ages;
$m_x =$ reciprocal of proportion of married women at age x.

8.8.4 BIRTH OUTSIDE MARRIAGE

In demography, births may be classified as legitimate and illegitimate. By ***legitimate births***, we mean births given by a legally married couple; and by ***illegitimate births***, we mean births outside a legal marriage. Corresponding to total marital fertility rate, we may calculate total illegitimate fertility rate.

In most developing countries, however, especially in Africa the problem of legitimacy or illegitimacy of births actually does not exist even though childbirth can and does take place outside marriage. Marriage is a long process and not a clearly defined event. The result of such a long process is that a pregnancy outside marriage can end up as a birth within marriage. This creates difficulty in the classification of birth and consequently the measurement. Again, there is nothing called illegitimate child in sub-Saharan Africa. The reason is that once the paternity of a child is acknowledged he/she is legitimate under the Customary Law. A more appropriate concept to use, therefore, is ***birth outside marriage***.

8.9 AVERAGE AGE OF CHILDBEARING

The average age of childbearing is the average age of mothers at the birth of their children if they were subjected throughout their lives to the age-specific fertility rates observed in a given year. It measures differences in the age pattern of childbearing. It describes the age pattern of childbearing of synthetic cohort of women, that is, a hypothetical group of women who are viewed as having in their lifetime the (fertility) experience recorded in a single calendar year.

8.9.1 METHOD OF COMPUTING AVERAGE AGE OF CHILDBEARING

The term *average* is a general term applying to three kinds of measures of central location, namely, the mean, median and the mode. Therefore, by *average age of childbearing* (also referred to as ***average age of mother***), we refer to the mean age of childbearing, the median age of childbearing and the modal age of childbearing. From the knowledge of age pattern of fertility, it is possible to calculate the average age of childbearing in its usual sense. The calculations are, however, based on the age-specific fertility rates (ASFR) than actual numbers of births in age groups. This is to eliminate the effect of differences in the age-sex composition of the populations being compared.

8.9.2 MEAN AGE AT CHILDBEARING

The mean age of childbearing may be calculated using

1) the actual age-specific fertility rates, f_a, or
2) the relative age-specific fertilities of the distributions, $f_a^{(r)}$.

When data are from a survey (sample or census), the class intervals for the age groups are 14.5–19.5, 19.5–24.5, $\cdots$, 44.5–49.5. This is because the women were half a year younger at the time of the birth of their children. The mid-points, X_{ma}, therefore become 17, 22, $\cdots$, 47. It is important to note that this method of defining class intervals is the same as defining class boundaries in statistical methods (refer to Nsowah-Nuamah, 2005).

When data are from vital registration, the mid-points, X_{ma}, of the age group should be 17.5, 22.5,$\cdots$, 47.5, because the registration should have taken place at the time of birth of a child.

Mean Age of Childbearing in Terms of Age-specific Fertility

The mean age of childbearing ($\overline{X}$) may be defined as:

$$\overline{X} = \frac{\sum_{a=15-19}^{45-49} f_a X_{ma}}{\sum_{a=15-19}^{45-49} f_a}$$

where

X_{ma} is the mid-point for each age interval[42];
f_a is the age-specific fertility rates for women whose age corresponds to age group a of which X is the mid-point.

Note

1) The formula follows the form of a weighted mean ages, the weights being the age-specific fertility rates, f_a.
2) The formula may be written as:

$$\overline{X} = \frac{\sum_{a=15-19}^{45-49} f_a (l + 2.5)}{\sum_{a=15-19}^{45-49} f_a}$$

l is the lower limit of each age interval.

3) In populations where there is early childbearing which continues till late ages and where the peak of fertility is also high, it is observed that the mean age of childbearing is usually high and a large fraction of fertility relates to the latter years of childbearing. On the other hand, in the population where childbearing starts and terminates early and where fertility is low, the mean age of childbearing is lower and a small fraction of total fertility occurs in latter years of childbearing.

Example 8.10

For the data in Example 8.3, calculate the mean age of childbearing, using the age-specific fertility rates, $f_a^{(a)}$.

Solution

Since the data were taken from vital registration, the mid-points are 17.5, 22.5, $\cdots$, 47.5.

Table 8.3 shows the preliminary calculations of the mean age of childbearing using the actual age-specific fertility rates.

Age-group	Mid-year women Population, P_a	Number of births, B_a	ASFR (f_a) per 1,000	Mid-point of age group, X_{ma}	$f_a X_{ma}$
1	2	3	$4 = \frac{(3)}{(2)}$	5	$6 = (4) \times (5)$
15–19	46,417	2,028	43.7	17.5	764.8
20–24	51,462	6,927	134.6	22.5	3,028.5
25–29	51,580	6,408	124.2	27.5	3,415.5
30–34	44,906	3,460	77.0	32.5	2,505.6
35–39	39,286	1,393	35.5	37.5	1331.3
40–44	27,741	303	10.9	42.5	463.3
45–49	22,841	19	0.8	47.5	38
Total	284233	20,538	426.7	-	11,543.8

TABLE 8.3 Calculation of mean age of childbearing

Thus,

$$\overline{X} = \frac{11,543.8}{426.7} = 27.0$$

That is, the mean age of childbearing in Mauritius in 1984 was 27 years.

Mean Age of Childbearing in Terms of Relative Age-specific Fertility

The mean age of childbearing may be given as:

$$\overline{X} = \sum_{a=15}^{49} f_a^{(r)} X_{ma}$$

where $f_a^{(r)}$ is the relative age-specific fertility of the distribution, defined as:

$$f_a^{(r)} = \frac{f_a}{\sum f_a}$$

Example 8.11

For the data in Example 8.3, calculate the mean age of childbearing, using the relative age-specific fertility of the distributions, $f_a^{(r)}$.

Solution

Table 8.4 shows the preliminary calculations of the mean age of childbearing using the relative age-specific fertility of the distributions.

Age-group	ASFR (f_a) per 1,000	Mid-point of age group, X_{ma}	$f_a^r = \frac{f_a}{\sum f_a}$	$f_a^r X_{ma}$
1	2	3	4	$5 = (3) \times (4)$
15–19	43.7	17.5	0.10241	1.79218
20–24	134.6	22.5	0.31544	7.09740
25–29	124.2	27.5	0.29107	8.00443
30–34	77.1	32.5	0.18046	5.86495
35–39	35.5	37.5	0.08320	3.12000
40–44	10.9	42.5	0.02554	1.08545
45–49	0.8	47.5	0.00187	0.08883
Total	426.8	-	1.00000	27.05323

TABLE 8.4 Calculation of mean age of childbearing

From column 6,

$$\overline{X} = \sum_{a=15-19}^{45-49} f_a^{(r)} X_{ma} = 27.1$$

8.9.3 MEDIAN AGE AT CHILDBEARING

The median age of childbearing may be defined as (Nsowah-Nuamah, 2005):

$$M_e = L_{me} + \frac{\frac{f_a}{2} - F}{f_{me}} \times h$$

where

M_e	=	median age of childbearing;
L_{me}	=	lower boundary of the median class (the median class is the age group upon which the median lies;
$\frac{f_a}{2}$	=	one-half the ASFR;
f_{me}	=	ASFR of the median class;
F	=	cumulative ASFR immediately before the median class, that is, sum of the observations on all age groups above L_{me}
h	=	width of median class.

Example 8.12

For the data in Example 8.3, calculate the median age of childbearing.

Solution

Table 8.5 shows the preliminary calculations of the median age of childbearing.

Age-group	Mid-year women Population	Number of births	ASFR (f_a) per 1,000	Cumulative frequency of ASFR
1	2	3	$4 = \frac{(3)}{(2)}$	6
15–19	46,417	2,028	43.7	43.7
20–24	51,462	6,927	134.6	178.3
25–29	51,580	6,408	124.2	302.5
30–34	44,906	3,460	77.0	379.5
35–39	39,286	1,393	35.5	415.0
40–44	27,741	303	10.9	425.9
45- 49	22,841	19	0.8	426.7
Total	284,233	20,538	426.7	–

TABLE 8.5 Calculation of median age of childbearing

We determine the class boundary that contains the median.

1) Since there are $\sum f_a = 426.7$, $\sum f_a/2 = 213.4$, and the median is that point in our distribution which has 213.4 age-specific fertility rate on each side of it.
2) Beginning at the first class boundary[43] and adding up the f_a in order, we find that class boundary 24.5–29.5, inclusive, contains the median. This is the median class.

Hence,

$$L_{me} = 24.5, \quad f_{me} = 124.2, \quad F = 178.3, \quad h = 5$$

Consequently,

$$\begin{aligned} M_e &= L_{me} + \frac{\frac{f_a}{2} - F}{f_{me}} \times h \\ &= 24.5 + \frac{213.4 - 178.3}{124.2} \times 5 = 25.9 \end{aligned}$$

That is, half of the women of childbearing age were less than 26 years and half more than that in Mauritius in 1984.

8.9.4 MODAL AGE AT CHILDBEARING

The modal age of childbearing may be defined as:

$$M_o = L_{m0} + \frac{f_{m0} - f_{m0-1}}{(f_{m0} - f_{m0-1}) + (f_{m0} - f_{m0+1})} h$$

Or

$$M_0 = L_{m0} + \frac{f_{m0} - f_{m0-1}}{2f_{m0} - (f_{m0-1} + f_{m0+1})} \times h$$

where

M_o	= mode age of childbearing;
L_{m0}	= lower boundary of the modal class;
h	= width of the age group;
f_{m0}	= ASFR of the modal class;
f_{m0-1}	= ASFR of the class before the modal class;
f_{m0+1}	= ASFR of the class after the modal class.

Example 8.13
For the data in Example 8.3, calculate the modal age of childbearing.

Solution

Step 1

a) The modal class is 19.5–24.5 since this interval has the highest age-specific fertility rate, 134.6.
b) The lower boundary of the age group is $L_{me} = 19.5$.
c) The width of the age group is $h = 5$.
d) The ASFR of the modal class is $f_{m0} = 134.6$.
e) The ASFR of the class before the modal class is $f_{m0-1} = 43.7$.
f) The ASFR of the class after the modal class is $f_{m0+1} = 124.2$.

Step 2

Using the formula we obtain:

$$M_o = 19.5 + \frac{134.6 - 43.7}{(134.6 - 43.7) + (134.6 - 124.2)} \times 5 = 24.0$$

That is, the modal age of childbearing was about 24 years in Mauritius in 1984.

8.10 MEASURES OF FERTILITY BASED ON CENSUS AND SURVEY DATA

In developing countries, especially in Africa, where vital registration system is inefficient, some measures of fertility can be derived from the census or survey data, using retrospective questions. In Africa, the Demographic and Health Surveys (DHS) and the Multiple Indicator Cluster Surveys (MICS) are two main national sample sample surveys together with the census where a special question is asked to women on the number of children born alive. From this question we can calculate the current fertility and lifetime fertility.

8.10.1 CHILD-WOMAN RATIO

The age and sex distribution of a population is the result of past force of fertility, mortality and migration. However, the overriding factor in giving shape to age distribution is the fertility. This is due to the fact that at the national level, migration is very negligible and also the mortality reduction in general does not relate to one or more specific age groups but affects the whole population. Consequently, the level and pattern of fertility in the past can, with reasonable accuracy, be inferred from the census age distribution with the help of analytical population models.

The measure of fertility calculated from census age-sex returns which is commonly used is the child-woman ratio (sometimes referred to as ***general fertility ratio*** or ***ratio of children to women***). It is defined as the ratio of the number of children to the number of women in reproductive age group. Various ages have been used, but the ages of children usually used are 0–4, 5–9 or 0–9 years while the ages of women are 15–44, 20–49 or 15–49 years. The child-woman ratio (CWR) then may be defined symbolically as follows:

a) $CWR = \frac{C_{0-4}}{W_{15-44}} \times k$

b) $CWR = \frac{C_{5-9}}{W_{20-49}} \times k$

c) $CWR = \frac{C_{0-9}}{W_{15-49}} \times k$

Example 8.14

In the 2000 census of Ghana, children under 5 years and those between 5 and 9 years inclusive were 2,769,421 and 2,775,206 respectively. The distribution of women of childbearing age is given in Table….

Calculate all the variants of the child-woman ratio.

Solution

The number of children in the various age groups are:

$$C_{0-4} = 2,769,421;\ C_{5-9} = 2,775,206;\ C_{0-9} = 5,544,627$$

The number of women in the various age groups are:

$$\begin{aligned} W_{15-44} &= 4,175,083 \\ W_{20-49} &= 3,595,534 \\ W_{15-49} &= 4,518,125 \end{aligned}$$

Age group	Women Population
15–19	922,591
20–24	837,769
25–29	791,805
30–34	604,370
35- 39	538,901
40–44	443,647
45–49	343,042

TABLE 8.5 Number of Women in Ghana by Age of Childbearing in 2000

Hence the three variants of the child-woman ratio are:

$$1)\ CWR = \frac{C_{0-4}}{W_{15-44}} \times k = \frac{2,769,421}{4,175,083} = 663.3$$

$$2)\ CWR = \frac{C_{5-9}}{W_{20-49}} \times k = \frac{2,775,206}{3,595,534} = 771.8$$

$$3)\ CWR = \frac{C_{0-9}}{W_{15-49}} \times k = \frac{5,544,627}{4,518,125} = 1,227.2$$

That is, in Ghana in 2000, there were 663 children per 1,000 women aged 15–44 years, 772 children per 1,000 women aged 20–49 years or 1,227 children per 1,000 women aged 15–49 years.

Advantages of Child-Woman Ratio

The child-woman ratio has a number of advantages as a measure of fertility.

1) It is simple, requires minimum data and is easily understandable.
2) It is a useful index where vital registration data are lacking; it reflects fertility performance in the five years preceding the census.

3) It does not require a special question or tabulation plan in census.
4) It can be calculated for various socio-economic and ethnic groups of the population and also for small areas (regions, districts, etc.).
5) It can be considered as the measure of effective fertility, as it does not consider the births of children dying early in life.

Disadvantages of Child-Woman Ratio

1) Like general fertility rate, the child-woman ratio is affected by distribution of women by age in the reproductive period. The broad range of ages used for women does not take into consideration the age distribution within this range.
2) It is affected by the under/over enumeration of children and women in reproductive ages. This comes about from the fact that
   i) ages of children and women may be mis-stated: ages of children in age group 0–4 are usually underenumerated while ages of children in age group 5–9 are overenumerated.
   ii) no consideration is given to mortality of children and women;

8.10.2 CHILDREN EVER BORN

The number of children ever born to a particular woman is a measure of her lifetime fertility experience up to the moment at which the data are collected. The number includes all live-born children[44], whether born in or out of marriage, whether born in the present or prior marriage, or in a de facto union[45], or whether living or dead at the time of the census.

Children ever born (CEB) to women in a particular age group is the mean number of children born alive to women in that age group. In most cases, it is computed as the ratio of the number of children born alive to all women in a particular age group to the number of women. In cases where the total number of children born to women in the age group is not provided but a tabulation is available on the distribution of women by age group and number of children ever born, the mean number of children ever born to women in the age group is obtained as:

$$CEB = \sum jP_j$$

where

j is the number of children;
P_j is the proportion of women in that age group who have given birth to a total of j children.

8.10.3 PARITY

Parity refers to the number of live births a woman has given birth to. In other text, it is referred to as the number of *children ever born*. A woman is said to be of the i^{th} parity if she has given birth to i children. A woman who has no children has zero parity, a woman who has given birth to only one child has one parity and so on. The children are identified by their order of birth (that is, first, second, and so forth).

Women can be classified according to their parity on the basis of the number of children ever born such as follows

Number of women	W_0	W_1	W_2	$\cdots$	W_i	$\cdots$	W_h
Parity	0	1	2	$\cdots$	i	$\cdots$	h

The average parity of women also referred to as the *average number of children ever born per woman* by age can be calculated as:

$$\bar{p}_a = \frac{p_a}{W_a}$$

where

p_a = total number of children ever born to women in age group a;
W_a= is the total number of women in age-group a.

The average parity, $\overline{p}_a$, provides the indication of cumulative fertility of women. It is expected to increase with age of mothers with the fertility performance of the youngest age group (15–19 years) almost negligible. When it happens that some of the higher age groups have lower average parity, we may attribute it to low quality of reporting of data on either children ever born or the age of mothers or both. The low quality of data on children ever born is usually a deliberate misreporting by mothers who have had live births which have later died. Usually people are not comfortable reminding themselves of such events and when asked to indicate total live births that have taken place, they try to conceal this fact.

Example 8.15

Table 8.7 (a) gives number of women older than 14 years and the total number of children ever born by them in a certain community.

Age Group (Years)	Total Number of Women	Total Number of Children Ever Born
15–19	82,356	39,889
20–24	75,919	152,366
25–29	75,688	259,373
30–34	59,646	265,451
35–39	47,425	237,562
40–44	38,352	188,642
45–49	29,993	170,077

TABLE 8.7 (a) Number of Women and Number of Children Ever Born of a Community

Calculate the mean number of children ever born per woman and comment on the distribution.

Solution

For the age group 15–19, the mean number of children ever born per woman by age is

$$\overline{p}_{(15-19)} = \frac{39,889}{82,356} = 0.4843$$

For the age group 20–24, the mean number of children ever born per woman by age is

$$\overline{p}_{(20-24)} = \frac{152,366}{75,919} = 2.007$$

and so on.

For the age group (45–49), the mean number of children ever born per woman by age is

$$\overline{p}_{(45-49)} = \frac{170,077}{29,993} = 5.6706$$

Column 4 of Table 8.7 (b) gives the results for all the other age groups.

We may observe from Table 8.7 (b) that the mean number of children per woman does not increase with age of mothers. For example, the number of children ever born to women in the age group 40–44 is 4.9 which is lower than those born to women in the age group 35–39 which is 5.

Age Group (Years)	Total Women	Total number of children ever born	Mean number of children per woman
1	2	3	4=(3)/(2)
15–19	82,356	39,889	0.4843
20–24	75,919	152,366	2.0070
25–29	75,688	259,373	3.4269
30–34	59,646	265,451	4.4504
35–39	47,425	237,562	5.0092
40–44	38,352	188,642	4.9187
45–49	29,993	170,077	5.6706

TABLE 8.7 (b) Calculation of Mean Number of Children per Woman

Uses of Average Parity

Measure of Completed Family Size

The completed family size, also referred to as ***cohort fertility rate*** or ***completed fertility rate***, is the average size of completed fertility. It is the number of children a woman would have ever borne when she completes her reproductive period of life, usually taken to be the age 50. Statistically, this involves following a group of women born in a particular year (birth cohort) or married in a particular year (marriage cohort) throughout their lifetime and recording the number of children they produce. By adding these period rates we obtain the average total number of children for the cohort.

The completed family size is equivalent to total fertility rate (TFR) of a calendar year and it exhibits much more stability than do age-specific rates from year to year. It is measured by the average parity in age group 40–44 or 45–49 if the values are equal or nearly equal or slightly higher in the age group 45–49.

If fertility remains unchanged for a long period (30 years or more), completed family size would be equal to TFR, assuming that the women who died had the same fertility as those who were alive.

Symbolically, the completed family size (cfs) is given as:

$$cfs = \sum_{i=0}^{35} f_{x+i}^{z+i}$$

where f_{x+i}^{z+i} = the age-specific fertility rate of age $x + i$ $(x = 15)$ during the calender year $z + i$.

If the average parity in age group 45–49 is significantly lower than that in the age group 40–44, TFR may be estimated as:

$$TFR = \frac{\overline{p}_3^2}{\overline{p}_2}$$

where $\overline{p}_2$ and $\overline{p}_3$ are the average parity in the age groups 20–24 and 25–29, respectively.

Example 8.16

For the data in Example 8.15, estimate the TFR.

Solution

From Example 8.15,

$$\overline{p}_2 = 4.9188; \quad \overline{p}_3 = 5.6706$$

Hence,

$$\begin{aligned}\overline{fs} &= TFR = \frac{\overline{p}_3^2}{\overline{p}_2} \\ &= \frac{(5.6706)^2}{4.9188} = 6.54\end{aligned}$$

This estimate can better represent the average parity than the values in the age group 40–44 or 45–49.

It is important to note that the completed family size is not an indicator of current fertility because the births to these women had occurred in the past at varying times in a period extending about 30 years.

In most developing countries, data on lifetime fertility by the ages of women show that it tends to rise steadily with age, reaching a maximum in the 45–49 age group, then a slight but generally consistent fall in the figures for the women over the age of 50 years.

Mean Age of Fertility Schedule

We may estimate the mean age of fertility schedule ($\overline{fs}$) from the average parity as:

$$\overline{fs} = 2.25\,\frac{\overline{p}_3}{\overline{p}_2} + 23.95$$

where $\overline{p}_2$ and $\overline{p}_3$ are as already defined.

Example 8.17

For the data in Example 8.15, calculate the mean age of fertility schedule.

Solution

From Example 8.15,

$$p_2 = 4.9188; \quad p_3 = 5.6706$$

Hence,

$$\begin{aligned} \bar{f}s &= 2.25\,\frac{p_3}{p_2} + 23.95 \\ &= 2.25 \times \frac{5.6706}{4.9188} + 23.95 = 26.54 \end{aligned}$$

8.10.4 BIRTH ORDER

When a woman is of the i^{th} parity, the birth itself is referred to as i^{th}-order birth ($i = 1, 2, \cdots, h$). Higher order births ought to be fewer in number. The order of birth receives much attention in modern times of high contraceptive usage. To evaluate the progress of birth control programmes, we use the birth order specific rates.

General Order Specific Fertility Rate

A simple way of analysing the order of birth statistics is to obtain the distribution of births by order, referred to as the general order specific fertility rate (GOSFR). The GOSFR is defined as:

$$GOSFR_i = \frac{B_i}{W_{15-49}} \times k$$

where

B_i = the number of births of order i born to women aged (15–49);
W_{15-49} = the mid-year (average) population of females in reproductive ages (15–49).

We may observe that the sum of all the GOSFR is nothing but the GFR. Thus

$$GFR = \sum_{i=1}^{h} GOSFR_i$$

Example 8.18

Table 8.8 (a) shows live birth order by age of mother and female population in the reproductive ages in the Republic of Mauritius in 2004. Calculate the general order specific fertility rate.

Age group (Years)	Mid-year women Population	Live Birth Order					
		First	Second	Third	Fourth	Fifth	Sixth
1	2	3	4	5	6	7	8
15–19	46,955	1473	277	28	1	0	1
20–24	53,308	3655	1853	341	53	10	5
25–29	53,955	2404	2626	837	181	32	18
30–34	46,591	778	1342	794	249	92	41
35–39	49,954	277	512	537	190	94	77
40–44	49,072	87	101	98	57	46	31
45–49	42,597	4	4	8	5	5	6

TABLE 8.8 (a) Live Birth Order by Age of Mother and Female Population in Reproductive Ages in Republic of Mauritius, 2004

Solution

$$GOSFR_1 = \frac{B_1}{W_{15-49}} \times 1,000 = \frac{8,678}{34,2432} \times 1,000 = 25.3$$

$$GOSFR_2 = \frac{B_2}{W_{15-49}} \times 1,000 = \frac{6,715}{34,2432} \times 1,000 = 19.6$$

$$GOSFR_3 = \frac{B_3}{W_{15-49}} \times 1,000 = \frac{2,643}{34,2432} \times 1,000 = 7.7$$

$$GOSFR_4 = \frac{B_4}{W_{15-49}} \times 1,000 = \frac{736}{34,2432} \times 1,000 = 2.1$$

$$GOSFR_5 = \frac{B_5}{W_{15-49}} \times 1,000 = \frac{279}{34,2432} \times 1,000 = 0.8$$

$$GOSFR_6 = \frac{B_6}{W_{15-49}} \times 1,000 = \frac{179}{34,2432} \times 1,000 = 0.5$$

The results are presented in the last row of Table 8.8 (b).

Age group (Years)	Mid-year women Population	Live Birth Order						Total Births
		First	Second	Third	Fourth	Fifth	Sixth	
1	2	3	4	5	6	7	8	9
15–19	46,955	1473	277	28	1	0	1	1,780
20–24	53,308	3655	1853	341	53	10	5	5,917
25–29	53,955	2404	2626	837	181	32	18	6,098
30–34	46,591	778	1342	794	249	92	41	3,296
35–39	49,954	277	512	537	190	94	77	1,687
40–44	49,072	87	101	98	57	46	31	420
45–49	42,597	4	4	8	5	5	6	32
Total	34,2432	8678	6715	2643	736	279	179	19,230
GOSFR		25.3	19.6	7.7	2.1	0.8	0.5	56.2

TABLE 8.8 (b) Calculation of General Order Specific Fertility Rate

We verify that, the sum of GOSFR is equal to GFR. Thus,

$$\begin{aligned} GFR &= \frac{B}{W_a} \times 1,000 \\ &= \frac{19,230}{34,2432} \times 1,000 = 56.2 \end{aligned}$$

which is the same as the sum of $GOSFR_i$ in the the last row in Table 8.8 (b).

Age and Order Specific Fertility Rate

The age and order specific fertility rate (AOSFR) is given by

$$AOSFR_{ai} = \frac{B_{a,i}}{W_a}$$

where

$B_{a,i}$ = the number of births of order i to women at age group a;
W_a = the mid-year (average) population of women at age group a.

If we sum all the AOSFR of births over all the orders of birth at a particular age group we get the age specific fertility rate. Thus

$$f_a = \sum_{i=1}^{h} AOSFR_{a,i}$$

Example 8.19

For the data in Example 8.14, calculate the age and order specific fertility rate.

Solution

We calculate the AOSFR for the age group 15–19:

$$AOSFR_{(15-19),1} = \frac{B_{(15-19),1}}{W_{15-19}} \times 1,000 = \frac{8,678}{46,955} \times 1,000 = 25.3$$

$$AOSFR_{(15-19),2} = \frac{B_{(15-19),2}}{W_{15-19}} \times 1,000 = \frac{6,715}{46,955} \times 1,000 = 19.6$$

$$AOSFR_{(15-19),3} = \frac{B_{(15-19),3}}{W_{15-19}} \times 1,000 = \frac{2,643}{46,955} \times 1,000 = 7.7$$

$$AOSFR_{(15-19),4} = \frac{B_{(15-19),4}}{W_{15-19}} \times 1,000 = \frac{736}{46,955} \times 1,000 = 2.1$$

$$AOSFR_{(15-19),5} = \frac{B_{(15-19),5}}{W_{15-19}} \times 1,000 = \frac{279}{46,955} \times 1,000 = 0.8$$

$$AOSFR_{(15-19),6} = \frac{B_{(15-19),6}}{W_{15-19}} \times 1,000 = \frac{179}{46,955} \times 1,000 = 0.5$$

For the remaining age group, the results are presented in Table 8.9.

Age group (Years)	Mid-year women Population	Live Birth Order						Total
		First	Second	Third	Fourth	Fifth	Sixth	
1	2	3	4	5	6	7	8	9
15–19	46,955	31.4	5.9	0.6	0.0	0.0	0.0	37.9
20–24	53,308	68.6	34.8	6.4	1.0	0.2	0.1	111.0
25–29	53,955	44.6	48.7	15.5	3.4	0.6	0.3	113.0
30–34	46,591	16.7	28.8	17.0	5.3	2.0	0.9	70.7
35–39	49,954	5.5	10.2	10.7	3.8	1.9	1.5	33.8
40–44	49,072	1.8	2.1	2.0	1.2	0.9	0.6	8.6
45–49	42,597	0.1	0.1	0.2	0.1	0.1	0.1	0.8
Total	34,2432	25.3	19.6	7.7	2.1	0.8	0.5	56.2

TABLE 8.9 Calculation of Age and Order Specific Fertility Rate

We can verify that the $\sum_{i=1}^{h} AOSFR_{ai}$ is equal to the age specific fertility rate for age group a. Thus for age group 15–19,

$$\begin{aligned} f_a &= \frac{B_a}{W_a} \times 1,000 \\ &= \frac{1,780}{46,955} \times 1,000 = 37.9 \end{aligned}$$

which is the same as the sum of $AOSFR_{(15-19)}$ given in column 9 of Table 8.9. We can verify this to be true for all the other age groups.

8.10.5 BIRTH INTERVALS

The interval between births provides a useful information on birth spacing patterns and is, therefore, useful for measuring family planning trends. Studies have shown that:

1) Maternal health is threatened by rapid childbearing;
2) Short (less than 24 months) birth intervals are detrimental to the health of both the mother and child.

Intervals between births may be closed or open. Intervals between two successive parities are called ***inter birth closed intervals***. Intervals between the last parity of the mother and the time of observation (sample survey or census) are called ***open birth intervals***. The average of these two intervals specified for parity indicates spacing between children and postponement of pregnancies.

8.10.6 PARITY PROGRESSION RATIO

Most often, in population analysis, we want to know what proportion of women of a particular parity will have an additional child. The measure that can be used to achieve this is the parity progression ratio (PPR). That is, PPR is designed to show the proportion, among the women who have had at least some specified number of births, who go on to have one more birth. It can be calculated from a special question asked in a census or survey on number of children ever born to all the women or to married women only. It is defined as

$$PPR_p = \frac{W_{p+1}}{W_p}$$

where

W_p is the number of women of parity p and above;
$p+1$ is the number of women of parity $p+1$ and above.

Thus,

$$\begin{aligned} PPR_0 &= \frac{W_1}{W_0} \\ PPR_1 &= \frac{W_2}{W_1} \\ \vdots & \quad \vdots \\ PPR_{p-1} &= \frac{W_p}{W_{p-1}} \end{aligned}$$

Example 8.21

Table 8.10 (a) gives the number of children ever born and the number of married women who gave birth to such children in an area as obtained in a census in 2004. Calculate the parity progression ratio and interpret the results.

Number of Live Born Children	Number of Married Women
0	29,777
1	42,593
2	39,489
3	26,242
4	20,452
5	15,983
6	12,809
7	10,563
8	5,480
9	3,007
Total	206,395

TABLE 8.10 (a) Number of Married Women and Live Births of an Area

Solution

There are 3,007 women who have had 9 children (column 3 of Table 8.10 (b).

Next we have 8487 (3,007+5,480) women who have had at least 8 children, 19,050 (8,487+10,563) women who have had at least 7 children; and so on till we get to 176,618 women who have had at least 1 child and finally 206,395 which is the total number of married women. Column (3) of Table 8.10 (b) gives the number of women who have had at least some specified number of births.

The proportion of the total number of married women who have had at least 1 child is:

$$PPR_0 = \frac{W_1}{W_0} = \frac{176,618}{206,395} = 0.856$$

Number of Live born Children	Number of Married Women, W_m	"Greater than or equal to" (Cumulative Number of Women)	Parity Progression Ratio
1	2	3	4
0	29,777	206,395	-
1	42,593	176,618	0.856
2	39,489	134,025	0.759
3	26,242	94,536	0.705
4	20,452	68,294	0.722
5	15,983	47,842	0.701
6	12,809	31,859	0.666
7	10,563	19,050	0.598
8	5,480	8,487	0.446
9	3,007	3,007	0.354
Total	206,395		

TABLE 8.10(b) Calculation of Parity Progression Ratio

The proportion of the total number of married women having had (at least) 1 child, who had (at least) a second child is:

$$PPR_1 = \frac{W_2}{W_1} = \frac{134,025}{176,618} = 0.759$$

That is, there is a 75.9% chance that a woman aged 45–49 has a second birth given that she already has had a first birth.

The proportion of the total number of married women who have had (at least) 2 children, having had (at least) a third child is

$$PPR_2 = \frac{W_3}{W_2} = \frac{94,536}{134,025} = 0.705$$

and so forth to finally:

The proportion of the total number of married women who have had (at least) 8 children, having had (at least) a ninth child is

$$PPR_8 = \frac{W_9}{W_8} = \frac{3,007}{8,487} = 0.354$$

Column (4) of Table 8.10 (b) gives the entire results.

Interpretation of PPR

That is 85.6 percent of the women having married gave birth to 1 child; 75.9 percent of the women having given birth to a first live-birth child also had a second live-birth child; and so on.

Uses of PPR

We may calculate from the PPR's the probability that a (married) woman will have in her life time at least n number of children. This can be obtained as:

$$Prob(n \text{ children}) = PPR_0 \times PPR_1 \times \cdots \times PPR_n$$

Example 8.21

For the data in Example 8.21, find the probability of a woman having 4 children in her life time.

Solution

This probability is given by

$$\begin{aligned} Prob(4 \text{ children}) &= PPR_0 \times PPR_1 \times PPR_2 \times PPR_3 \times PPR_4 \\ &= 0.856 \times 0.759 \times 0.705 \times 0.722 \\ &= 0.3307 \end{aligned}$$

Interpretation

That is, it is expected that out of 1,000 women born in the area in 2004, only 331 will have at least 4 children

Note

1) PPR can be considered in terms of age cohort and marital cohort. We can also make PPR specific for both age and marriage duration.
2) The parity progression ratio is not specific to calender year and hence it can only be meaningfully interpreted when applied to women who have completed their fertility behaviour. Consequently, it only measures cohort fertility. It is of limited value as a measure of period fertility.

Period Parity Progression Ratios

When we calculate PPR for cohorts that are yet to complete the reproductive span, we have period parity progression ratio (PPPR):

$$PPPR = \frac{W_{p+1}^{t}}{W_{p}^{t}} \times k$$

where

W_p^t = Number of women in a cohort who were of the ith parity at the beginning of the year t.
W_{p+1}^t = Number of women of a cohort who had their $(i+1)th$ order births in a year t;

This measure can be considered as the probability that an ith parity woman of a given cohort gives birth to the $(i+1)th$ order child in a year.

Generally, PPRs are always less than unity but the PPPR can sometimes be greater than unity, especially, at the lower parities of 1 and 2. This situation arises if spacing between ith and $(i+1)th$ parity differs widely among cohorts, thus making the $(i+1)th$ order births of several cohorts to overlap. This will present a larger figure than the ith parity women of the youngest cohort resulting in a value greater than one for the PPPR.

8.10.7 DISTRIBUTION OF FAMILY SIZE

The PPR's characterise the distribution of childbearing in a population. From Table 8.7, we can know how different family sizes are distributed, namely, the number of women with ***at least*** B_i children and the number of women with ***exactly*** B_i children. In both cases we do the computation either through the PPR or directly from the data.

Computation through PPR

The number of women with at least B_i children $PP_{i+1}^{at\ least} = PP_i$ may be obtained from PPR as

$$PP_{i+1}^{at\ least} = PP_i \times PPR_{i+1}$$

The number of women with exactly B_i children may be obtained as

$$PP_{i+1}^{exactly} = PP_i - PP_{i+1}$$

Example 8.22

Refer to Example 8.18. Calculate

1) the number of women with at least B_i children;
2) the number of women with exactly B_i children;

with the aid of PPR and interpret the results.

Solution

1) The number of women with at least B_i children.
   In Table 8.7 (b) of Example 8.21, $PPR_1 = 0.856$.
   This indicates that out of 1,000 women, 856 had had at least 1 child.

   $PPR_2 = 0.759$ indicates that among the 856 women who had at least 1 child, 649 (856×0.759) had at least 2 children.

   $PPR_3 = 0.705$ indicates that among the 649 women who had at least 2 child, 458 (649×0.705) had at least 3 children.

   and so forth and;
   $PPR_9 = 0.354$ indicates that among the 41 women who had at least 8 children, 15 (41×0.354) had at least 9 children.

   In a more systematic manner, among 1,000 married women we have

$$
\begin{array}{rcl}
PP_0: & 1000 & 1000\,\text{had at least 0 children;} \\
PP_1: & 1000 \times 0.856 & 856\,\text{had at least 1 child;} \\
PP_2: & 1000 \times 0.856 \times 0.759 & 649\,\text{had at least 2 children;} \\
 & \vdots & \\
PPR_9: & 1000 \times 0.856 \times 0.759 \times \cdots \times 0.354 & 15\,\text{had at least 9 children;}
\end{array}
$$

   The results are given in Column (5) of Table 8.11.

2) The number of women with exactly B_i children.
   There were 1,000 married women and 856 had 1 child. Therefore 144 $(1,000 - 856)$ women never had a child. Similarly, among the 856 women who had at least 1 child, 207 $(856 - 649)$ had exactly 1 child. Among the 649 women who had at least 2 children, 191 $(649 - 458)$ had exactly 2 children; and so on. Finally, among the 41 women who had at least 9 children, 26 $(41 - 15)$ had exactly 8 children. Fifteen women had 9 or more children.

Number of Live born Children	Number of Women	Greater than or equal to Cumulative Number of Women	Parity Progression Ratio	Number of Women with at least B_i children	Number of Women with exactly B_i children
1	2	3	4	5	6
0	29,777	206,395	-	1000	144
1	42,593	176,618	0.856	856	207
2	39,489	134,025	0.759	649	191
3	26,242	94,536	0.705	458	127
4	20,452	68,294	0.722	331	99
5	15,983	47,842	0.701	232	78
6	12,809	31,859	0.666	154	62
7	10,563	19,050	0.598	92	51
8	5,480	8,487	0.446	41	26
9	3,007	3,007	0.354	15	15
Total	206,395				

TABLE 8.11 Calculation of Parity

Direct Computation

The number of women with at least B_i children may be obtained directly as

$$PP_i = \frac{W_i^c}{\sum W_i}$$

where W_i^c is the cumulative number of women at i number of live children born.

The number of women with exactly B_i children may be obtained as:

$$PP = \frac{W_i}{\sum W_i} \times k$$

where W_{im} is the total number of (married) women.

Example 8.23

Refer to Example 8.21. Calculate

1) the number of women with at least B_i children;
2) the number of women with exactly B_i children;

using the direct method.

Solution

1) The number of women with at least B_i children.
   We can obtain the results in column (5) of Table 8.8 directly by dividing each of the numbers in Column (3) by the total number of the married women which is 206,395. That is

$$\begin{aligned} PP_0 &= \frac{206,395}{206,395} \times 1000 = 1000 \\ PP_1 &= \frac{176,618}{206,395} \times 1000 = 856 \\ PP_2 &= \frac{134,025}{206,395} \times 1000 = 649 \\ \vdots & \quad \vdots \\ PP_9 &= \frac{3007}{206,395} \times 1000 = 15 \end{aligned}$$

2) The number of women with exactly B_i children.
   Similarly, we can obtain the results in column (6) of Table 8.8 directly by dividing each of the numbers in column (2) by the total number of (married) women which is 206,395. Thus,

$$\begin{aligned} PP_0 &= \frac{29,777}{206,395} \times 1000 = 144 \\ PP_0 &= \frac{42,593}{206,395} \times 1000 = 206 \\ PP_0 &= \frac{39,489}{206,395} \times 1000 = 191 \\ \vdots & \quad \vdots \\ PP_0 &= \frac{3007}{206,395} \times 1000 = 15 \end{aligned}$$

8.10.8 PREGNANCY RATE

The pregnancy rate (PR) may be defined as:

$$PR = \frac{C}{M} \times k$$

where

$C =$ Number of conceptions;
$M =$ Number of months of exposure to risk;
$k = 1,200$ taken as equivalent to 100 years.

PR has significance because it provides a new approach to measuring risk. It agrees closely with the definition of a rate (see Chapter 3). To obtain data for both the numerator and the denominator is difficult and this makes its application difficult.

8.10.9 FECUNDABILITY

Fecundability (F) is a measure of fecundity (see Section 8.1). It is the probability that a woman in a given group will conceive during a month, assuming that she completes a menstrual cycle during the month. Fecundability is defined as:

$$F = \frac{W^c}{W^e}$$

where

$W^c =$ Number of women conceiving during the month;
$W^e =$ Number of women exposed to becoming pregnant during the month.

8.11 STANDARDISED FERTILITY RATE

Generally, the rate of childbearing, as indicated earlier, varies with age. It is low before age 20 and after age 35 with a peak at about 25 to 29. If two populations were to differ radically in the way their female population is distributed between ages 15 and 49, it would affect the GFR; the population with the greatest proportion in the ages 20 to 29 would tend to have a higher GFR. Just as crude death rate may be standardised or adjusted, so is general fertility rate to control differences in age composition of the female population of childbearing age by the process of direct and indirect standadisation[46].

By the interpretation of standardised rate, we may use any population distribution as the standard for standardised fertility rate. Thus, even though we shall be using the women population as the standard population, we could also use the distribution of the total population in the specified age group (15–49) as the standard population.

8.11.1 DIRECT STANDARDISED FERTILITY RATE

Standardised fertility rate, by the direct procedure, is defined as:

$$DSFR = \frac{\sum f_a W_a^s}{\sum W_a^s} \times k$$

where

f_a = age-specific fertility rate for age group a,
W_a^s = mid-year population of women in age group a in the standard population.

Or

$$DSFR = \frac{B_E}{W^s} \times k$$

B_E = total number of expected births defined as:

$$B_E = \sum f_a W_a^s$$

W^s = total number of women in the standard population, which is the sum of women in all the age groups a:

$$W^s = \sum W_a^s$$

Example 8.24

Table 8.12 (a) shows the births and estimated mid-year population of women by age for Mauritius in 1989. The table also includes the distribution of the Tunisia's women population by age for the same year.

Compute the direct standardised fertility rate for Mauritius using her total population as standard.

Age group of women (years)	Number of women in Mauritius population in 1989	Number of births to women of specified age groups	Number of women in Tunisia population in 1989
1	2	3	4
15–19	46,417	2,028	409,337
20–24	51,462	6,927	379,663
25–29	51,580	6,408	324,720
30–34	44,906	3,460	266,050
35–39	39,286	1,393	217,155
40–44	27,741	303	157,295
45–49	22,841	19	140,143
Total	284,233	20,538	1,894,363

TABLE 8.12 (a) Number of Women, Number of Births and Total Population of Mauritius in 1989

Solution

The procedure is illustrated in Table 8.12 (b).

Age group of women (years)	Number of women in Mauritius population in 1989 W_a	Number of birth to women of specified age group in Mauritius B_a	Age specific fertility rate per woman in Mauritius f_a	Number of women in Tunisia Population in 1989 (Standard) W_a^s	Number of Expected birth in Mauritius based on Tunisia Women population f_a
1	2	3	$4 = (3)/(2)$	5	$6 = (4) \times (5)$
15–19	46,417	2,028	0.0437	409,337	17,884.3
20–24	51,462	6,927	0.1346	379,663	51,104.2
25–29	51,580	6,408	0.1242	324,720	40,341.3
30–34	44,906	3,460	0.0770	266,050	20,499.1
35–39	39,286	1,393	0.0355	217,155	7,699.9
40–44	27,741	303	0.0109	157,295	1,718.0
45–49	22,841	19	0.0008	140,143	116.6
Total	284,233	20,538		1,894,363	139,363.5

TABLE 8.12 (b) Calculation of Direct Standardised Fertility Rate.

Direct standardised fertility rate is:

$$\begin{aligned} DSFR &= \frac{\text{Total number of expected births}}{\text{Total standard population}} \times 1,000 \\ &= \frac{139,363.5}{1,894,363} \times 1,000 = 73.6 \end{aligned}$$

Interpretation

The fertility rate in 1989 that could have been expected in Mauritius if the age composition of the women of the country had been the same as that of Tunisia was 73.6 per 1,000 standard population.

Alternative Formula for Computing DSFR

Similar to direct standardised death rate, an alternative formula for computing direct standardised fertility rate may be defined as:

$$DSFR = \sum_a f_a w_a^s \times k$$

where f_a = age-specific fertility rate for the population being studied;
w_a^s = the proportion of female population of the standard population for each age group a, defined as

$$w_a^s = \frac{W_a^s}{W^s}$$

Note that

$$\sum_a w_a^s = 1$$

Example 8.25

Re-work Example 8.24 using the alternative formula for computing direct standardised fertility rate.

Solution

The preliminary calculations are done in Table 8.13.

$$\begin{aligned} DSFR &= \sum_{a=2}^{n} f_a w_a^s \times k \\ &= 0.07357 \times 1000 = 73.6 \end{aligned}$$

Age group of women (Year)	Age specific fertility rate per woman f_a	Number of women in Tunisia Population in 1989 (Standard) W_a^s	$w_a^s = \frac{W_a^s}{W^s}$	$w_a^s \times f_a$
1	2	3	4	$5 = (2) \times (4)$
15–19	0.0437	409,337	0.21608	0.00944
20–24	0.1346	379,663	0.20042	0.02698
25–29	0.1242	324,720	0.17141	0.02130
30–34	0.0770	266,050	0.14044	0.01082
35–39	0.0355	217,155	0.11463	0.00406
40–44	0.0109	157,295	0.08303	0.00091
45–49	0.0008	140,143	0.07398	0.00006
Total		1,894,363	1.00000	0.07357

TABLE 8.13 Calculation of Direct Standardised Fertility Rate of Example 8.24

8.11.2 INDIRECT STANDARDISED FERTILITY RATE

When only the number of total births is available and a tabulation of births by age of mother is lacking we use an indirect procedure to compute the fertility rate. The formula is:

$$ISFR = \frac{B}{\sum f_a^s W_a} \times b^s \times 1,000$$

where

B = observed births (total births) in the study population;
f_a^s = "standard" set of age-specific fertility rates;
W_a = women population by age for the study population;
b^s = crude birth rate for the standard population;

Note

$f_a^s W_a =$ expected number of births in the particular population on the basis of the standard set of age-specific birth rates.

The indirect standardised fertility rate is computed in the same way as the indirect standardised death rate.

Example 8.26

Table 8.14 (a) contains women populations for Mauritius and Tunisia in 1989 and the number of births in Mauritius by age for the same year.

Compute the indirect standardised fertility rate for Tunisia using Mauritius' age specific fertility rate as standard if the total number of births in Tunisia in 1989 was 199,095.

Age group of women (year)	Number of women in Population of Mauritius in 1989	Number of births to women in Mauritius in 1989	Number of women population in Tunisia in 1989
1	2	3	4
15–19	46,417	2,028	409,337
20–24	51,462	6,927	379,663
25–29	51,580	6,408	324,720
30–34	44,906	3,460	266,050
35–39	39,286	1,393	217,155
40–44	27,741	303	157,295
45–49	22,837	19	140,143
Total	284,229	20,538	1,894,363

TABLE 8.14 (a) Number of Women Mauritius and Tunisia, and Number of Births in Mauritius in 1989

Solution

The calculations are shown in the Table 8.14 (a).
Crude birth rate for Mauritius (standard population):

$$\begin{aligned} b^M &= \frac{B^M}{W_a^M} \\ &= \frac{20,538}{284,299} \times 1,000 = 72.3 \end{aligned}$$

The observed births in Tunisia (study population) in 1989 is 199,095.

The procedure for computing the indirect standardised fertility rate for Tunisia (study population) is illustrated in Table 8.14 (b).

$$\begin{aligned} ISFR &= \frac{\text{Observed birth in Tunisia}}{\text{Expected birth in Tunisia}} \times b^s \\ &= \frac{199,095}{139,342.4} \times 72.3 = 103.3 \end{aligned}$$

Interpretation

The fertility rate for Tunisia that could have been expected if its age-specific death rate had been the same as that of Mauritius is 103.3 per 1,000 population.

Age group of women (year)	Number of women Population of Mauritius in 1989 W_a^M	Number of births to women in Mauritius in 1989 B^M	Age-specific fertility rate for Mauritius (standard) f_a^M	Number of women in Population of Tunisia in 1989 W_a	Number of Expected births in Tunisia based on Mauritius Age specific fertility rate $f_a^M W_a$
1	2	3	$4 = (3)/(2)$	5	$6 = (4) \times (5)$
15–19	46,417	2,028	0.0437	409,337	17,888.0
20–24	51,462	6,927	0.1346	379,663	51,102.6
25–29	51,580	6,408	0.1242	324,720	40,330.2
30–34	44,906	3,460	0.0770	266,050	20,485.0
35–39	39,286	1,393	0.0355	217,155	7,709.0
40–44	27,741	303	0.0109	157,295	1,714.5
45–49	22,837	19	0.0008	140,143	112.1
Total	284,229	20,538		1,894,363	139,342.4

TABLE 8.14 (b) Preliminary Calculation of Indirect Standardised Fertility Rate

Alternative Formula for Indirect Standardised Birth Rate

Similar to indirect standardised death rate, suppose that the data available are:

1) for study population:
   i) composition of female across the various segments (in this case across ages), w_a;
   ii) the crude birth rate, b.

3) for standard population:
   i) age specific fertility rates, $f^s a$;
   ii) the crude birth rate, b^s.

Then the indirect standardised birth rate is given by:

$$ISFR = \frac{b}{\sum \frac{W_a}{W} \times f_a^s} \times b^s \times k$$

or

$$ISFR = \frac{b}{\sum w_a \times f_a^s} \times b^s \times k$$

where w_a is as defined above.

Example 8.27

Suppose that in addition to the Table 8.11 (a), we were given the crude birth rate of Mauritius to be 0.0723 and the number of births in Tunisia to be 199,095. Calculate the indirect standardised fertility rate for Tunisia using the age-specific birth rate of Mauritius as standard.

Solution

The birth rate of Tunisia is:

$$b = \frac{199,095}{1,894,363} = 0.10510$$

The preliminary calculations are done in Table 8.15.

Age Group (Years)	Age-Specific Fertility rate of Mauritius f_a^M	Population of Women in Tunisia W_a	$\frac{W_a}{W}$	$\frac{W_a}{W} \times f_a^M$
1	2	3	4	$5 = (2) \times (4)$
15–19	0.0437	409,337	0.21608	0.00944
20–24	0.1346	379,663	0.20042	0.02698
25–29	0.1242	324,720	0.17141	0.02129
30–34	0.0771	266,050	0.14044	0.01081
35–39	0.0355	217,155	0.11463	0.00407
40–44	0.0109	157,295	0.08303	0.00091
45–49	0.0008	140,143	0.07398	0.00006
Total		1,894,363	1.00000	0.07356

TABLE 8.15 Calculation of ISDR for Example 8.24

Therefore, the indirect standardised fertility rate, ISFR, is

$$
\begin{aligned}
ISFR &= \frac{b}{\sum \frac{W_a}{W} \times b_a^M} \times b^M \times k \\
&= \frac{0.10510}{0.07356} \times 0.0723 \times 1,000 = 103.3
\end{aligned}
$$

8.11.3 UNITED NATIONS SEX-AGE ADJUSTED BIRTH RATE

In other situations, all the data available to us are only:

1) the total number of births, and
2) the age distribution of women in the reproductive span.

In such a case we resort to a practical application of the indirectly standardised rate which is the United Nations sex-age adjusted birth rate (UNSAABR). It is defined as

$$
UNSAABR = \frac{B}{\sum q_a W_a} \times k
$$

where q_a's are the standard weights $1, 7, 7, 6, 4, 1$ representing the relative fertility rates by age for the five-year age groups 15–19, 20–24, 25–29, 30–34, 35–39, 40–44, respectively. These weights were deduced by the United Nations Population Division, on the basis of observations of the age-specific fertility rates of 52 countries, representing both the low and high fertility populations. The UNSAABR is expressed as the number of births per 1,000 of the weighted sum of women in the reproductive ages (15–44).

Example 8.27

For the data in Example 8.3, calculate the UNSAABR.

Solution

The preliminary results are presented in Table 8.16.

Therefore the UNSAABR is:

$$UNSAABR = \frac{B}{\sum q_a W_a} \times k$$
$$= \frac{20,538}{1,222,032} \times 1,000 = 16.8$$

Age group	Mid-year Women Population	Number of Births	Standard Weights w_a	$w_a W_a$
1	2	3	4	5=(3) × (4)
15–19	46,417	2,028	1	46,417
20–24	51,462	6,927	7	360,234
25–29	51,580	6,408	7	361,060
30–34	44,906	3,460	6	269,436
35–39	39,286	1,393	4	157,144
40–44	27,741	303	1	27,741
45–49	22,841	19	...	...
Total	284,233	20,538		1,222,032

TABLE 8.16 Number of Births by Age of Women in Mauritius, 1984

EXERCISES

8.1 The mid-year population of the Republic of Mauritius in 2002 was 1,210,196; in 2003 was 1,222,811 and in 2004 was 1,233,386. In these years the number of live births were 19,799; 19,165 and 19,230 respectively. Calculate:

1) Crude birth rate for each year;
2) Mean annual crude birth rate for the entire period;
3) Annual average crude birth rate for the entire period;
4) Mid-period crude birth rate for the entire period.

8.2 The mid-year population of a community is 50,000 with 25,500 women in the ages 15 and 49 years. In that year 800 children were born. Calculate the:

1) crude birth rate;
2) general fertility rate.

8.3 The following were data for a community in a certain year:

Vital Events	Number		
	Total	Black	White
Estimated population			
Total as of July 1	100,000	60,000	40,000
Women between ages 15–49			
as given by the census	49,000	28,000	21,000
Women between ages 25–29	8,000	5,000	3,000
Women ever married of			
ages between 15–49	38,000	20,000	18,000
Married women	34,500	18,000	16,500
Births			
Total of all ages	4,000	3,000	1,000
Births outside marriage	3,700	2,800	900

Calculate, for the each of the racial group and combined, the following:

1) the crude birth rate;
2) the general fertility rate;
3) the general marital fertility rate;
4) the general legitimate fertility rate;
5) the general illegitimate fertility rate;
6) the illegitimacy ratio.

8.4 The following table shows the mid-year female population and the number of births of the Republic of Mauritius by age of women in 2004:

Age-group	Mid-year Women Population in Mauritius in 2004	Number of births in Mauritius in 2004
15–19	46,955	1,735
20–24	53,308	5,900
25–29	53,955	6,080
30–34	46,591	3,287
35–39	49,954	1,682
40–44	49,072	419
45–49	42,597	27

Calculate the:

1) General fertility rate;
2) Age-specific fertility rates;
3) Cumulative fertility rate for:
   i) Age group 25–34,
   ii) Age group 35–49,
3) Total fertility rate;
4) Mean age at childbearing;
5) Median age at childbearing;
6) Child-woman ratio:

$$CWR = \frac{C_{0-4}}{W_{15-44}} \times k$$

7) Children ever born;
8) Parity Progression Ratio.

8.5 The table below shows live birth order by age of mother and female population in the reproductive ages in the Republic of Mauritius in 2003. Calculate:

1) General order specific fertility rate;
2) Age and order specific fertility rate.

Age group (Years)	Mid-year Women Population in Mauritius in 2003	Live Birth Order in Mauritius in 2003					
		First	Second	Third	Fourth	Fifth	Sixth
1	2	3	4	5	6	7	8
15–19	46,391	1481	225	14	2	0	1
20–24	55,496	3721	1975	334	54	5	0
25–29	52,327	2167	2616	770	172	45	17
30–34	45,784	764	1288	835	267	90	43
35–39	51,452	348	580	531	210	100	72
40–44	47,563	91	71	103	54	43	48
45–49	41,402	7	3	5	3	3	5

8.6 The following table shows the births and estimated mid-year women population by age for Mauritius in 2003. It also includes the estimated mid-year women population of South Africa in the same year.

Age group of women (year)	Number of women Population in Mauritius in 2003	Number of births to women in Mauritius in 2003	Number of women Population in South Africa in 2003
15–19	46,391	1,692	2,448,341
20–24	55,496	6,077	2,320,855
25–29	52,327	5,776	2,111,636
30–34	45,784	3,281	1,843,665
35–39	51,452	1,837	1,449,763
40–44	47,563	411	1,312,067
45–49	41,402	26	1,157,891
Total	40,415	19,130	12,644,218

Compute the direct standardised fertility rate for the Republic of Mauritius using the female population of South Africa as standard.

8.7 The following table shows the births and women population of reproductive ages for Mauritius as enumerated in 2000. The table also includes the women population of reproductive ages for Ghana as enumerated in 2000. Compute the direct standardised fertility rate for Mauritius using the Ghana's women population as standard.

Age group of women (year)	Women Population in Mauritius in 2000	Births to women of Mauritius of specified age group in 2000	Women Population in Ghana in 2000
15–19	50,424	1,682	922,591
20–24	55,790	6,607	837,769
25–29	47,054	5,493	791,805
30–34	49,557	3,368	640,370
35–39	50,331	1,810	538,901
40–44	44,613	389	443,647
44–49	38,803	24	343,042
Total	336,572	19,373	4,518,125

8.8 The data below show some demographic data on two areas A and B in 2002.

Age group of women (year)	Number of women Population in Area A in 2002	Births to Women of reproductive age group in Area A	Number of women population in Area B in 2002
15–19	1,350	640	6,540
20–24	1,320	750	6,440
25–29	1,290	760	5,300
30–34	1,230	620	4,190
35–39	1,180	510	3,900
40-44	1,080	405	3,600
45-49	1,500	120	1,750

Compute the indirect standardised fertility rate for Area B using Area A's age specific fertility rate as standard if the total number of births in Area B was 989.

8.9 The table below shows some demographic data on Mauritius and South Africa for 2004.

Age group of women (year)	Mid-year Women Population in Mauritius in 2004	Births in Mauritius to Women of reproductive age group in 2004	Mid-year Women Population in South Africa in 2004
15–19	46,955	1,735	2,448,341
20–24	53,308	5,900	2,320,855
25–29	53,955	6,080	2,111,636
30–34	46,591	3,287	1,843,665
35–39	49,954	1,682	1,449,763
40–44	49,072	419	1,312,067
45–49	42,597	27	1,157,891

Compute the indirect standardised fertility rate for South Africa, using the Mauritius age specific fertility rate as standard if the total number of live births registered in South Africa in 2004 was 1,475,809.

8.10 For the table below, calculate the UN sex-age adjusted birth rate.

Age group	Mid-year Women Population	Number of Births
15–19	546,417	5,683
20–24	651,462	8,665
25–29	651,580	7,932
30–34	544,906	5,671
35–39	439,286	3,306
40–44	327,741	2,001
45–49	322,841	900

8.11 For the data in Exercise 8.9, calculate the UN sex-age adjusted birth rate for Mauritius.

9 MEASURES OF REPRODUCTIVITY

9.1 INTRODUCTION

So far we have considered the measures of mortality and fertility separately. But fertility and mortality rates are inadequate in giving us any idea about the extent at which the population is replacing itself since they ignore the sex of the newly born children and their mortality. How then can fertility and mortality be combined so that we can assess whether one is more or less offsetting the other, whether fertility is sufficient to balance mortality. Measures that are appropriate for this purpose are reproductivity rates.

Reproductivity or population replacement refers to the extent to which a cohort is replacing its own numbers by the natural processes. It is the net force of fertility and mortality and is usually expressed in terms of a generation rather than a year or other brief period of time.

9.2 TYPES OF REPRODUCTIVITY MEASURES

Reproductivity may be measured using simple and conventional rates.

9.2.1 SIMPLE MEASURES OF REPRODUCTIVITY

Simple measures of reproductivity are measures of comparing the number of births and deaths on an annual (instead of a generation) basis. These measures include;

1) natural increase;
2) crude rate of natural increase;
3) vital index.

Natural Increase

Natural increase (NI) is the change in population due to the net effect of fertility and mortality. It is defined as the difference between the number of births and the number of deaths in a given year. Thus,

$$NI = B - D$$

where

B = births during a calendar year;
D = deaths during a calendar year.

When the $B - D$ is negative, the change is referred to as natural decrease.

Example 9.1

In Mauritius, 20,875 children were born and 6,946 people died in 1989. Calculate the natural increase.

Solution

$$\begin{aligned} B &= 20,875; \quad D = 6,946 \\ NI &= B - D \\ &= 20,875 - 6,946 = 13,929 \end{aligned}$$

That is, the population of Mauritius increased naturally by 13,929 people in 1989.

Crude Rate of Natural Increase

The crude rate of natural increase (CRN), also referred to as ***crude rate of reproductive change***, is the simplest measure of population growth. It is defined as the (algebraic) excess of births over deaths per 1,000 of the population, that is,

$$CRN = \frac{B - D}{P} \times 1,000$$

Example 9.2

In addition to the data in Example 9.1, the mid-year population of Mauritius was 1,064,000. Calculate the crude rate of natural increase.

Solution

$$\begin{aligned} B &= 20,875; \quad D = 6,946; \quad P = 1,064,000 \\ CRN &= \frac{B - D}{P} \times 1,000 \\ &= \frac{20,875 - 6,946}{1,064.000} \times 1,000 = 13.1 \end{aligned}$$

That is, the population of Mauritius increased naturally by 13.1 per 1,000 persons in 1989.

Alternative Definition of Crude Rate of Natural Increase

Depending on the data at hand the crude rate of natural increase may also be defined as the difference between the crude birth rate and the crude death rate:

$$CRN = b - d$$

where
$b =$ the birth rate;
$d =$ the death rate.

Example 9.3

Refer to Example 9.Calculate the crude rate of natural increase.

Solution

$$
\begin{aligned}
P &= 1,064,000 \qquad B = 20,875 \qquad D = 6,946 \\
b &= \frac{20,875}{1,064,000} \times 1,000 = 19.619 \\
d &= \frac{6,946}{1,064,000} \times 1,000 = 6.528 \\
\therefore \quad CRN &= b - d \\
&= 19.619 - 6.528 = 13.1
\end{aligned}
$$

Advantages of Crude Rate of Natural Increase

1) It is simple and easy to compute.
2) The rate of natural increase is the most direct indication of how rapidly a given population actually grew during a given year as a result of vital processes. It is the current annual rate of growth, exclusive of migration. If births exceed deaths, there is growth and the rate is positive. If deaths outnumber births, the population fails to increase during the year, and the rate is negative.

Disadvantages of Crude Rate of Natural Increase

1) This rate represents only the change due to the balance of births and deaths and does not include the effect of migration. Hence it does not give the actual population change.
2) It suffers from the drawbacks of crude birth rate and crude death rate.
3) The rate of natural increase may be affected indirectly by migration. If migrants have higher or lower fertility and mortality than the remainder of the population their movement will have some influence on the annual birth and death rates.

In the absence of migration, although the rate of natural increase and the rate of growth both represent the same thing, they are not identical. They are calculated differently. The rate of natural increase is based on the mid-year population while the rate of growth is based on the population at the beginning of the year.

Vital Index

The vital index (VI) is the ratio of the number of births to the number of deaths, times 100, that is,

$$VI = \frac{B}{D} \times 100$$

or equivalently, the ratio of the birth rate to the death rate times 100:

$$VI = \frac{b}{d} \times 100$$

Example 9.4

Refer to Example 9.3 and calculate the vital index.

Solution

$$
\begin{aligned}
B &= 20,875; \qquad D = 6,946 \\
VI &= \frac{B}{D} \times 100 \\
&= \frac{20,875}{6,946} \times 100 = 300.5
\end{aligned}
$$

Or from Example 9.3

$$
\begin{aligned}
b &= 19.619 \quad d = 6.528 \\
VI &= \frac{b}{d} \times 100 \\
&= \frac{19.619}{6.528} \times 100 = 300.5
\end{aligned}
$$

That is, for every 100 deaths in Mauritius in 1989, there were about 301 births.

Advantages of Vital Index

The vital index has some advantages.

1) It is simple and easy to calculate.
2) It is regarded as a fairly reliable statistical constant reflecting the net biological status of the population as a whole. Thus:
   i) If the ratio $VI > 100$ then the population is regarded as having good medical care.
   ii) If $VI > 100$ then the population is not holding its own.
3) It can be computed for an area for which post-censal population estimates are not available.
4) It does indicate the extent to which the force of fertility exceeds that of mortality at a given time.

Disadvantages of Vital Index

There are two main disadvantages of vital index.

1) It merely gives a measure of whether births exceed deaths or not. It certainly fails to give us any idea about the trend in the population growth, that is, it does not tell us anything about whether population has a tendency to increase or decrease.
2) It suffers from the drawbacks of crude birth and crude death rate.

9.2.2 CONVENTIONAL REPRODUCTION RATES

The simple measures discussed in section 9.1 may be highly misleading since such figures, as we have explained, may be influenced by temporary factors. For example, if owing to high birth rates in the past there is a relatively young population, for the time being, there will be a low death rate and relatively few deaths and abnormally large number of births because of the high proportion of young marriageable people in the population. We may want to know whether births will balance deaths given the continuation of fertility and mortality trends, once this abnormality has passed.

The earliest and most commonly used measure is that which eliminates only age and sex structure. This can be done easily by combining figures of total fertility and the measurement of mortality as calculated in a life table. The total fertility rate shows how many children would be born to a group of women if they experienced the specific fertility rates of the year or series of years which are being examined. These children born will include both male and female, but if we are considering whether births are sufficient to replace the existing generation of women we must include only female births.

Conventional reproductivity measures attempt to eliminate such "abnormalities" and tell us whether each generation will replace itself on the basis of the fundamental factors influencing mortality and fertility. Conventional reproduction rates measure the replacement of the female population only. Such measurements of replacement tell us only whether in the long run the population will increase or decrease, given existing mortality or fertility rates. Conventional reproduction rates are usually expressed per woman, however they could be expressed per 100 or 1,000 women. The conventional reproduction rates include the gross reproduction rates and the net reproduction rates.

9.3 GROSS REPRODUCTION RATE

9.3.1 CONCEPT AND INTERPRETATION OF GROSS REPRODUCTION RATE

Gross reproduction rate (GRR) is a measure of population replacement which describes the rate of increase of population over a generation. It tackles directly the problem of estimating how quickly a population is reproducing itself. Exhibiting the rate at which mothers would be replaced by daughters and the old generation by the new, it assumes that,

1) all newly born daughters survive to the end of the childbearing period,
2) all newly born daughters experience throughout the reproductive period, the current level of fertility.

GRR can be interpreted as:

1) the average number of daughters that a woman would bear by the time she reaches her forty-ninth birthday period if she conforms to the age-specific fertility rates of a given year;
2) the average number of daughters that a group of k females starting life together would bear if all the initial group of females survived the childbearing period.

9.3.2 METHODS OF CALCULATING GROSS REPRODUCTION RATE

GRR is the ratio of the hypothetical total of female babies, who would be born to a group of women who start their childbearing period together and who neither die nor migrate until they have reached the end of that period, to the total number of women considered. It is the age specific fertility rates calculated from female births for each single year of childbearing age.

Definition of GRR for Single Years of Age

If the data are for single years of age then

$$GRR = \sum_{a=15}^{m=49} \frac{B_a^w}{W_a} \times k$$

where $k =$ 1; 100 or 1,000.

Definition of GRR for Equal Interval of Age-groups

GRR for equal class interval is computed as:

$$GRR = n \sum_{a=15-19}^{m=45-49} \frac{B_a^w}{W_a} \times k$$

where

B_a^w = female live births to mothers of childbearing age;
W_a = mid-year population of women of the childbearing age;
n = class width for the age group.

If the data are for 5-year age group, as is usually the case $(n = 5)$ then

$$GRR = 5 \sum f_a^w \times 1,000$$

where $f_a^w =$ fertility rate for female births.

Example 9.5

Refer to Table 9.1 (a).

Age-group (years)	Mid-year women population	Female births
15–19	46,417	993
20–24	51,462	3,424
25–29	51,580	3,129
30–34	44,906	1,660
35–39	39,286	706
40–44	27,741	142
45–49	22,837	6

TABLE 9.1 (a) Mid-year Women Population and Female Live Births of Mauritius, 1989
Source: United Nations Demographic Yearbook (1980–1993), (New York: Statistics Office, Department of Economic and Social Affairs)

Calculate the gross reproduction rate.

Solution

The procedure is illustrated in Table 9.1 (b):

Age-group (years)	Mid-year women population W_a	Female births B_a^w	Age-specific fertility rate of females
1	2	3	$4 = (3)/(2)$
15–19	46,417	993	0.02139
20–24	51,462	3,424	0.06653
25–29	51,580	3,129	0.06066
30–34	44,906	1,660	0.03697
35–39	39,286	706	0.01797
40–44	27,741	142	0.00512
45–49	22,837	6	0.00026
Total	284,229	10,060	0.2089

TABLE 9.1 (b) Preliminary Calculation of Gross Reproduction Rate

$$\begin{aligned} GRR &= 5\sum f_a^w \times 1,000 \\ &= 5(0.2089)(1,000) = 1045 \end{aligned}$$

Interpretation

That is, 1,045 daughters per 1,000 women would be born by a group of females starting life together if all the initial group of females survived the childbearing periods. or 1.045 daughters per woman if she survived the childbearing periods.

Definition of GRR for Unequal Interval of Age Groups

If the age-groups do not have equal intervals, then

$$GRR = \sum_{a=15-19}^{m=45-49} n_a \times \frac{B_a^w}{W_a} \times k$$

Example 9.6

Table 9.2 (a) presents Table 9.1 (a) by unequal age-groups. Calculate the gross reproduction rate.

Age-group (years)	Mid-year women population	Female births
15–19	46,417	993
20–24	51,462	3,424
25–34	96,486	4,789
30–49	89,864	854

TABLE 9.2 (a) Mid-year Women Population and Female Live Births of Mauritius by Unequal Age-group, 1989

Solution

The procedure is illustrated in Table 9.2 (b).

Age-group (years)	Mid-year women population	B_a^w	Class width n_a	Birth rates of Daughters only $f_a^w = \frac{B_a^w}{W_a}$	Expected female births per woman $n_a \times \frac{B_a^w}{W_a}$
1	2	3	4	5	$6 = (4) \times (5)$
15–19	46,417	993	5	0.02139	0.10695
20–24	51,452	3,424	5	0.06653	0.33265
25–34	96,486	4,789	10	0.04963	0.49630
30–49	89,864	854	15	0.00950	0.14250
Total	284,229	10,060			1.07840

TABLE 9.2 (b) Preliminary calculation of the gross reproduction rate

$$\begin{aligned} GRR &= \sum_{a=15-19}^{m=45-49} n_a \times \frac{B_a^w}{W_a} \times 1,000 \\ &= 1.07840 \times 1,000 = 1078.4 \end{aligned}$$

Interpretation

That is, 1,078.4 daughters would be born by a group of 1,000 women at the current fertility level if all the initial group of 1,000 women survived the childbearing age; or about one daughter on the average would be born by a woman if she survived the childbearing age at the current fertility levels.

Computation of GRR from the TFR

The GRR is a special case of the TFR In fact, it is a modified form of TFR for an one-sex model. TFR has been defined in Chapter 8 as "the sum of age-specific fertility rates of women of childbearing ages". GRR is the sum of age-specific fertility rates of women of childbearing ages, restricted to female births only. Hence, whereas the numerator of the TFR is based on total births, the numerator of the GRR is based on female births only. Otherwise, this rate is essentially the same as the total fertility rate. It is, therefore, easy to convert TFR to a GRR simply by multiplying TFR by the proportion of the total births (during the corresponding period) that were female. Thus,

$$GRR = \delta TFR$$

where

$$\delta = \frac{B^w}{B}$$

That is, δ is the proportion of all births B which are females B^w under the assumption that the sex-ratio at birth remains more or less constant at all the ages of the women in the reproductive period.

Alternatively,

$$\delta = \frac{1}{1 + \text{Sex Ratio at birth}}$$

Example 9.7

Using the data in Example 9.5, compute GRR if 49 percent of the births by women in the reproductive period were females.

Solution

For convenience we reproduce the data here as Table 9.3.

$$\begin{aligned} TFR &= 5\sum f_a^w \\ &= 5(0.42678) = 2.1339 \text{ babies per woman} \end{aligned}$$

Not all these babies are females. The proportion of female births $\delta = 0.49$, hence

$$GRR = 0.49(2.1339) = 1.0456$$

Age-group	Mid-year women population	Total births	Age-specific fertility rate
1	2	3	$4 = (3)/(2)$
15–19	46,417	2,028	0.04369
20–24	51,462	6,927	0.13460
25–29	51,580	6,408	0.12423
30–34	44,906	3,460	0.07705
35–39	39,286	1,393	0.03546
40–44	27,741	303	0.01092
45–49	22,837	19	0.00083
Total	284,229	20,538	0.42678

TABLE 9.3 Preliminary calculation of the Gross Reproductive Rate

Note

GRR may be computed for males on the basis of male births rather than female births though such a measure is rather rare.

9.3.3 ADVANTAGES AND DISADVANTAGES OF GRR

Advantage of Gross Reproduction Rate

1) GRR, as a measure of fertility, is quite useful for comparing the fertility in different regions or in the same region at different periods of time.
2) GRR can be considered as a replacement index
3) Implicit in GRR is the concept of growth.

Disadvantage of Gross Reproduction Rate

GRR would overestimate growth as it takes no account of the deaths of females at all ages from 15–49. It is computed on the hypothesis that mortality through the childbearing period is zero, that is, none of the newly born female babies is subject to the risk of mortality till the end of the reproductive period. This is a very serious limitation of GRR since all the girls born do not survive till the end of the childbearing span. Accordingly, GRR leads to fallacious conclusions as it inflates the number of potential mothers.

9.4 NET REPRODUCTION RATE

9.4.1 CONCEPT OF NET REPRODUCTION RATE

As already pointed out, the principal drawback of GRR is that it completely ignores the current mortality and takes into account only the current fertility. This limitation is overcome in net reproduction rate (NRR).

NRR is nothing but GRR adjusted for the effects of mortality. NRR takes into account the fact that out of a number of girls born to 1,000 women some die in infancy or before completing their childbearing years (either before attaining the age of reproduction or during the reproductive span). Others will live to complete their reproductive life. NRR consists of a hypothetical cohort of women, their deaths and all their female births during the childbearing period. It measures the number of daughters that a cohort of newborn female babies will bear during their lifetime assuming a fixed schedule of age-specific fertility rates and a fixed set of mortality rates; in other words, NRR measures the extent to which a generation of newly born girls survive to reproduce themselves as they pass through the childbearing age group under given schedules of age-specific fertility and mortality rates. Some girls will die before attaining the age of reproduction, others will die during the reproductive span, and others will live to complete their reproductive life. Also some of the women do not give birth at all and it is only the remaining females who pass through the reproductive period and thus add to the population growth.

Interpretation of Net Reproduction Rate[47]

NRR may be interpreted as the

1) average number of daughters who will be born by a woman if she survived the childbearing age conforming to the age-specific fertility and mortality rates of a given year.
2) average number of daughters who will be born by a cohort of *k* females starting life together with fixed age-specific fertility and mortality rates of a given year.

If all the new born girls survive at least till the end of the reproductive period NRR = GRR. Otherwise NRR $<$ GRR. Since NRR $\leq$ GRR then GRR provides an upper limit to NRR. Consequently,

$$0 \leq NRR \leq 5$$

$NRR = 1$ (or 100 or 1,000), depending on the value of *k* means exact replacement, that is, if the current fertility and female mortality rates prevail in future, then a cohort of new born girls will exactly replace itself in the next generation. In other words, present female generation will exactly maintain itself. The population, therefore, has a tendency to remain more or less constant.

$NRR > 1$ indicates that the women are bearing more than adequate daughters to represent them in the next generation of mothers. That is, the population is more than replacing itself, having a tendency to increase.

$NRR < 1$ means that the women are not bearing adequate daughters to represent them in the next generation of mothers. That is, the population is not replacing itself, indicating a declining population.

9.4.2 METHODS OF CALCULATING NET REPRODUCTION RATE

NRR computed from Unabridged Life Table

When the data available are for single years of age NRR is defined as:

$$NRR = \sum_{a=15}^{m=49} f_a^w \times \frac{L_a^w}{l_o^w} \times k$$

where $f_a^w = \frac{B_a^w}{W_a}$ is age-specific female fertility rate at each age a;

$l_o^w = 100,000$ (a benchmark called the radix of the life table);

$L_a^w = \frac{1}{2}(l_a^w + l_{a+1}^w)$

= the number of person-years that would be lived within the indicated age interval (a to $a+1$) by the cohort of 100,000 female births assumed, that is, the number of years lived by the females who have reached age a, before they reach the next higher age;

l_a^w = female survivors at age a;

l_{a+1}^w = number of female survivals from age-group a to $a+1$;

$\frac{L_a^w}{l_o^w}$ = life-table female survival rate.

This formula means "the sum of the daughters per *k* women in the cohort, beginning at age 15, up to the end of age 49".

NRR Computed from Abridged Life Table

Using age in completed years

When 5-year age groups are used ($n = 5$), the NRR may be defined using the conventional rate that relates to women of the same age in completed years. The numbers of survivors are related to 15 years, 20 years, 25 years, 30 years, 35 years, 40 years, 45 years, and 50 years, respectively from the unabridged life table.

We have

$$NRR = \sum_{a=15-19}^{m=45-49} f_a^w \times \frac{{}_5L_a^w}{l_o^w}$$

where

$$_5L_a^w = \frac{5}{2}\left[l_a^w + l_{(a+5)}^w\right]$$

or

$$NRR \quad = \quad \sum_{a=15-19}^{m=45-49} f_a^w \times \frac{_5L_a^w}{l_o^w} \times k$$

for a cohort of k daughters.

Example 9.8

The mid-year women population, and the number of female births (all in thousands) and the life table female survivors of a country for 1986 are shown in the table below. If female survivors for age group 50–54 are 81,000 per 100,000 women calculate the net reproduction rate. (Hint: $l_0 = 100,000$).

Age-group (years)	Mid-year women population	Female births	Life table survivors
15–19	4,000	210	93,200
20–24	3,800	240	91,500
25–29	3,500	230	89,400
30–34	3,400	200	86,600
35–39	3,100	150	85,300
40–44	3,000	100	84,000
45–49	2,500	70	82,700

TABLE 9.4 (a) Hypothetical Data for Calculation of Net Reproductive Rate

Solution

The calculations are illustrated in Table 9.4 (b). In column 5, for example,

$$\begin{aligned} _5L_{15-19}^w &= \frac{5}{2}(93,200 + 91,500) = 461,750 \\ _5L_{20-24}^w &= \frac{5}{2}(91,500 + 89,400) = 452,250 \\ &\vdots \\ _5L_{20-24}^w &= \frac{5}{2}(82,700 + 81,000) = 409,250 \end{aligned}$$

Hence,

$$NRR = \sum_{a=15-19}^{m=45-49} {}_5f_a^w \times \frac{{}_5L_a^w}{l_o^w} \times 1,000 = 1,528.2$$

Interpretation

That is, 1,528.2 daughters will be born by a group of 1,000 females starting life together, assuming a fixed schedule birth rates and death rates.

Age group (year)	Mid-year women population	Female births	Life table survivors for females	Age-specific female fertility rate per		Expected Survival rate per woman	Expected female births per woman
		L_a^w	l_a^w	${}_5f_a$	${}_5L_a^w$	$\frac{{}_5L_a^w}{l_o^w}$	${}_5f_a \times \frac{{}_5L_a^w}{l_o^w}$
1	2	3	4	5	6	$7 = (6)/(4)$	$8 = (5) \times (7)$
15–19	4,000	210	93,200	0.0525	461,750	4.6175	0.2424
20–24	3,800	240	91,500	0.0632	452,250	4.5225	0.2858
25–29	3,500	230	89,400	0.0657	440,000	4.4000	0.2891
30–34	3,400	200	86,600	0.0588	429,750	4.2975	0.2527
35–39	3,100	150	85,300	0.0484	423,250	4.2325	0.2049
40–44	3,000	100	84,000	0.0333	416,750	4.1675	0.1388
45–49	2,500	70	82,700	0.0280	409,250	4.0925	0.1146
Total				0.3499			1.5282

TABLE 9.4 (b) Preliminary calculation of the Net Reproductive Rate

Note

Though, throughout, NRR has been expressed per 1,000 woman, it is usually expressed per woman. Thus, the total of column 8 gives the average number of daughters per woman, that is, an average of 1.5 daughters per woman would be born.

NRR at age group "a" for women who reach age group "$a+n$"

We consider the NRR at age group "a" for women who reach age group "$a+n$" during the year in question. The number of survivors are those relating to 17, 22, 27, 32, 37, 42, and 47 years from the unabridged table. That is, we use the female survival rate to the mid-point of each age interval $\frac{l^w(a+n)}{l_o^w}$. For example, the survival rate of age 17 is

$$\frac{l_{(17)}^w}{l_o^w} = \frac{93,200}{100,000} = 0.932$$

In addition, the fertility rate for the 5-year age group is a mean fertility rate valid for 5 years. Therefore, in this approach, the result needs to be multiplied by 5 to produce the NRR. Hence the NRR is defined as

$$NRR = 5 \sum_{a=15-19}^{m=45-49} f_a^w \times \frac{L_a^w}{l_o^w}$$

Explanation of Technique

It is assumed that during the five years the women spend in the age group of, say, 30–34 years, 1,000 women will bring, on the average, 58.8 (multiplying column 3 by 1,000) female children into the world each year but according to the life table, only 866 of the 1,000 women (i.e. 86,600 of 100,000) at birth will have female children while in this age group (column 4). Consequently, it is necessary to correct the number of female births that will occur (0.0588×0.866). Thus, we have 0.05092 births per female births or 50.9 births per 1,000 female births. Continuing in this way, for all the age groups and adding the results, we obtain 0.30883 daughter per woman (from birth) per year. However, as the women remain fixed in each 5-year age group, it is necessary to multiply these 0.30883 female births by 5, to give 1.54414 total female births per woman per cohort or 1,5441.4 daughters per 1,000 women.

Example 9.9

For the data in Example 9.8, calculate the net reproduction rate.

Solution

The calculations are in Table 9.5 (b).

$$\begin{aligned} NRR &= 5\sum_{a=15-19}^{m=45-49} f^w a \times \frac{L_a^w}{l_o^w} \\ &= 5(0.30883) = 1.54415 \end{aligned}$$

Age group (year)	Life table survivors for females L_a^w $l_o^w = 100,000$	Age-specific female fertility rate ${}_5f_a$	Life table Survival rate per woman $\frac{{}_5L_a^w}{l_o^w}$	Expected female births per woman ${}_5f_a \times \frac{{}_5L_a^w}{l_o^w}$
1	2	3	$4 = (2)/100,000$	$5 = (3) \times (4)$
15–19	93,200	0.0525	0.932	0.04893
20–24	91,500	0.0632	0.915	0.05783
25–29	89,400	0.0657	0.894	0.05874
30–34	86,600	0.0588	0.866	0.05092
35–39	85,300	0.0484	0.853	0.04129
40–44	84,000	0.0333	0.840	0.02797
45–49	82,700	0.0280	0.827	0.02316
Total		0.3499		0.30883

TABLE 9.5 Preliminary calculation of the Net Reproductive Rate

Note

The two methods for computing NRR from the abridged life table may be used depending on which of ${}_nL_a^w$ or L_a^w is available. The results may not be exactly the same but very close.

Computation of NRR from Total Population

Sometimes instead of female births (f_{aw}) being given, only the total births (f_a) and total number of the general population are given. In this case NRR will be computed as:

$$NRR = \delta \sum_{a=15-19}^{m=45-49} f_a \times \frac{L_a}{l_0} \times k$$

where

$f_a \quad = \frac{B_a}{P_a}$ – age-specific fertility rate at age a;

$\delta \quad$ = proportion of all births which are females;

$L_a \quad$ = the number of years lived by the persons who have reached age a, before they reach the next higher age.

Example 9.10

Table 9.6 (a) gives the population and number of total births of a country for 1988. The life table survivors for age group 50–54 were 80,026.

Age-group (year)	Total Pop- ulation	Total births	Life Table Survivors $l_0 = 100{,}000$
15–19	495,426	6,200	94,888
20–24	491,378	17,520	93,750
25–29	486,260	14,840	92,218
30–34	480,300	10,850	90,255
35–39	471,121	9,870	87,829
40–44	465,967	5,050	84,734
45–49	360,703	4,200	80,802

TABLE 9.6 (a) Data for calculation of the Net Reproductive Rate

Calculate the net reproduction rate, assuming that the sex-ratio at birth was 103.4 males to 100 females.

Solution

In this example if we are assuming that the age groups are in completed years then the calculations are similar to those of Example 9.8.

$$NRR = \delta \sum_{a=15-19}^{m=45-49} f_a \times \frac{{}_5L_a}{l_0}$$

where

$$\delta = \frac{1}{1.034 + 1} = 0.492$$

All preliminary calculations are given in Table 9.6 (b).

From column 8,

$$\sum_{45--49}^{a=15--19} f_a \times \frac{{}_5L_a}{l_0} = 0.648$$

$$\therefore \qquad NRR = 0.492(0.648) = 0.3188$$

That is, an average of 318.8 daughters will be born by 1,000 women starting life together, assuming a fixed schedule of birth and death rates.

NRR for Males

NRR may be computed similarly for males on the basis of male mortality and fertility. However, NRR for the two sexes for the same period *t* will generally not be alike and, therefore, there will be two figures for current population growth. Such difficulty may be attributed to the following:

1) Factors affecting the proportions of married people in the two sexes are different and they will provide fortuitous influences in their current fertility rates.
2) Since the NRR measures the rate of growth per generation, it can hardly be expected that the current situation for the two sexes will yield the same result simply because the length of the male generation is generally greater than the female.

Note

NRR, though is derived from the data of a loose period, is not the same as vital-statistics rate of births or deaths "per year". It covers the whole reproductive period of the hypothetical cohort and might be regarded as a rate "per cohort" or "per generation" of these women.

9.4.3 ADVANTAGES AND DISADVANTAGES OF NRR

Advantage of Net Reproduction Rate

NRR takes into account both the current mortality and current fertility in its computation.

Age group (year)	Total population	Total births	Life table survivors $l_0 = 100,000$	Age-specific fertility rate per person f_a	${}_5L_a$	Survival rate per person $\frac{{}_5L_a}{l_0}$	Expected birth survived per person $f_a \times \frac{{}_5L_a}{l_0}$
1	2	3	4	$5 = (3)/(2)$	6	7	$8 = 5 \times 7$
15–19	495,426	6,200	94,888	0.0125	471,595	4.71595	0.059
20–24	491,378	17,520	93,750	0.0357	464,920	4.64920	0.166
25–29	486,260	14,840	92,218	0.0305	456,183	4.56183	0.139
30–34	480,300	10,850	90,255	0.0226	445,210	4.45210	0.101
35–39	471,121	9,870	87,829	0.0210	431,408	4.31408	0.091
40–44	465,967	5,050	84,734	0.0108	413,840	4.13840	0.045
45–49	360,703	4,200	80,802	0.0116	402,070	4.02070	0.047
Total				0.1447			0.648

TABLE 9.6 (b) Preliminary calculation of the Net Reproductive Rate

Disadvantages of Net Reproduction Rate

1) NRR assumes that current mortality and fertility rates prevail in future. In practice, this assumption is not valid because the rates are changing from time to time.
2) Changes in

   i) the proportion of the population who marry,
   ii) the age and rate of marriage and
   iii) the rate of family building, will influence specific fertility rates and therefore the NRR, even though the size of family which each married woman has during the whole course of her married life remains unchanged.

3) Net migration may affect population growth but NRR overlooks the factor of migration.
4) It is not desirable to use NRR to forecast future population changes. This is evident from (a) and (c), above.

9.5 REPRODUCTION-SURVIVAL RATIO

The reproduction-survival ratio (RSR) is defined as

$$RSR = \frac{NRR}{GRR}$$

It is interpreted as the proportion of potential reproductivity that survived the effects of mortality.

Example 9.10

For the data in Example 9.8, calculate

1) Gross reproductive rate;
2) Net reproduction rate;
3) Reproduction-survival ratio.

Comment on your results.

Solution

The preliminary calculations are in Table 9.4 (b).
From column 5 of 9.4 (b),

$$\begin{aligned} GRR &= 5\sum f_a^w \\ &= 5 \times 0.3499 = 1.7495 \end{aligned}$$

From column 8 of 9.4 (b), $NRR = 1.5283$.
Therefore,

$$RSN = \frac{NRR}{GRR} = \frac{1.5282}{1.7495} = 0.87$$

That is, if 1986 fertility levels were to continue, a woman in that country would produce 1.7 daughters, on the average, during her childbearing age. However only 1.5 of the daughters would survive through the childbearing years. That is, 87 percent of the daughters would survive the effects of mortality at their childbearing age.

EXERCISES

9.1 The mid-year population of Egypt in 1987 was 49,089,000. In that year 1,902,604 children were born and 466,261 people died. Calculate the

1) Natural increase;
2) Crude rate of natural increase;
3) Vital index.

9.2 The following table shows the mid-year women population and female live births of a country in 2003 by age.

Age-group (years)	Mid-year women population	Female births
15–19	1,289,437	2,544
20–24	1,369,423	14,424
25–29	1,352,571	15,390
30–34	1,282,809	9,076
35–39	1,226,915	1,706
40–44	1,035,037	502
45–49	962,356	109

Calculate the gross reproduction rate and interpret the result.

9.3 The following are data on the distribution of mid-year women population and the female live births of Mauritius in 1989 by unequal age-group:

Age-group (years)	Mid-year women population	Female births
15–19	640,510	1,803
20–24	769,521	6,209
25–34	778,739	7,683
30–49	573,939	1,001

Calculate the gross reproduction rate and interpret the result.

9.4 The mid-year population and the number of births and deaths of a country for 1986 are shown in the table below.

Age-group (years)	Mid-year women population	Female births	Life table female survivors ($l_o = 100,000$)
15–19	587,000	3,892	96,400
20–24	683,800	4,840	94,800
25–29	679,100	4,730	91,500
30–34	684,900	4,690	89,600
35–39	630,000	3,870	88,200
40–44	593,000	1,300	86,900
45–49	568,200	860	83,800

Calculate the:

1) Total fertility rate;
2) Gross reproduction rate;
3) Net reproduction rate if
   i) from the unabridged life table the average female survivors for age group 50-54 are 81,000 per 100,000 women;
   ii) the female survivors are taken from the abridged life table.
4) Reproduction-survival ratio.

Interpret the results in each case.

10 NUPTIALITY

10.1 INTRODUCTION

Nuptiality is the study of the entire process of marriages and their dissolutions as a population phenomenon. It includes the rate at which marriages occur, the characteristics of people united in marriage, and the dissolution of such unions through divorce, separation, widowhood, and annulment. Marriage is a milestone stage in the growth of human evolution and it remains the social requirement for the formation of family[48]. It is generally considered a primary indicator of exposure to the risk of pregnancy and, therefore, a relevant factor in the analysis of demographic changes (mortality, fertility and migration).

In Chapter 1, we have defined family as the group of people who are related by blood, marriage or adoption. It is the residential, social and economic set-up in which children are born and raised. Again, in the previous chapter on fertility, we considered marital status as a characteristic essential for identifying groups of women exposed to the risk of child bearing. The need to construct rates specific for marital status was also pointed out. We shall here discuss some of the important measures.

10.1.1 DEFINITION OF CONCEPTS IN NUPTIALITY

The definition of the concept of marriage and divorce as recommended by the Statistical Commission of the United Nations (1955) and World Health Organisation (1950) may not satisfy most African countries as customs differ from culture to culture. In this book, we shall attempt to confine ourselves to the concepts that are widely understood in most African countries. They are sufficient for statistical purposes.

Marriage

The definition of marriage as the "union of a man and woman" by social consent, whatever be the form, contract or obligations is giving way to a new one with the recognition of same sex marriage in some countries.

Marriage may, therefore, be defined as the act, ceremony or process by which the legal relationship of husband and wife is constituted. The legality of the union may be established by civil, religious, or other means as recognized by the laws of the country. In some countries, especially in Africa, an informal union, also known as consensual union, or cohabitation is also permitted. Both marriage and cohabitation are generally considered primary indicators of exposure to the risk of pregnancy. However, in most African countries, a union is not necessarily a prerequisite to child bearing as some child bearing occurs outside marital union.

Divorce

Divorce is the final legal dissolution of a marriage through legal procedure, simple consent or other means according to the laws of the country. It is the separation of husband and wife which confers on the parties the right to remarry under civil, religious and/or other provisions.

Widowhood

Widowhood is the resulting marital status when one of the spouses dies. The surviving husband is known as the widower and the surviving wife the widow.

10.1.2 USES OF STUDY OF NUPTIALITY

The study of nuptiality helps us to understand the social dynamics of a society and how it changes over time.

1) Marriage is the major determinant of fertility, especially in a country where the large majority of children are born in wedlock. Knowing the age at which people tend to get married and how many people are in union enables us to understand more about the dynamics of the population.
2) The comparison of the distribution of marital status at different periods provides information on how a society is evolving.

10.2 CLASSIFICATION OF MARITAL STATUS

The concept of marital status starts with unmarried person. Every person starts life as an unmarried person until his or her status changes to married. In-between these occurrences (unmarried and married) are common classification used to study marital composition of the population.

In a narrower sense, marital status may be classified as never married and ever married. The never married indicates that the person's marital status has never changed since birth. Ever married includes people who are currently married, separated, or once married but currently divorced or widowed. In a broader sense, marital status may be classified as unmarried (not married) or married. Unmarried, also referred to as single includes persons who have never married, are divorced or widowed. Married includes currently married as well as separated but not divorced or widowed. The discussion so far identifies marital status as a stock with proportion as the appropriate tool for analysis. Thus, we can calculate the proportion ever married, proportion unmarried and so forth.

Marriage is an event as opposed to marital status which is stock as indicated earlier. The ***marital event*** leads to a transition from one marital state to another and this includes first marriage, divorce, death of a marital partner and remarriage. As vital events, we can construct certain rates which will be discussed in this chapter.

10.3 SOURCES OF NUPTIALITY DATA

Data on marital status of persons are obtained from ***census***. The question usually asked for those of particular ages is: "What is the current marital status of [NAME]?" For instance, the 2010 Population and Housing Census of Ghana asked a question on the current marital status for those who were 12 years or older.

Data on marital events in a year, on the other hand, are obtained from ***civil registration system*** since marriages and divorces have to be registered. Data on divorces may also be obtained from court records, following national practice. However, in most developing countries, especially those in Africa, marriages are not always registered and divorces do not usually end in court. The high incidence of consensual unions in some developing countries presents special difficulties for the analysis of marriage patterns.

The third source of nuptiality data is the ***sample survey***. A survey, as a source of data on nuptiality, has an advantage over census and registration systems:

1) Survey questionnaires are able to deal with the marital history of persons in much more detail.
2) Surveys are more capable of relating nuptiality to issues such as fertility, economic status, etc.

The Demographic and Health Survey is one major sample survey which provides information on nuptiality.

Table 10.1 shows the Nigeria population by the marital status classification. It is important to note that the information in the table cannot be used to calculate the Marriage rates discussed in this text. It is better used to calculate the proportion of marital status.

It is difficult to obtain data on marital events and we generate a hypothetical data from Table 10.1 with the assumptions that:

1) 20% of the marital status in 1991 took place in that year;
2) the population is the mid-year population for 199

We then obtain Table 10.2 for the purpose of illustration in this chapter.

10.4 CRUDE MARRIAGE RATE

10.4.1 DEFINITION OF CRUDE MARRIAGE RATE

The crude marriage rate (CMR) is the number of marriages per 1,000 mid-year population in a given year. It is given as:

Age Group	Males	Females	Both Sexes	Never Married	Currently Married	Separated	Divorced	Widowed	Total
0 - 4	7344,454	6,999,435	14,343,889						
5 - 9	7,374,314	7,126,144	14,500,458						
10 14	5,812,538	5,336,143	11,148,681	10,497,408	595,938	8,945	17,119	29,271	11,148,681
15 19	4,528,811	4,806,977	9,335,788	7,290,725	1,965,218	21,617	29,897	28,331	9,335,788
20 24	3,314,303	4,357,267	7,671,570	3,865,139	3,687,304	45,370	46,657	27,100	7,671,570
25 29	3,304,739	4,006,932	7,311,671	2,117,690	5,020,333	70,265	61,883	41,500	7,311,671
30- 34	2,808,629	3,105,298	5,913,927	771,863	4,915,265	81,029	72,702	73,068	5,913,927
35 39	2,206,871	2,008,062	4,214,933	287,504	3,709,519	66,817	59,210	91,883	4,214,933
40 44	1,971,197	1,874,721	3,845,918	158,604	3,374,718	73,479	71,429	167,688	3,845,918
45 49	1,355,101	1,061,602	2,416,703	79,862	2,073,576	50,436	48,409	164,420	2,416,703
50 54	1,388,650	1,182,149	2,570,799	72,113	2,107,143	60,092	60,354	271,097	2,570,799
55 59	638,375	481,394	1,119,769	28,962	883,185	26,442	25,811	155,369	1,119,769
60 64	898,801	791,573	1,690,374	44,982	1,238,726	46,136	46,163	314,367	1,690,374
65 69	408,540	387,400	703,040	18,110	528,216	19,567	19,141	178,906	763,940
70 74	492,186	394,116	886,302	26,608	587,851	25,424	25,447	220,972	886,302
75 79	195,455	156,368	351,823	9,587	224,248	8,940	8,741	100,307	351,823
80 84	258,059	222,627	480,686	15,467	294,754	14,402	14,435	141,628	480,686
85+	230,585	194,404	424,989	14,199	239,073	12,534	11,773	147,410	424,989
Total	44,531,608	44,492,612	88,931,320	25,298,823	31,445,067	631,495	619,171	2,153,317	60,147,873

TABLE 10.1 Population Distribution of Nigeria by Marital Status, Age and Sex, 1991
Source: Annual Abstract of Statistics 2006, (Nigeria: National Population Commission)

Age Group	Males	Females	Both Sexes	Never Married	Marital Events in 1991			
					Marriages	Separation	Divorce	Widowhood
15 19	4,528,811	4,806,977	9,335,788	7,290,725	393,044	4,323	5,979	5,666
20 24	3,314,303	4,357,267	7,671,570	3,865,139	737,461	9,074	9,331	5,420
25 29	3,304,739	4,006,932	7,311,671	2,117,690	1,004,067	14,053	12,377	8,300
30- 34	2,808,629	3,105,298	5,913,927	771,863	983,053	16,206	14,540	14,614
35 39	2,206,871	2,008,062	4,214,933	287,504	741,904	13,363	11,842	18,377
40 44	1,971,197	1,874,721	3,845,918	158,604	674,944	14,696	14,286	33,538
45 49	1,355,101	1,061,602	2,416,703	79,862	414,715	10,087	9,682	32,884
50 54	1,388,650	1,182,149	2,570,799	72,113	421,429	12,018	12,071	54,21
55 59	638,375	481,394	1,119,769	28,962	176,637	5,288	5,162	31,074
60 64	898,801	791,573	1,690,374	44,982	247,745	9,227	9,233	62,873
65 69	408,540	387,400	703,040	18,110	105,643	3,913	3,828	35,781
70 74	492,186	394,116	886,302	26,608	117,570	5,085	5,089	44,194
75 79	195,455	156,368	351,823	9,587	44,850	1,788	1,748	20,061
80 84	258,059	222,627	480,686	15,467	58,951	2,880	2,887	28,326
85+	230,585	194,404	424,989	14,199	47,815	2,507	2,355	29,482
Total	24,000,302	25,030,890	48,938,292	14,810,415	6,169,826	124,510	120,410	424,809

TABLE 10.2 Population Distribution of Nigeria by Hypothetical Marital Events: Age and Sex, 1991

$$CMR = \frac{M}{P} \times k$$

where

$M =$ the number of marriages during a calendar year;
$P =$ the mid-year population;
$k =$ the radix, usually taken as 1,000.

The number of marriages M in a calendar year includes both first marriages and remarriages after divorce, widowhood or annulment.

Example 10.1

For data in Tables 10.1 and 10.2, calculate the crude marriage rate.

Solution

Number of marriages during the year (from Table 10.2), $M = 6,169,826$;
Mid-year population (from Table 10.1), $P = 88,931,320$

Thus the crude marriage rate is:

$$\begin{aligned} CMR &= \frac{M}{P} \times k \\ &= \frac{6,169,826}{88,931,320} \times 1,000 = 70.8 \end{aligned}$$

Interpretation

For every 1,000 population of Nigeria in 1991, there were 71 marriages.

10.4.2 ADVANTAGES AND DISADVANTAGES OF CRUDE MARRIAGE RATE

Advantages of Crude Marriage Rate

Similar to the crude birth rate, the crude marriage rate:

1) is simple to calculate;
2) is a simple concept and can easily be explained to laymen;
3) gives a general overview of marriage in an area.

Disadvantages of Crude Marriage Rate

CMR suffers from the usual defects of a crude rate in addition to the following.

1) There is no complete correspondence between the numerator and the denominator. A marriage involves two persons so it would be better to represent the numerator and denominator by only one sex, usually the female.
2) It does not take into account people who cannot marry. The denominator (mid-year population) includes the very young and those who are already married (in the case where one woman for one man is the situation).
3) It is influenced by population age structure. Husbands tend to be older than wives in general, so the age specific rates will be different for the sexes.

10.5 GENERAL MARRIAGE RATE

The crude marriage rate, as indicated earlier, suffers from the limitation that it relates the total number of marriages to the total mid-year population. But we know that the total number of marriages depends on the population of marriageable ages and not on the entire population. To overcome this limitation, we calculate general marriage rate.

10.5.1 DEFINITION OF GENERAL MARRIAGE RATE

General marriage rate (GMR) is a single ratio of all marriages to the number of persons of marriageable age (15 years and older):

$$GMR = \frac{M}{P_{15+}} \times k$$

where

M = as defined above;
P_{15+} = mid-year population of persons of marriageable age (15 years and older);
k = 1,000.

Example 10.2

For the data in Example 10.1, calculate the general marriage rate.

Solution

Number of marriages during the year, $M = 6,169,826;$
Mid-year population of marriageable age, $P_{15+} = 48,938,292.$

The general marriage rate is:

$$GMR = \frac{M}{P_{15+}} \times k$$
$$= \frac{6,169,826}{48,938,292} \times 1,000 = 128.6$$

Interpretation

For every 1,000 marriageable population of Nigeria in 1991, there were 129 marriages.

10.5.2 ADVANTAGES AND DISADVANTAGES OF GENERAL MARRIAGE RATE

Advantage of General Marriage Rate

If marital status distributions are available, GMR can specify with greater accuracy.

Disadvantages of General Marriage Rate

1) GMR is not adequate for a very detailed examination of marriage patterns because it ignores the age distribution of married persons.
2) The denominator includes all married persons who are not at risk of marriage and remarriage. In the one woman for one man situation,[49] the general marriage rate may be refined as:

$$GMR^u = \frac{M}{P^u_{15+}} \times k$$

where

GMR^u = the refined general marriage rate for unmarried persons;
P^u_{15+} = the mid-year population of unmarried persons of nubile age.

Example 10.3

For the data in Table 10.2, calculate the refined general marriage rate.

Solution

Number of marriages during a year, $M = 6,169,826$;
Total population of unmarried persons of nubile age[50]:

$$\begin{aligned} P_{15+}^{u} &= NM_{15+} + DV_{15+} + WD_{15+} \\ &= 14,801,415 + 120,410 + 424,809 = 15,346,635 \end{aligned}$$

where NM = never married, DV = divorced and WD = widowed

Hence, the refined general marriage rate is:

$$\begin{aligned} GMR^{u} &= \frac{M}{P_{15+}^{u}} \times k \\ &= \frac{6,169,826}{15,346,635} \times 1,000 = 410.1 \end{aligned}$$

Interpretation

For every 1,000 unmarried population aged 15 years and above, there were 410 marriages.

10.6 SPECIFIC MARRIAGE RATE

The defects of the CMR and GMR call for calculating a marriage rate specific for age, sex and order of marriage. However, the information in Table 10.2 is not sufficient to calculate these rates because there are no data on marriage according to male and female separately.

Let us generate a hypothetical data from the data in Table 10.2 on the assumption that

1) a man can marry more than one but a woman cannot;
2) there is 40% to 60% marriages of men to women up to age 39 and the reverse is the case thereafter;
3) 80% of all marriages in the year were first marriages.

This produces Table 10.3. We remind readers that this information is for illustration purpose only as it does not represent real data.

Age Group	Mid-year Population		Marriages		Order of Marriages	
	Males	Females	Males	Females	First Marriages	Second Marriages
15–19	4,528,811	4,806,977	157,217	235,826	373,391	19,652
20–24	3,314,303	4,357,267	294,984	442,476	700,588	36,873
25–29	3,304,739	4,006,932	401,627	602,440	953,863	50,203
30–34	2,808,629	3,105,298	393,221	589,832	933,900	49,153
35–39	2,206,871	2,008,062	296,762	445,142	704,809	37,095
40–44	1,971,197	1,874,721	404,966	269,977	641,196	33,747
45–49	1,355,101	1,061,602	248,829	165,886	393,979	20,736
50–54	1,388,650	1,182,149	252,857	168,571	400,357	21,071
55–59	638,375	481,394	105,982	70,655	167,805	8,832
60–64	898,801	791,573	148,647	99,098	235,358	12,387
65–69	408,540	387,400	63,386 4	2,257	100,361	5,282
70–74	492,186	394,116	70,542 4	7,028	111,692	5,879
75–79	195,455	156,368	26,910 1	7,940	42,607	2,242
80–84	258,059	222,627	35,370 2	3,580	56,003	2,948
85+	230,585	194,404	28,689 1	9,126	45,424	2,391
Total	24,000,302	25,030,890	2,929,990	3,239,836	5,861,335	308,491

TABLE 10.3 Population Distribution of Nigeria by Hypothetical Marital Events: Age, Sex and Order for Marriage, 1991

10.6.1 ORDER-SPECIFIC MARRIAGE RATE

The order-specific marriage (first and remarriage) rate is another refinement of the marriage rate. We shall observe that the denominator in the formula for calculation the first marriage and remarriage rates are different.

First Marriage Rate

The first marriage rate is defined as:

$$FMR = \frac{M_1}{P^n_{15+}} \times k$$

where

M_1 = the number of first marriages in a calendar year;
P^n_{15+} = the mid-year nubile (never married) population of age 15 years and older in a calendar year.

Example 10.4

For the data in Table 10.3, calculate the first marriage rate.

Solution

From Table 10.2, $P^n_{15+} = 14,810,415$.
From Table 10.3, $M_1 = 5,861,335$.

Hence

$$\begin{aligned} FMR &= \frac{M_1}{P^n_{15+}} \times k \\ &= \frac{5,861,335}{14,810,415} \times 1,000 = 395.8 \end{aligned}$$

There were 396 marriages among every 1,000 persons who had never married in 1991.

Remarriage Rate

The remarriage rate is defined as:

$$RMR = \frac{M_{2+}}{P^{d+w}_{15+}} \times k$$

where

M_{2+} = the number of second or more marriages in a calendar year;
P^{d+w}_{15+} = the mid-year divorced and widowed populations of age 15 years and older in a calendar year.

Example 10.5

For the data in Table 10.3, calculate the remarriage marriage rate.

Solution

From Table 10.2,

$$P_{15+}^{d+w} = 120,410 + 424,809 = 545,219$$

From Table 10.3, $M_2 = 308,491$.

Hence

$$\begin{aligned} RMR &= \frac{M_{2+}}{P_{15+}^{d+w}} \times k \\ &= \frac{308,491}{545,219} \times 1,000 = 565.8 \end{aligned}$$

There were 566 remarriages among every 1,000 divorced and widowed persons in 1991.

10.6.2 SEX-SPECIFIC MARRIAGE RATE

The GMR are usually calculated separately for both sexes s (males and females) because men can be in union with more than one woman in some part of the world but women cannot.

This rate which may also be referred to as sex-specific marriage rate, is given as:

$$SSMR^s = \frac{M^s}{P^s_{15+}} \quad \times \quad k$$

Example 10.6

For the data in Table 10.3, calculated the sex-specific marriage rates.

Solution

For the male population:

$$\begin{aligned} SSMR^m &= \frac{M^m}{P^m} \times k \\ &= \frac{2,929,990}{24,000,302} \times 1,000 = 122.1 \end{aligned}$$

For the female population:

$$\begin{aligned} SSMR^f &= \frac{M^f}{P^f} \times k \\ &= \frac{3,239,836}{25,030,890} \times 1000 = 129.4 \end{aligned}$$

There were 122 and 129 marriages among 1,000 marriageable male and female populations, respectively.

10.6.3 AGE-SPECIFIC MARRIAGE RATE

The GMR may be calculated separately for each single age x or each age group a because timing of marriage is different for the various age groups. This may be referred to as age-specific marriage rate (ASMR) and is given as:

$$ASMR_a = \frac{M_a}{P_a} \times k$$

Example 10.7

For the data in Table 10.2, calculate the age-specific marriage rates.

Solution
For age group 15–19:

$$\begin{aligned} ASMR_{15-19} &= \frac{M_{15-19}}{P_{15-19}} \times k \\ &= \frac{393,044}{9,335,788} \times 1,000 = 42.1 \end{aligned}$$

For age group 20–24:

$$\begin{aligned} ASMR_{20-24} &= \frac{M_{20-24}}{P_{20-24}} \times k \\ &= \frac{737,461}{7,671,570} \times 1000 = 96.1 \end{aligned}$$

The age specific marriage rates for the other age groups are similarly calculated and the results presented in Table 10.4. We observe from the table that the rates increase steadily to age 40–44 before it starts reducing.

Age Group	Population	Married	Age specific Marriage Rate $(ASMR(a))$
15–19	9,335,788	393,044	42.1
20–24	7,671,570	737,461	96.1
25–29	7,311,671	1,004,067	137.3
30–34	5,913,927	983,053	166.2
35–39	4,214,933	741,904	176.0
40–44	3,845,918	674,944	175.5
45–49	2,416,703	414,715	171.6
50–54	2,570,799	421,429	163.9
55–59	1,119,769	176,637	157.7
60–64	1,690,374	247,745	146.6
65–69	703,040	105,643	150.3
70–74	886,302	117,570	132.7
75–79	351,823	44,850	127.5
80–84	480,686	58,951	122.6
85+	230,585	47,815	112.5
Total	48,938,292	6,169,826	2,079

TABLE 10.4 Age-specific Marriage Rate of Hypothetical Data of Nigeria, 1991

10.6.4 AGE-SEX SPECIFIC MARRIAGE RATE

The age-specific marriage rate is usually calculated for both sexes (women and men) separately because timing of marriage is different for both sexes.

The age-sex specific marriage rate (ASSMR) calculated separately for single ages x can be defined as:

$$ASSMR_x^s = \frac{M_x^s}{P_x^s} \times k$$

where

M_x^s = the number of marriages among a specific sex s (male or female) for single age x in a calendar year;
P_x^s = the mid-year population of the specific sex s and single age x category in a calendar year.

Limitation of Age-Sex Specific Marriage Rates for Single Ages

A set of marriage rates calculated by single years is:

1) too detailed and clumsy for comparative purposes, and
2) affected by the misreporting of ages by spouses.

The age-sex specific marriage rate $(ASSMR_a^s)$ calculated for age-groups, usually the 5-year age groups a is given as:

$$ASSMR_a^s = \frac{M_a^s}{P_a^s} \times k$$

where M_a^s and P_a^s are as defined earlier but for age group a.

We observe that the age-specific marriage rate is calculated as the ratio of the number of women contracting marriages and all the women in the age group and not only the unmarried ones.

Example 10.8

For the data in Table 10.2, calculate the age-sex specific marriage rates.

Solution

For age group 15–19 for males:

$$\begin{aligned} ASSMR_{15-19}^m &= \frac{M_{15-19}^m}{P_{15-19}^m} \times k \\ &= \frac{157,217}{4,528,811} \times 1,000 = 35 \end{aligned}$$

For age group 15–19 for females:

$$\begin{aligned} ASSMR^f_{15-19} &= \frac{M^f_{15-19}}{P_{15-19}} \times k \\ &= \frac{235,826}{4,806,977} \times 1,000 = 49 \end{aligned}$$

The age-sex specific marriage rates for the others are similarly calculated and the results presented in Table 10.5.

Age Group	Mid-year Population		Marriages		Age-sex specific Marriage Rates $(ASMR_a^s)$	
	Males	Females	Males	Females	Males	Females
15–19	4,528,811	4,806,977	157,217	235,826	35	49
20–24	3,314,303	4,357,267	294,984	442,476	89	102
25–29	3,304,739	4,006,932	401,627	602,440	122	150
30–34	2,808,629	3,105,298	393,221	589,832	140	190
35–39	2,206,871	2,008,062	296,762	445,142	134	222
40–44	1,971,197	1,874,721	404,966	269,977	205	144
45–49	1,355,101	1,061,602	248,829	165,886	184	156
50–54	1,388,650	1,182,149	252,857	168,571	182	143
55–59	638,375	481,394	105,982	70,655	166	147
60–64	898,801	791,573	148,647	99,098	165	125
65–69	408,540	387,400	63,386 4	2,257	155	109
70–74	492,186	394,116	70,542 4	7,028	143	119
75–79	195,455	156,368	26,910 1	7,940	138	115
80–84	258,059	222,627	35,370 2	3,580	137	106
85+	230,585	194,404	28,689 1	9,126	124	98
TOTAL	24,000,302	25,030,890	2,929,990	3,239,836		

TABLE 10.5 Population Distribution of Nigeria by Hypothetical Marital Events: Age and Sex, 1991

10.6.5 AGE-SEX-ORDER SPECIFIC MARRIAGE RATE

The age-sex-order specific marriages rate $(ASOSMR)$ can be defined as:

$$ASOSMR = \frac{M_{ia}^s}{P_{(i-1)a}^s} \times k$$

where

M_{ia}^s = the number of i^{th} marriage among a given sex, s (male or female) in the age group a;

$P^s_{(i-1)a}$ = the mid-year population of the same sex and of the same age group a but whose $(i-1)^{th}$ marriage remains dissolved.

Limitation of Age-sex-order Specific Marriage Rate

It is difficult to get appropriate data for its calculation. However, when $i-1$, then P^s_{0a} identifies the number of unmarried single individuals and the rate can be easily obtained. In particular,

$$ASOSMR = \frac{M^s_{1a}}{P^s_{0a}} \times k$$

are the s^{th} (female or male) first marriage rates among those in age group a.

10.7 TOTAL MARRIAGE RATE

10.7.1 DEFINITION OF TOTAL MARRIAGE RATE

The total marriage rate (TMR) is an important marriage measure which summarises the pattern of marriage exhibited by the age-sex specific marriage rates. It gives an indication of the extent of marriages in the population, based on the marriage trends according to age at a specific time. It presents a single index of total marriage which answers as nearly as possible the question: "How many marriages are being contracted nowadays"?

The total marriage rate describes the proportion of women (generally per 1,000 persons) that contract marriage in their lives, on condition that

1) none of them die;
2) the marriage rate in the pertinent cohort remains constant throughout the period;
3) these persons belong to the age groups of 15 to 49 years.

In other words, it measures the total number of marriages a person will have at the end of his/her marriageable age (before age 50) if he/she follows the given schedule of marriage.

TMR is computed by adding up the age-specific marriage rates of first marriages in the whole age period. It may be computed using single years x or age groups a and may be expressed as per marriage or per 1,000 marriages.

For single years, x:

$$TMR^s_{(x)} = \sum_{x=15}^{49} \frac{M^s_x}{P^s_x} \times k$$

For age groups a with equal interval:

$$TMR^s_{(a)} = \sum_{a=15-19}^{45-49} \frac{M^s_a}{P^s_a} \times k$$

or

$$TMR^s_{(a)} = n \sum_{a=15-19}^{45-49} ASSMR^s_a$$

and for age groups with unequal interval:

$$TMR^s_{(a)} = \sum_{a=15-19}^{\infty} n_a \frac{M^s_a}{P^s_a} \times k$$

10.7.2 TOTAL FIRST MARRIAGE RATE

Total first marriage rate ($TFMR$) is the proportion of men or women who would have married at least once by age 50 if they had been subject throughout their lives to the age specific first marriage rates observed in a given year.

$TFMR$ is calculated the same way as the TMR except that, in the case of $TFMR$, only first marriages are taken into account. Similar to TMR, $TFMR$ may be computed using single years x or age groups a:

$$\begin{aligned} TFMR^{ns}_{(x)} &= \sum_{x=15}^{49} \frac{M^1_x}{P^s_x} \times k \\ TFMR^{ns}_{(a)} &= \sum_{a=15-19}^{45-49} \frac{M_a}{P^s_a} \times k \end{aligned}$$

where $TFMR^{n(s)}_{(a)}$ is the total first marriage rate of the never married n for age a and for sex s.

Note

1) $TFMR$ may be expressed as the number of first marriages per person. A $TFMR = 1$ for women, for instance, indicates that all women in the population will contract a marriage before their 50th birthday if present trends should prevail. Since the TFMR is sensitive to changes in the timing of marriage, it may exceed 1.
2) $TFMR^{ns}_a = 5 \times (ASFMR^{n(s)}_{(15-19)} + \cdots + ASFMR^{n(s)}_{(45-49)})$

   where $ASFMR^{n(s)}_{(a)}$ is the age-specific first marriage rate among the never married n for age a and for sex s. It is multiplied by 5 if 5-year age group are used.
3) $TMR_a = TFMR^{n(s)}_a + TRMR^{n(s)}_{(a)}$

10.8 STANDARDISED MARRIAGE RATE

Marriages rates can be standardised using the direct and indirect methods, depending on which data are available.

10.8.1 DIRECT STANDARDISED MARRIAGE RATE

The direct standardised marriage rate ($DSMR$)is defined as:

$$DSMR = \frac{\sum_{15-19}^{45-49} ASMR_a^s P_a^{*s}}{\sum_{15-19}^{45-49} P_a^{*s}} \times k$$

where

$DSMR$ = the direct standardised marriage rate;

$ASMR_a^s$ = the age-specific marriage rate population for sex s of age 15 years and older of the ***study population***;

P_a^{*s} = the mid-year population of male or female or both for age group a in the ***standard population***;

$ASMR_a^s P_a^{*s}$ = the number of expected marriages of male, female or both for age group a in the ***standard population***;

k = $1,000$.

10.8.2 INDIRECT STANDARDISED MARRIAGE RATE

The indirect standardised marriage rate ($ISMR$)is defined as:

$$ISMR = \frac{M}{\sum ASMR_a^{*s} P_a} \times CMR^* \times 1,000$$

where

M = observed marriages (total marriagess) in the study population;

$ASMR_a^{*s}$ = "standard" set of age-specific fertility rates for sex s;

P_a = mid-year population by age for the study population;

CMR^* = crude marriage rate for the standard population;

k = $1,000$.

If we observe that $ASMR_a^{*s}P_a$ is the expected number of marriages in the particular population on the basis of the standard set of age-specific marriage rates, then the indirect standardised marriage rate may be defined as:

$$ISMR = \frac{M}{M_E} = \frac{\text{Observed birth in the study population}}{\text{Expected birth in the study population}} \times CMR^{*s}$$

10.9 MEASURES OF MARRIAGE TIMING

10.9.1 AGE AT MARRIAGE

Age at marriage usually refers to first marriages and whenever remarriages are involved, that has to be specified.

Age at first marriage is probably the most useful fact about women's marital history for the study of their fertility. It is a better measure of marriages for the study of demographic trends in a society with universal marriage[51] than some of the earlier marriage rates discussed.

10.9.2 AVERAGE AGE AT FIRST MARRIAGE

The average age at marriage for each year in a population usually concerns first marriages, as indicated earlier, and whenever remarriages are involved a separate average is calculated for each order. It is derived from the distribution of first marriages by age group of husband or wife derived from civil registration data. It is usually computed separately for males and females, because females typically marry at younger ages.

The study of the average age at first marriage depicts in the population

1) the marriage pattern; and
2) the possible trends in fertility[52].

Two of the averages often used in demography, namely, the arithmetic mean (or simply the mean) and the median.

Mean Age at First Marriage

Mean age at first marriage is the average age at which men or women marry.

There will always be a few who will not get married at all and hence the mean age at marriage may not be completely meaningful. In such situations the median age at marriage may be calculated.

Median Age at First Marriage

The median age at first marriage indicates that half the people marrying for the first time in a given year got married before the median age and half after.

10.9.3 SINGULATE AGE AT FIRST MARRIAGE

It is easy to calculate the average age of first marriage if the relevant reliable data are available from civil registration. These data, however, are difficult to obtain, especially in Africa. It is rather easy to obtain data on marital status by age from census data and for this, the singulate mean age at marriage (SMAM) is calculated.

10.10 DIVORCE RATES

All the various marriage rates earlier defined have their corresponding divorce rates. The numerator for calculating the rates are the number of divorces in a calendar year. The denominator can be:

1) Mid-year population;
2) Total number of marriages during the same year;
3) Total number of married people at the middle of the year;
4) Total number of currently married couples at the middle of the year.

The radix k is usually taken as 100.

It is difficult to define an ideal measure for divorce where the risk of divorce is properly determined.

1) It is better to calculate divorce rates for men and women separately because ages of husband an wife need not be equal.
2) In most cases, the order of the divorce is important in calculating the divorce rate; otherwise there will be confusion with the calculation of rates as defined here when one or both partners get remarried during the very year the divorce took place.
3) The appropriate divorce rate should be for marriage cohorts or equivalently for duration of marriage since marriage has to take place before a divorce can happen.

A suitable measure for divorce will be

$$\frac{D}{MC}$$

where

D as defined above and MC is the number of a marriage cohort.

This measure requires that we have data on each marriage as to whether the marriage

1) ended in divorce; or
2) got terminated due to death of either or both the spouses; and
3) the time and ages of the spouses at divorce.

This type of information required for calculating standard divorce rates is rarely collected because of the waiting time involved in following through the marriage cohorts. Whenever the need arises, an appropriate rates should be derived.

Total Divorce Rate

Total divorce rate (TDR) is the number of divorces that men or women would have gone through by age 50 if the age-specific divorce rates observed in a given year applied throughout their life. This measure is expressed as number of divorces per person.

10.11 WIDOWHOOD

The difficulties encountered in the construction of divorce rates are applicable as such in the case of widowhood also. A useful measure to study widow will be the age at widowhood. The difficulty, as before is the paucity of relevant data. And, hence these areas remain inadequately measured.

11 MEASURES OF MIGRATION

11.1 INTRODUCTION

Migration is the third vital process which alters the size of a population, the other vital process being births and deaths already discussed. These two processes do not take into consideration space. Population change through these natural increase has been viewed independently from the area in which the population lives. Migration considers area, since it implies moving from one area to another. However, the records of migration do not lend themselves so well to study as those of births and deaths.

11.1.1 USES OF MIGRATION

1) Migration influences the socio-demographic structure of a population. As a consequence, it

   i) influences the growth of population by influencing fertility and mortality of the areas of origin and destination (for example, it can influence birth rates by altering the proportion of women of child-bearing ages in the population);
   ii) affects the characteristics of the labour force of the areas of origin and destination.

2) The measurement and analysis of migration are important in the preparation of population estimates and projections for a nation or parts of a nation.
3) Migration data (sex, age, citizenship, etc, of the migrant) are required together with census data and vital statistics for

   i) analysing changes in the structure of the population and labour force of an area;
   ii) the assessment of other basic demographic characteristics.

3) Migration facilitates an understanding of the nature and magnitude of the social and cultural problems that often result in areas with heavy immigration.

11.1.2 CHARACTERISTICS OF MIGRATION

1) Like death, migration can occur at any age. However, its study is fundamentally different from the study of natality and mortality.

   i) Migration takes into account both time and space;
   ii) Migration is an event which may be repeated several times within the lifetime of an individual.

2) The terminology in migration is not yet as well standardised as that in fertility and mortality; the present definitions are mostly supported by the consensus of users.
3) In most developed countries, there is a permanent system of recording births and deaths, as well as marriage dissolutions in registers, such as registers of births and deaths. In the case of migration, specific registers usually do not exist, though changes of residence are recorded in some countries having population registers.
4) Most of the vital processes in demography are slow, and take a considerable amount of time in working out the consequences. Migration, on the other hand, can vary rapidly in its effects (transferring millions of persons in a matter of months, and altering significantly the distribution of people and their activities). With these potentialities, migration is associated with large and rather sudden changes. For this reason it is often unpredictable and difficult to study.

11.1.3 CHARACTERISTICS OF MIGRATION STUDIES IN AFRICA

The records of migration do not lend themselves so well to study as those of birth and deaths for the reason that collection of migration data is poorly organised.

In the first place, migration data are collected by various agencies and are seldom kept together with ordinary vital registration. Secondly, migration data are not given enough attention to maintain a high standard of quality because they are collected as a minor by-products of administration. Thirdly, Governments interest themselves primarily in recording the migrants from other countries and do little to keep data on movement from place to place within the same country. Consequently much is written on international migration and rather little about internal migration. Finally, information recorded on international migrants is inaccurate and inconsistent, especially if data from different countries are compared.

11.2 DEFINITION OF CONCEPTS

11.2.1 PROBLEMS IN DEFINITION OF MIGRATION

The definition and analysis of migration present some peculiar problems.

1) Migration is a vague statistical concept with few uniform criteria to distinguish who is a migrant and who is not. A migrant, first of all, is a person who travels. This is the only unambiguous element in the entire subject. "Migrants" are obviously some smaller category of travellers, but the difficulty is encountered whenever the limits of that category are to be defined.
2) Although a single individual may make several moves within his or her lifetime, most censuses and surveys only record one of these moves. Even if all moves are described in some sample surveys, the data are incomplete because a person remains a potential migrant until death. Thus, the analysis of population movement from any type of data source can only provide an extremely complex phenomenon.

In spite of these difficulties we shall attempt to define some concepts in migration.

11.2.2 MOBILITY

Some writers suggest that since migration cannot be defined exactly, it should be considered as part of a continuum covering all types of population movements.

Population Mobility

Population mobility or spatial mobility is defined as the ability to move in space. These movements may be either

1) daily commuting[53] journeys (between a given place of residence and some other point in space), from a few metres to thousands of kilometres, or
2) long run shifts of residence (the length of stay at destination from minutes to the remainder of lifetime). Since commuting does not directly affect the level and the structure of the population of a given territory, it is not, strictly speaking, a demographic phenomenon and will not be considered further in this chapter.

Mover

A mover is a person who moved from one address (house or apartment) to another.

Short-distance mover

A short-distance or local mover is a person who moved only within a political or administrative area.

Mover from abroad

A mover from abroad is a person who has moved from outside the country.

11.2.3 MIGRATION

Migration

Not all geographic movements qualify as migration. Moves which are included in the migration process are generally considered to be at least semi-permanent and to take place across definite geographical boundaries. The operational definition of migration is that "migration is a change from one administrative spatial unit to another, made between two points in time". The concept of residence is of particular significance to the definition of migration for it combines the two elements on which the definition must hinge. There are, however, some changes of residence which we exclude from migration. These are

1) Temporary changes which do not involve changes in usual residence: these are usually excluded from "migration", which include brief visits, excursions, vacation, or business travels even across national boundaries;
2) Other changes in residence although permanent are short-distance movements, and hence, are also excluded from "migration".

Thus, the term migration has, in general usage, been restricted to relatively permanent changes in residence between specifically designated political or statistical areas or between type-of-residential areas. (Shryock, H.S. *et al*, 1970).

Migrant

A migrant is a person who changes the political area of his usual place of residence.

Area of Origin (Departure)

The area of origin is the area from which a migrant moves. It may be either (a) the area of residence at the beginning of the migration interval, or (b) the area of residence from which the move was made.

Area of Destination (Arrival)

The area of destination (sometimes referred to as the receiving area) is the area to which a migrant moves.

Migration Stream

A migration stream (sometimes referred to as ***migration flow*** or the ***dominant stream***) consists of all moves or all migrants having a common area of origin and destination in a given migration period. Thus, a migration stream from i to j, denoted by M_{ij}, is the movement of people from an area of origin i to the area of destination j during a given interval of time.

Counterstream

A counterstream (also referred to as ***reverse stream***) consists of movement in the opposite direction to a migration stream. Thus, if the migration stream is from area of origin i to area of destination j, the counterstream is that from j to i during the same period and denoted by M_{ji}. In general sense, we can think of a counterstream as the smaller of the two movements.

Net Stream

The net stream (also referred to as net interchange) between two areas is the difference between a stream and its counterstream.

Gross Stream

The gross stream (or gross interchange) between two areas is the sum of the stream and the counterstream.

Return Migrant

A return migrant is a person who moved back to the area where he formerly resided.

11.3 TYPES OF MIGRATION

Migration may be classified by the following types:

1) By type of boundary, namely,

   i) international migration;
   ii) internal migration.

2) By *duration*, that is,

   i) regular (daily or periodic) migration which is a regular trip to place of work or studies beyond the person's place of residence;
   ii) seasonal migration;
   iii) temporary migration;
   iv) permanent migration.

3) By *direction:*

   i) rural-rural migration;
   ii) rural-urban migration;
   iii) urban-rural migration;
   iv) urban-urban migration.

4) According to *decision to migrate*:

   i) voluntary migration;
   ii) involuntary migration.

The sources of data, the type of data available, and the techniques of estimation and analysis are sufficiently different for international and internal migration, and so we shall treat them separately.

11.4 INTERNATIONAL MIGRATION

11.4.1 DEFINITION OF CONCEPTS

International Migration

International migration is the movement of people across national boundaries. This movement may be designated emigration or immigration.

Immigration

Immigration (I) is a movement of people into a country.

Immigrant

An immigrant is a person who has resided abroad for a year or more and on entering the country has declared an intention to stay for a year or more.[54]

Emigration

Emigration (E) is a movement of people out of a country.

Emigrant

An emigrant is a person who has resided in a place for a year or more and on leaving the country has declared an intention to stay away for a year or more.

It should be noted that every international migrant is an emigrant with respect to the country of departure and an immigrant with respect to the country of arrival.

Net Migration

Net migration (NM) is the balance of immigration and emigration for a country. It may be referred to as net immigration (NI) if immigration is larger than emigration and net emigration (NE), if emigration is larger than immigration. That is,

$$\begin{aligned} NI &= I - E, \quad if\ I > E, \\ NE &= E - I, \quad if\ E > I \end{aligned}$$

The term "net migrant" as far as possible must be avoided because it is not reasonable to think of a group of people as being net migrant.

Gross Migration

Gross migration (GM) is the sum of total immigration and total emigration. That is,

$$= I + E$$

This figure, also called migration turnover, is intended to represent the total movement across the borders of a country during a period of time.

Example 11.1

In 1986, the number of persons who departed from Nigeria to various countries was 635,397, while in that same year 647,763 persons arrived in Nigeria.

Calculate:

a) net migration;
b) gross migration.

Solution

$$I = 647,763; \qquad E = 635,397$$

1) The net migration is

$$\begin{aligned} NM &= I - E \\ &= 647,763 - 635,397 = 12,366 \end{aligned}$$

That is, in 1986 the Nigerian population increased by 12,366 through migration.

2) Gross migration is

$$\begin{aligned} GM &= I + E \\ &= 647,763 + 635,397 = 1,283,160 \end{aligned}$$

That is, the migration turnover of Nigeria in 1986 was 1,283,160. In other words, 1,283,160 people crossed the Nigerian border in 1986.

11.4.2 SOURCES OF INTERNATIONAL MIGRATION DATA

International migration data may be obtained from a variety of different sources such as the following:

1) *Administrative Records*
   Administrative records may be:
   i) Statistics collected by immigration officials at national frontiers but these are rarely disseminated.
   ii) Passenger statistics obtained from lists of passengers on sea and air transport manifests.
   iii) Population registers tracing the movements of foreigners.

2) *Population Censuses*
   Migration related questions included in the census questionnaire: country of birth, country of citizenship, country of usual residence, or country of residence at a previous specific date.

3) *Sample Surveys*
   Sample surveys may be periodic or special.
   i) Periodic surveys such as the Demographic and Health Survey (DHS), the Living Standards Measurement Survey (LSMS), the Core Welfare Indicators Questionnaire (CWIQ) Survey may have specific questions regarding migration: previous and present residence, or citizenship.
   ii) Special inquiries on migration.

11.4.3 METHODS OF ANALYSIS OF INTERNATIONAL MIGRATION DATA

Intercensal Component Equation

In Chapter 4, we stated the intercensal component equations as:

$$P_1 = P_0 + (B - D) + (I - E)$$

or

$$(P_1 - P_0) = (B - D) + (I - E)$$

where

$(P_1 - P_0)$	is net change in population during the period;
$(B - D)$	is natural increase/decrease during the period;
$(I - E)$	is mechanical increase/decrease (net immigration) during the period.

The general formula for estimating the total volume of net migration is, therefore, to rearrange the elements of the intercensal component equation as:

$$M = (P_1 - P_0) - (B - D)$$

where $M = I - E$ is the net migration.

Thus, net migration equals population change during the intercensal period minus natural increase during that period. The result is the difference between the total number of persons moving into a country and the number moving out from the country during a given intercensal period.

Although there is an interest in separate figures on immigration and emigration, this formula does not permit making adequate estimates of immigration and emigration separately.

Example 11.2

The census population of a country was 5,668,000 in 1982 and 6,986,000 in 1992. Estimate the country's net migration for the period if the number of births and deaths within the period were 2,235,000 and 1,347,000 respectively.

Solution

$$\begin{aligned} P_0 &= 5,668,000;\ P_1 = 6,986,000;\ B = 2,235,000;\ D = 1,347,000 \\ I - E &= (P_1 - P_0) - (B - D) \\ &= (6,986,000 - 5,668,000) - (2,235,000 - 1,347,000) \\ &= 1,318,000 - 888,000 = 430,000. \end{aligned}$$

The population of the country in 1992 increased by 430,000 through migration.

Migration Ratios and Proportions

Migration Index

Migration Index (MI) may be defined as

$$MI = \frac{E}{I}, \qquad \text{where} \quad E > I$$

or

$$MI = \frac{I}{E}, \qquad \text{where} \quad I > E$$

Example 11.3

For the data in Example 11.1, calculate the migration index.

Solution

$$I = 647,763; \quad E = 635,397$$

$$\begin{aligned} MI &= \frac{I}{E} \\ &= \frac{647,763}{635,397} = 1.019 \end{aligned}$$

That is, for every 100 persons that left Nigeria in 1986 about 102 persons entered it.

Net Migration Ratio

Net Migration Ratio (NMR) may be defined as:

$$NMR = \frac{I - E}{I}, \qquad \text{where} \quad I > E$$

This definition is used to measure the proportion of immigration which is not compensated by emigration.

Net migration ratio may also be defined as

$$NMR = \frac{E - I}{E}, \qquad \text{where} \quad E > I$$

This definition is used to measure the proportion of emigration that is not compensated by immigration.

Example 11.4

Using the data in Example 11.1, calculate the net migration ratio.

Solution

$$I = 647,763; \quad E = 635,397; \quad I > E$$

$$\begin{aligned} NMR &= \frac{I - E}{I} \\ &= \frac{647,763 - 635,397}{647,763} = 0.019 \end{aligned}$$

Migration Effectiveness Ratio

Migration Effectiveness Ratio (MER) is defined as the ratio of net migration to gross migration (migration turnover):

$$MER = \frac{I - E}{I + E}$$

This ratio measures the relative difference between the effective addition or loss of population through migration and the overall gross movement.

Example 11.5

For the data in Example 11.1, calculate the migration effectiveness ratio.

Solution

$$I = 647,763; \quad E = 635,397$$

$$\begin{aligned} MER &= \frac{I - E}{I + E} \\ &= \frac{647,763 - 635,397}{647,763 + 635,397} = 0.01 \end{aligned}$$

Immigration Proportion

Immigration Proportion (IP) is defined as

$$IP = \frac{I}{I + E}$$

Emigration Proportion

Emigration Proportion (EP) is defined as:

$$EP = \frac{E}{I + E}$$

Example 11.6

For the data in Example 11.1, calculate immigration and emigration proportions.

Solution

$$I = 647,763; \quad E = 635,397$$

$$\begin{aligned} IP &= \frac{I}{I + E} \\ &= \frac{647,763}{647,763 + 635,397} = 0.505 \end{aligned}$$

$$\begin{aligned} EP &= \frac{E}{I + E} \\ &= \frac{635,397}{647,763 + 635,397} = 0.495 \end{aligned}$$

Crude Migration Rates

The *crude immigration rate (CIR)* is defined as:

$$CIR = \frac{I}{P} \times 1,000$$

where P is the mid-year population.

The *crude emigration rate (CER)* is defined as

$$CER = \frac{E}{P} \times 1,000$$

Example 11.7

For the data in Example 11.1, calculate

1) the crude immigration rate;
2) the crude emigration rate.

if the mid-year population of Nigeria in 1986 was 98,168,000.

Solution

$$I = 647,763; \quad E = 635,397; \quad P = 98,168,000$$

1) The crude immigration rate is

$$\begin{aligned} CIR &= \frac{I}{P} \times 1,000 \\ &= \\ &= \frac{647,763}{98,168,000} \times 1,000 = 6.599 \end{aligned}$$

That is, there were about 6.6 new immigrants to Nigeria per 1,000 mid-year population.

2) The crude emmigration rate is

$$\begin{aligned} CER &= \frac{E}{P} \times 1,000 \\ &= \frac{635,397}{98,168,00} \times 1,000 = 6.473 \end{aligned}$$

That is, there are about 6.6 new emigrants from Nigeria per 1,000 mid-year population. In other words, for every 1,000 mid-year population, about 6.5 persons left Nigeria in 1986.

The *net migration rate (NMR)* is defined as:

$$NMR = \frac{I - E}{P} \times 1,000$$

The gross migration rate (GMR) is defined as:

$$GMR = \frac{I + E}{P} \times 1,000$$

Example 11.8

For the data in Example 11.7, calculate:

1) the net migration rate;
2) the gross migration rate.

Solution

$$I = 647,763; \qquad E = 635,397; \qquad P = 98,168,000$$

1)

$$\begin{aligned} NMR &= \frac{I - E}{P} \times 1,000 \\ &= \frac{647,763 - 635,397}{98,168,000} \times 1,000 = 0.12597 \end{aligned}$$

That is, in 1986, Nigeria experienced a net increase of 0.13 person per 1,000 (or 13 per 100,000)) population through migration.

2)

$$\begin{aligned} GMR &= \frac{I + E}{P} \times 1,000 \\ &= \frac{647,763 + 635,397}{98,168,000} \times 1,000 = 13.07. \end{aligned}$$

That is, about 13 per 1,000 Nigerian population crossed the Nigerian border in 1986.

Specific Migration Rates

Migration rates may also be specific for, say, age, sex, race, or other characteristics of the migrants. For example, an age-specific net migration rate is:

$$NMR(a) = \frac{I(a) - E(a)}{P(a)} \times 1,000$$

where
$I(a)$ and $E(a)$ represent immigration and emigration, respectively at age a;

$P(a)$ is the mid-year population at age a.

Measures of Migrantion Based on Place of Birth Data

Migrant Component

The migrant component of a population (MC) is the total number of persons living in an area who are migrants in relation to the total population at a given time. It is given by:

$$MC = \frac{M_t}{P_t} \times 100$$

where
M_t are migrants at time t in a population; P_t is the size of the population at time t.
This measure is useful when studying:

1) Rate of migration;
2) Rate of life-time migration.

Example 11.9

An area had a migrant population of 590 in the population of 12,000 in 2001. What is the migrant component of a population.

Solution

$$P_{2001} = 12,000; \qquad M_{2001} = 590$$

Therefore,

$$\begin{aligned} MC &= \frac{M_{2001}}{P_{2001}} \times 100 \\ &= \frac{590}{12,000} \times 100 = 4.9 \text{ percent} \end{aligned}$$

Rate of Migration

The rate of migration from area i to area j is defined as:

$$R_M = \frac{M_{ij}}{P_i} \times 100$$

where
M_{ij} = Migration stream from area of origin i to area of destination j;
P_i = Population of the area of origin i.

Example 11.10

Suppose that 63 out of an estimated population of 1,800 left area A to settle in area B in 2003. Calculate the rate of migration from area A to area B.

Solution

$$M_{AB} = 63; \qquad P_A = 1,800$$

Therefore,

$$\begin{aligned} R_M &= \frac{M_{AB}}{P_A} \times 100 \\ &= \frac{63}{1,800} \times 100 = 3.5 \text{ percent} \end{aligned}$$

Efficiency of Streams

The efficiency of migration stream (EMS) from i to j is defined as the ratio of stream to counter stream.

$$EMS = \frac{M_{ij}}{M_{ji}} \times 100$$

where M_{ij} is as defined above and M_{ij} is the counterstream from area of origin j to area of destination i.

Example 11.11

Refer to Example 11.10. Suppose that in that year, 39 persons went from area B to settle in area A. Calculate the efficiency of streams.

Solution

$$M_{AB} = 63; \qquad M_{BA} = 1,800$$

Therefore,

$$\begin{aligned} EMS &= \frac{M_{ij}}{M_{ji}} \times 100 \\ &= \frac{63}{39} \times 100 = 161.5 \text{ percent} \end{aligned}$$

That is, the migration stream from area A in 2003 was 62 percent more than the counterstream. In other words, for every person who migrated from area B to area A, there were 1.6 persons who migrated from area A to area B.

We notice that EMS is net migration between two areas in relative term. The advantage of this measure is that it is useful for studying "push-pull" factors between areas, for example, rural to urban migration. The measure has one main disadvantage. The ratio using absolute numbers of migrants can be misleading when the population of areas i and j are vastly different in which case it may not exactly reflect the power of attraction.

Relative Efficiency of Streams

To overcome the deficiency associated with the efficiency streams defined above, we weight the streams by their base population. This gives a measure called relative efficiency of migration stream (REMS), defined as:

$$REMS = \frac{M_{ij}}{M_{ji}} \times \frac{P_j}{P_i} \times 100$$

Example 11.12

Refer to Example 11.11. Suppose that the population of area A was 850 and that of area B was 5,000 in 2003. Calculate the relative efficiency of stream.

Solution

$$M_{AB} = 63; \quad P_A = 850 \quad M_{BA} = 1,800; \quad P_B = 5,000$$

Therefore,

$$\begin{aligned} REMS &= \frac{M_{ij}}{M_{ji}} \times \frac{P_j}{P_i} \times 100 \\ &= \frac{63}{39} \times \frac{5000}{850} \times 100 = 950.2 \text{ percent} \end{aligned}$$

Index of Preference

The index of preference of migrants (IPM) from area i to area j is the number of emmigrant from area i to area j in relation to all emmigrants from area i:

$$IPM = \frac{M_{ij}}{M_i} \times 100$$

Example 11.13

Refer to Example 11.10. Suppose that in all only 96 people had migrated from area i. Calculate the index of preference of migrants for area B.

Solution

$$M_{AB} = 63; \quad M_A = 96$$

Therefore,

$$\begin{aligned} REMS &= IPM = \frac{M_{ij}}{M_i} \times 100 \\ &= \frac{63}{96} \times 100 = 65.6 \text{ percent} \end{aligned}$$

11.5 INTERNAL MIGRATION

11.5.1 DEFINITION OF CONCEPTS

Internal Migration

Internal migration refers to changes of residence within the boundaries of a given nation. Residential moves are considered to have taken place across the boundary of minor subdivisions of districts or regions of the country. Residential moves which do not result in crossing such a boundary are usually termed *mobility*. Thus, with internal migration not only intent of movement but also the distance is considered.

Internal migration may be designated out-migration or in-migration.

Out-migration

Out-migration is a movement of people out of a migration-defining area to a place outside it but within the same country during the time period under consideration.

Out-migrant

An out-migrant is a person who leaves a migration-defining area by crossing its boundary for a point outside it, but within the same country during the time period under consideration.

In-migration

In-migration is a movement of people into a migration-defining area by crossing its boundary from some point outside the area but within the same country during the time period under consideration.

In-migrant

%vspace11pt An in-migrant is a person who enters a migration-defining area by crossing its boundary from some point outside the area, but within the same country during the time period under consideration.

Note

It should be noted that every internal migrant is an out-migrant with respect to the area of departure and an in-migrant with respect to the area of arrival.

Net Migration

Net migration is the balance between in-migration and out-migration. It may be characterised as

1) net in-migration if in-migration is greater than out-migration, and
2) net out-migration if out-migration is greater than in-migration.

Migration turnover

Migration turnover (in relation to internal migration) is the sum of in-migration and out-migration with respect to a given area. That is,

Migration turnover = in-migration + out-migration.

11.5.2 SOURCES OF INTERNAL MIGRATION DATA

Generally, the sources of internal migration data are the following.

1) *Administrative Records*
   i) Continuous population register (deployed in a few developed countries including Denmark, Netherlands, Norway, Japan, Sweden);
   ii) Miscellaneous sources that include the national social security records, some of the other partial population registers (compulsory military service, aliens, etc.), electoral rolls, employment records, school records, public utility records, and city directory listings.

2) *National Population Census*
   The census is for most countries the only way to obtain, either directly or indirectly, some information on internal migration.

3) *Sample Surveys*
   Demographic surveys for collecting internal migration data generally take two forms: specialised migration surveys and general demographic surveys that include questions on migration.

In Africa, however, the national population census and, to a limited extent, the sample surveys are the most important internal migration data sources. Together with the evidence from censuses, sample surveys provide a much more complete description of the operation of internal migration, and ideally the two approaches should be used together.

In both the population censuses and sample surveys, data on internal migration are obtained directly or indirectly. The usual direct questions on internal migration relate to:

1) place of birth;
2) place of last residence;
3) duration of residence in the place of enumeration;
4) last prior residence;
5) mobility history; and so forth.

Generally, if place of residence is different from place of birth then migration has taken place, or if current place of residence is different from usual place of residence. There are other similar definitions that make use of the questions listed above.

Migration data obtained indirectly are through estimation procedures that use data presumably obtained for other purposes.

Advantages of Census data on migration

Census data are

1) potentially the most important source of data on internal migration in Africa since nearly all the countries now take censuses;
2) fairly standard in form;
3) most suitable for macro-analysis since they cover the national population;
4) used for both direct and indirect estimation of migration.

Disadvantages of Census data on migration

1) Censuses do not show the extent of return migration.
2) Censuses underestimate the significance of short distance movement.

Advantages of Sample Survey Data on migration

1) Small sample surveys adopt more flexible definitions in space and time and therefore can describe the entire migration process for each individual.
2) Well-designated migration surveys would provide both macro and micro-level data for development planning.

Disadvantage of Sample Survey Data on migration

Though some small sample surveys are based on random samples, most have been conducted among highly specific groups and, hence, the results cannot be generalised.

11.5.3 METHODS OF ANALYSIS OF INTERNAL MIGRATION

Statistics on internal migration are quite limited in most countries. It is however, possible to estimate internal migration as a residual, similar to international migration. Thus,

$$IM = (P_1 - P_o) - (B - D)$$

In-, out-, gross and net migration rates may be expressed by formulae analogous to those for international migration.

EXERCISES

11.1 52,536 persons in 1990 left a certain country to various parts of the world while 54,442 persons entered the country. Calculate the:

1) net migration;
2) gross migration;
3) migration rate;
4) net migration ratio;
5) migration effectiveness;
6) immigration proportion;
7) emigration proportion.

112 If the mid-year population of the country in Question 1 was 14,674,000, calculate

1) the crude immigration rate;
2) the crude emigration rate;
3) the net migration rate;
4) the gross migration rate.

11.3 The mid-year population of a country in 1985 was 17,146,553 and 19,885,132 in 1990. Estimate the country's net migration for the period 1985–1990 if the number of births and deaths within the period were 2,001,442 and 1,957,603 respectively.

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ENDNOTES

1. Births and deaths are vital events because of their direct biological relevance to life itself.
2. Marriages and dissolutions are sometimes considered vital events because of their relationship to childbearing and reproduction, even though they are not directly connected to population changes.
3. Demographic variables include mortality, fertility, age at first birth, age at first marriage and sex. Non-demographic variables include income, education, occupation, ethnicity and religion.
4. For details on the organisation of census, we refer the reader to the UN Handbook on Census (2006).
5. Retrospective Surveys may be used to collect vital statistics.
6. As we shall observe later, ratios and proportions are concepts for describing vital events while fractions, decimals and percentages are ways of expressing ratios and proportions.
7. To be discussed further in Chapter 6.
8. The population is usually not the same at the beginning and end of the year, hence the use of the average of the population at the beginning and end of the one year period.
9. The concepts of direct and indirect standardization, as well as the term standard population were first to introduced by Neison in 1844.
10. The interval of time taken to study population change in a fairly large population is usually one year.
11. A population is said to be closed when it is not subject to immigration and emigration; otherwise it is open.
12. A sequence of population figures

    $$P_1, P_2, \cdots, P_n$$

    follows a geometric progression if there exists a nonzero constant *r*, called a common ratio, such that

    $$\frac{P_n}{P_{n-1}} = r$$

    That is,

    $$P_n = rP_{n-1} \quad \text{for every } n \geq 1$$

13. The logarithm of a number N is the exponent to which a base b must be raised to obtain N. The base is usually the number 10. This is denoted as log_{10} or simply log. When the base used is the number $e \approx 2.718$, then it is called natural log, written log_e or usually as ln.
14. In demography, the word "growth" is used to mean "change" because of the fact that most populations are increasing in size.
15. We may also use logarithm base 10 instead and this is denoted by log_{10}.
16. Sex is not the same as gender. Gender refers to the roles and relationships of men and women in a specific society or culture. Society assigns roles based on a person's sex. Some of these roles are arbitrarily assigned and others are shaped by history, ideology, culture, religion and economic development
17. In other texts, it is defined as ratio of females per 100 males in the population.

18. Epidemiology is the study of the distribution of health and diseases in groups of people and the factors that influence this distribution.
19. A progeny is an individual born simultaneously to a definite set of parents.
20. The figure 98 may sometimes be reserved as a code for "age 98 or older".
21. In some text, MI is defined without $\frac{1}{2}$, in which case, the maximum value of the index is 180.
22. For details, see Nsowah-Nuamah, 2005.
23. We may observe that the mean age and the median age are not the same. They will be the same only when the age composition data are normally distributed, that is, bell-shaped.
24. In this text x is used for single age x and a for age group a. However, we may use x to refer to both single ages or age groups.
25. By normal situations we mean that there is neither irregular demographic events such as wars, epidemics nor differential migration. Hence, unless the history of the population is well documented, there is a likelihood of misinterpretation of irregular sex ratios and age ratios.
26. Although there are exceptions, in most cases the transfer of maternal deaths will balance out in a given year.
27. We may define infant death rate under one year as:

$$IDR = \frac{D_0}{P}$$

    but this is a crude rate and we shall not advise its usage.
28. The standard population may be the population of one of the area segment or date of the population being studied or a different external population.
29. Ernst Louis tienne Laspeyres, 1834–1913, was a German economist and statistician.
30. Hermann Paasche was a German economist.
31. That is that the population and the numbers in each age and sex group did not change over many decades.
32. Stable age distribution means that the proportion of persons at each age group in respect of the total population will remain constant despite the growth in absolute numbers.
33. The values of the life table functions are dependent on the value of the radix chosen; otherwise they do not have any absolute meaning.
34. The symbol d_x is a standard one in life tables for number of deaths. Unfortunately, it has also been used in Chapter 6 to represent age specific death rate. We implore the reader not to confuse the two.
35. If a person lives one complete year from the beginning of the year to the end of the year he/she has lived one person-year. But if a person lived at the beginning of the year and died at the end of March, that person would have lived

$$\frac{31 + 28 + 31}{365} = 0.25 \text{ person-year}$$

    The total of all person-years lived by all l_x persons is L_x
36. For a brief account, see Spiegelman, 1968, pages 127–128.
37. We use M_x to represent age-specific death rate, instead of the usual symbol d_x used in Chapter 6 just to avoid confusion.

38. In a complete life table, all intervals are equal to 1 and it is dropped so that we have, say, d_x.
39. Medical science has made it possible to ignore this stage in some cases.
40. $a = 15 - 19;\ 20 - 24;\ 25 - 29;\ 30 - 34;\ 35 - 39;\ 40 - 44;\ 45 - 49.$
41. In other texts, this rate is referred to as total legitimate fertility rate.
42. As a convention, the following seven five-year age groups are utilized: 15 to 19; 20 to 24; 25 to 29; 30 to 34; 35 to 39; 40 to 44; and 45 to 49. Hence the mid-points for the age intervals are 17.5, 22.5, 27.5, 32.5, 37.5, 42.5, 47.5).
43. To determine a class boundary from the class interval 25–29, refer to Example 5.17.
44. Stillbirths, stepchildren, and adopted children are excluded from the number of children ever born.
45. A de facto union is a marriage in practice or actuality, but not officially established. The individuals reside together and represent themselves as spouses to the outside world.
46. TFR is also a standardised rate, because it refers to a constant base of 1,000 women at each age group (equal weight). Therefore, all TFR have the same age composition and, therefore, are not affected by differences in age composition. In the study of differential fertility, it is desirable to use either TFR or age standardised GFR.
47. The net reproduction rate, though is derived from the data of a loose period, is not the same as vital-statistics rate of births or deaths "per year". It covers the whole reproductive period of the hypothetical cohort and might be regarded as a rate "per cohort" or "per generation" of these women.
48. This is not always the case in most African countries where people can form families without marriage.
49. Of course, in most part of the world especially in Africa, it is not uncommon to have married men going into another marriage.
50. When a couplennis separated, they are, in principle, not single.
51. Universal marriage is when all women in get married before age 50.
52. This is because delayed marriages tend to delay pregnancies.
53. Commutating involves the daytime population (for instance, the population at its place of work, whereas the night time population (i.e., the population at its place of residence) is the one commonly recorded as the *de jure* population and the one to which demographic analysis refers. As long as there is no migration (no change in the place of residence), the level and the structure of the night time population do not change by definition.
54. The cut-off point varies from country to country. In some countries, a 6-month period is used to identify migrants.

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